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EMPIRICAL MODE DECOMPOSITION IN DISCRETE TIME SIGNALS DENOISING

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Abstract

This study explores the data-driven properties of the empirical mode decomposition (EMD) for signal denoising. EMD is an acknowledged procedure which has been widely used for non-stationary and nonlinear signal processing. The main idea of the EMD method is to decompose the analyzed signal into components without using expansion functions. This is a signal dependent representation and provides intrinsic mode functions (IMFs) as components. These are analyzed, through their Hurst exponent and if they are found being noisy components they will be partially or integrally eliminated. This study presents an EMD decomposition-based filtering procedure applied to test signals, the results are evaluated through signal to noise ratio (SNR) and mean square error (MSE). The obtained results are compared with discrete wavelet transform based filtering results.

Key words:empirical mode decomposition, signal processing, denoising

1. Introduction

Denoising could be the most common signal processing procedure in order to obtain a cleaner signal. Through denoising it is possible to remove unwanted components, usually noises and keeping or strengthen others. The most accepted denoising procedures use transforms in time, frequency or time-frequency domains. All of these make use of expansion functions which more or less expect priori observation-based information about the signals to process [3].

An adaptive procedure as the Empirical Mode Decomposition (EMD) which is strongly dependent on the analyzing signal could be an alternative solution. The EMD was proposed as the first part of the Hilbert–Huang transform (HHT) which decomposes a signal in the time domain through an iterative sifting process [1]. The results are a finite number of amplitude and frequency modulated zero-mean oscillations called intrinsic mode functions (IMFs). In contrast to transform based decomposition, IMFs are expressed as the signal dependent semi-orthogonal basis functions. Briefly the method detects local maxima and minima in a signal, interpolates these values in upper and lower envelopes and removes their instantaneous mean value [2], [5]. This is made repetitively until the IMFs reach a stoppage criterion, usually based on the standard deviation of two consecutive IMFs. This paper presents an EMD based denoising procedure using test signals and different types and values of noises[7].

The paper is formed as follows. The second section introduces the theoretical knowledge about the decomposition algorithm. The third describes the de-noising algorithm based on IMFs thresholding and selective reconstruction of the signal and the different types of noises. The experimental results are presented in the fourth section. Finally, come the conclusions and valuable new directions for further work.

2. Empirical Mode Decomposition

The EMD decomposes a multi-component signal into its mono-component constituents. The algorithm is based on obtaining interpolated envelope (covering) curves defined by local maxima and minima of a discrete signal and iterative subtraction of the mean of these curves from the initial signal. For these is obvious to identify all local minima and maxima in the signal [1]. The upper and lower covering curves are then obtained by interpolating these extrema through (usually) cubic spline functions. Sequentially removing the instantaneous mean value of these two envelopes leads to obtain IMFs. The maxima and minima of the next level depend on the subtraction of the previous level. So, EMD can be done only sequentially. The IMFs are obtained through this procedure named sifting process. Sifting is a wide used expression in signal processing related to separating out components of a signal one at a time. It is often used related to signal decomposition since this process is very similar. The IMFs are not set analytically and are instead determined only by the analyzed sequence, meaning that the basis functions are in this case derived adaptively directly from input data [4]. An IMF resulting from the sifting process must satisfy two basic requirements. At first, the number of extrema and the number of zero-crossings must either be equal or differ at most by one and the second, at any point of an IMF the mean value of the envelope defined by the local maxima and the envelope defined by the local minima shall be zero [9]. For a given discrete signal the first prototype component is computed by extracting the mean value from the signal. In the second sifting process, the first prototype is treated as the data, and the mean extraction is repeated. The next IMF can be obtained by subtracting the previously extracted IMF from the original signal and repeating the already mentioned procedure until all IMFs are extracted. The sifting process usually stops when the residue satisfies a stoppage criterion [6].

