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Research on the Pricing Model about Online Celebrity Economy

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Abstract

With the continuous development of China's Internet economy, the model of grasping Internet economy benefit is diversified. Since 2016, the online celebrity economy has formed a trend of Internet marketing, and the online celebrity economic industry chain further has established along with the further development of MCN institutions. The relationship among the overall industry upstream and downstream sectors become closer and closer and even appear the trend of vertical cooperation. This paper is aimed at expressing the pricing mechanism of the online celebrity industrial chain and applies game theory and mathematical analysis to propose a reasonable pricing model

Keywords: Online celebrity economy; Bilateral market; Dynamic game; Sensitivity analysis**AMS 2010 codes:** 03Bxx.

1 Introduction

1.1 Basic situation of online celebrity economic development

According to Insight Report on Chinese Internet Celebrities' Economic Development by I-research, by the end of April 2018, the number of online celebrities who attract more than 100,000 fans had increased by 51% comparing with 2017's. At the same time, the total number of Chinese online celebrity fans maintained a growing trend, reaching at 588 million people. With growth of the number of online celebrities and fans, the scale of the online celebrity market, as well as the ability of participants in industrial chain to get profit are further enhanced. In the process, advertising becomes the main way for online celebrities to get benefit. On this basis, the MCN platform is an abbreviation of multi-channel network platform. This platform is a bilateral market based on information flow. In this platform, offline manufactures (especially advertisers) can find suitable influencers for the promotion of goods and services. Economic benefits can be achieved through cooperation with content providers, and the entire platform benefits by charging appropriate commissions or dividends;

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MCN platform enterprises are expressed as network channel providers for e-commerce, advertising enterprises and live broadcast industries and communicate with network consumers to get the profit. In the process, MCN platform cooperates with the online celebrity group to provide business support and training, exchanging for registration fee and profit bonus. On the other hand, the manufactures of goods and services need to improve the power of influence through the Internet, and also depends on the MCN platform to explore suitable online celebrity endorsement. Under the push of two-side market demands, MCN platform become the backbone of commercial chain and become the link between the upstream and downstream industry trading. Therefore, the research on platform transaction pricing under online celebrity economy requires with practical guiding significance. In this paper, online celebrity refers to a group of young entrepreneurs who rely on the publicity channels of the Internet to improve their popularity, and then equips with the temperament of personalizing idols, at the time, gain benefits from network consumers and some platform enterprises. On the other hand, MCN platform enterprises are expressed as network channel providers for e-commerce, advertising enterprises and live broadcast industries and communicate with network consumers to get the profit.

1.2 Literature review

This paper mainly studies the domestic and foreign literature on platform pricing. Due to the characteristics of two-side market, most scholars focus on the pricing behavior of the theory of two-side market.

In terms of foreign literature, Caillaudet [1] proposed a price strategy aiming at solving the excessive competition of the platform in the industrial chain. Rochet etc. [2] discussed the optimal pricing problem of platform under monopoly platform and competitive platform, and draw the conclusion that the price distribution is proportional to the elasticity of bilateral price demand. Armstrong [3] studied the price decision problem about three kinds of platform competition: monopoly platform, bilaterally owned competition platform and unilateral owned competition platform, which concluded that the equilibrium price is related to the cross network effect, charging method and market structure.

Economides etc. [4] pointed out that the net neutrality of the Internet is analyzed with the price tool of two-sided market theory, and it is found that the net neutrality platform increases the total surplus in a certain range of parameter about cross-network externality compared with private platform, while decreases the total surplus in another parameter range.

In terms of domestic literature in China, Ji [5] studied the registration fee, transaction fee and two-step charge could impact the profit and welfare. Wang etc. [6] analyzed the pricing and profit distribution of the platform and content provider in the new video business operation. Xing etc. [7] discussed that the pricing problem of logistics information platform in the two-side market between the vehicle owner and the good owner, and puts forward a relatively perfect pricing model.

2 Two-side market of the online celebrity economy

As there are obvious two-sided markets characteristics for platform economy, the main features of the two-side market naturally exist in the network system of the multi-channel network platform (MCN) recognized as the trading platform [8–10]. In order to simplify the analysis, in the relatively closed network architecture, we need the following structure figure of the two-side market under the online celebrity economy.