Decomposition results in a family of frequency ordered IMF components. Each successive IMF contains lower frequency oscillations than the preceding one. The fact is that an IMF has an oscillatory behavior, having variable amplitude and frequency along the time axis. These components are synthetic; their extraction helps to better understand the structure of the signal and allows its analysis.

The EMD method, as well as the Hilbert–Huang transform, is intended for analyzing non-stationary and nonlinear data, but it can be successfully applied to linear and stationary sequences.

3. The denoising procedure

In this paper, the proposed algorithm and the computational procedures are carried out in MATLAB. The used ECG signals are taken from specific toolboxes. At first, the EMD procedure is applied to the input noisy signal $x_n$in order to obtain the set of IMFs. For every IMF the Hurst exponent is estimated, if this is above then a previewed threshold it will be taken as noise t and eliminated. A Hurst exponent value between 0 and 0.5 is indicative of anti-persistent behavior and the closer the value is to 0, the stronger is the tendency for the time series to revert to its long-term means value [14]. Denoising means a thresholding process for each IMF and a summing of them. [8].Usually the noisy signal x_n is assumed to be the superposition of a clean signal x and a noise n

$$x_n = x + n \tag{1}$$

This is the simplest approach; usually the signal can be correlated with several types of noises.

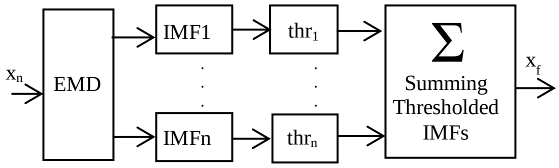

Fig. 1: The proposed procedure

In order to evaluate the denoising, different types of noise are used, gaussian white noise, red, pink, and blue noise. Every type of these has specific characteristics as white noise has equal power throughout all frequencies, while the power in pink noise decreases as the frequency increases [].Blue noise increases in volume with increasing frequencies, the spectrum of red noise is strongly weighted toward low frequencies, dropping off like $1/f^2$[12].

The filtering procedure should eliminate the noise and preserve the clear signal. In order to evaluate the filtering results added Gaussian white noise, pink, was used. The level of added noise was varied. The followed parameters are the obtained signal to noise ratio (SNR) and the mean square error (MSE). The power of signal and noise are defined as

$$\mathrm{P_S} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i=1}}^{\mathrm{N}} \mathrm{x_i^2}, \mathrm{P_n} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i=1}}^{\mathrm{N}} \mathrm{n_i^2} \tag{2}$$

The initial signal to noise ratio SNR_i (with known noise) and the obtained signal to noise ratio SNR (where x_f is the filtered signal) are

$$SNR_i = 10lg(P_S/P_n) \tag{3}$$

$$P_{Sf} = \frac{1}{N}\sum_{i=1}^{N}\left(x_{fi}\right)^2, P_{ne} = \frac{1}{N}\sum_{i=1}^{N}\left(x_{ni} - x_{fi}\right)^2 \tag{4}$$

$$SNR = 10lg\left(\frac{P_{Sf}}{P_{ne}}\right) \tag{5}$$

Where N is the length of the signal.

4. Simulation results

This study uses test signals from MATLAB toolbox, added noise are of different types (white noise with normal distribution, pink, red and blue) and values. These are representedtogether with their spectra on Fig. 2. The noise was added in different quantities and the mentioned parameters were computed in every case [10].

$$x_n = x + k \cdot n \tag{6}$$

The coefficient k takes different values in order to produce different levels of added noise. The main parameters are mean squared error (MSE), mean absolute error (MAE), signal to noise ratio (SNR). The thresholding procedure was performed starting from the fact that the noise is located in lower order IMFs, all IMFs were evaluated through their Hurst exponent. The Hurst exponent of a data set provides a measure of whether the data is a pure white noise random process or has underlying trends. If the Hurst exponent is less than 0.5 then it can be considered as a noisy component and can be eliminated from the signal.

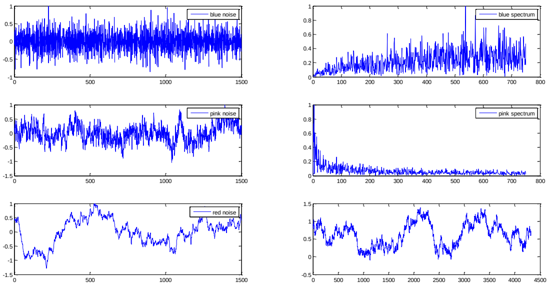

Fig. 2: The used colored noises and their spectrum

Both test signals ('testi', 'leleccum') and their noisy versions are presented on Fig. 3. The two signals have different lengths

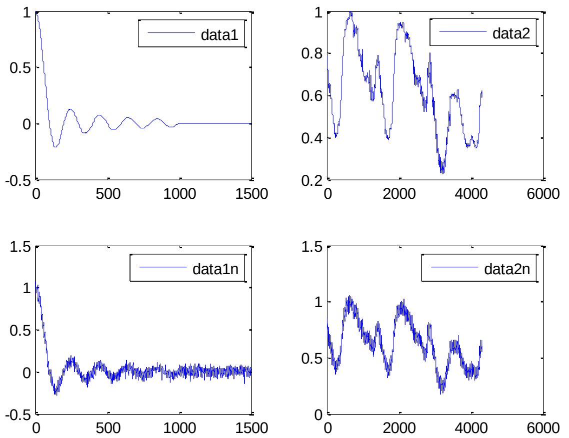

Fig. 3: The original and the noise corrupted data ('testi', 'leleccum')

The test signals and the applied noises are normed before denoising procedure in order to be compared. The EMD based denoising effect on the second signal can be seen on Fig. 4. The level of applied noise is 10% from signals value.

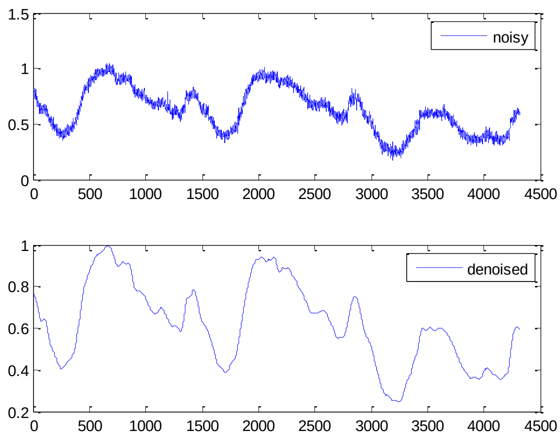

Fig. 4: The noisy and the denoised signal ('leleccum')

Different levels of Gaussian white noise were added. Parameter k indicates the percentile level of noise, which is presented also in form of initial signal to noise ratio, following the expression (7).

$$SNR_i = 10lg(P_S/P_n) = 10lg\left(\frac{1}{k}\right)^2 = -20lgk \qquad (7)$$

Table 1: Parameters for different levels of noises

k	0.1	0.12	0.14	0.16	0.18	0.2
SNRi	20	18.4163	17.0774	15.9176	14.8945	13.9794
MSE	0.00114	0.00119	0.00141	0.00209	0.00287	0.00332
MAE	0.02723	0.02753	0.02990	0.03655	0.04294	0.04582
SNR	25.6899	25.5058	24.7843	23.0738	21.6957	21.1177

The measured mean squared erroris presented on Fig. 5 for different values of k. of added gaussian white noise

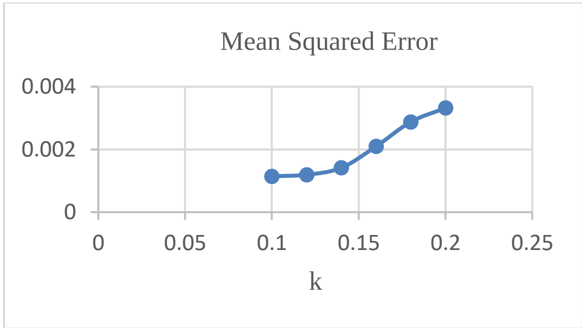

Fig. 5: MSE obtained for various gaussian white noise levels

The signal to noise ratio (SNR) obtained in case of added gaussian white noise is presented on Fig. 6

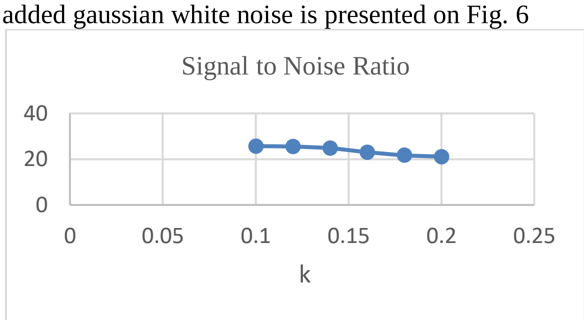

Fig. 6: SNR obtained for various gaussian white noise levels

To evaluate the denoising results, this study used Gaussian white(g), pink(p), red(r) and blue(b) noises. The SNR parameters obtained for different types of noise of the same level are presented in Table 2 and Fig.7.

Table2

k	0.1	0.12	0.14	0.16	0.18	0.2
white G	25.6899	25.5059	24.7843	23.0738	21.6958	21.1177
blue	8.01543	8.62235	7.85659	7.42176	8.81133	7.99612
pink	12.3504	10.7151	12.8325	10.9202	11.4723	10.8602
red	23.0794	28.3939	23.8352	27.0272	27.3685	19.8614

In order to evaluate the proposed method, a discrete wavelet transform (DWT) based denoising was performed, using a third level decomposition, a ‘db4’ type wavelet function and soft thresholding [8]. This means that the signal is decomposed with a basis function into time-frequency components which are selectively thresholded and resumed again, obtaining the filtered signal.

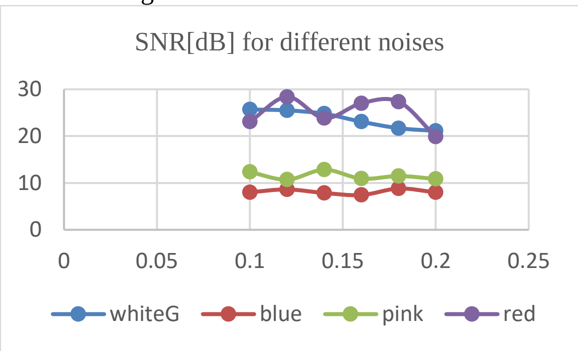

Fig. 7: SNR obtained for various noises with the same level

The obtained results are presented on Fig.8, where the obtained SNRs are presented.

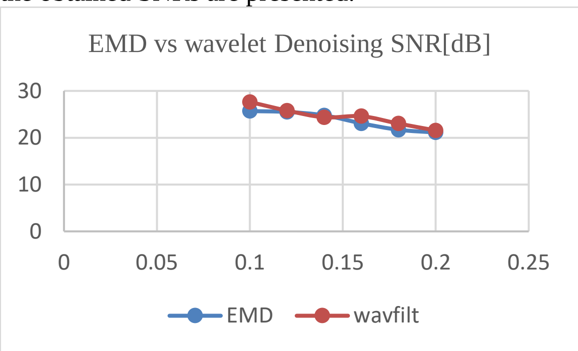

Fig.8. SNR obtained for various noises with the same level

The results show a slightly better signal to noise ratio in the case of wavelet filtering but the used computational resources are huge compared to the EMD method.

5. Conclusions

This study shows that EMD based filtering procedure can perform denoising if the noise is supposed to be an additive signal. The obtained simulation results show that EMD based denoising can offer in certain conditions good, even better results than othermore expensive procedures. A criterion for an efficient thresholding of relevant IMFs could improve the denoising results

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