Through analysis of the relationship between the three parties, offline manufacturers of goods and services need network window to expand business and improving reputation. Therefore, looking for appropriate online celebrity from MCN platform is needed for offline manufactures. At the same time, online celebrities recognize MCN as a platform and cooperation object, which could cultivate and support the group of online celebrity and help this group match appropriate offline manufactures of goods and services and gain benefits from this trade. In order to simplify the analysis, the possibility of collusion about the offline manufactures and online celebrities in private is too low. On the one hand, this collusion is limited by the high transaction cost. On the other hand,

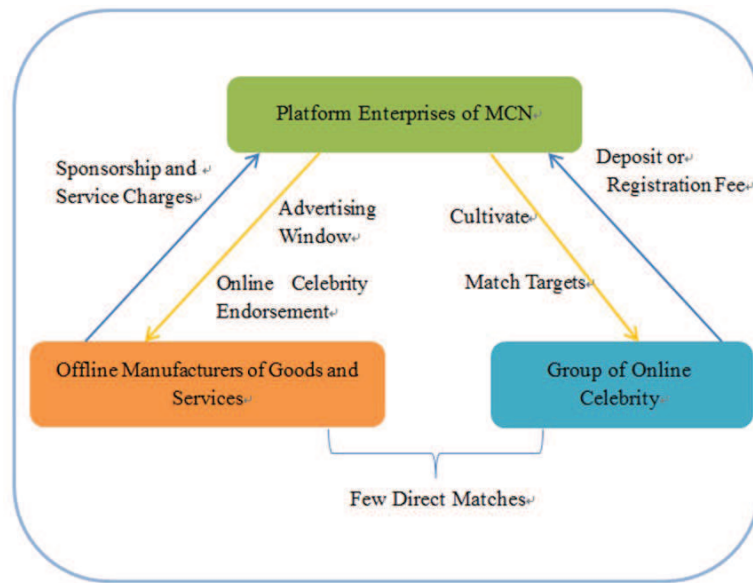


Fig. 1 The structure of two-side market in online celebrity economy

the MCN platform could require both sides meet exclusive contract and improve the penalty fees for them.

In this process, the MCN platform, as an information intermediary institution, could get benefits because of the two-side market advantages. For offline manufactures of goods and services, the MCN platform get of-line sponsorship and service cost by providing online advertising window and matching online celebrity endorsements. On the other hand, the MCN platform could get registration fee or deposit (even bonus from the cooperation between potential costumer and online celebrities) from the group of online celebrity.

3 Pricing of the monopoly platform

The ecosystem of Multi-channel Networks (MCN), aimed at providing network publicity window for of-line manufactures of goods and services and providing a channel to match online celebrity endorsements with potential customers, is a comprehensive trading system with multiple stakeholders, including MCN platform enterprises, offline manufactures of goods and services, the group of online celebrity, advertising agencies, third-party platforms, training institutes, and Internet users. But the core of this ecosystem is the MCN organization, the manufactures of goods and services and the group of online celebrity. As mentioned in the previous section, MCN helps these offline manufactures to find appropriate online celebrities to improve their reputation in the space of the Internet and encourage the group of the online celebrity to match these potential customers to achieve values. Furthermore, advertising agencies, third-party platforms and training institutions will be attracted to serve for bilateral users (online manufactures of goods and services and the group of online celebrity) to form an ecosystem of platform business. The MCN platform charges online manufactures of goods and services and the group of online celebrity, and they decide whether to join the platform according to the charges and the benefit of cross network effect on the platform (cross-network effect refers to the effect of the number of one- side uses on the utility of another side uses in the platform).

3.1 Platform stakeholders

(1) Offline manufactures of goods and services

Offline manufactures of goods and services contains two different kinds of types. One is the type of direct manufacturers and another type is the business platform of various manufacturers. Although business platform

is superior to the direct manufacturer on scale effect, there is a common need for them to cooperate with MCN enterprises in order to enhance the publicity and influence of the network window. Both parties need online celebrities to play their role of network spokespersons for improving the popularity of products and services, and reducing advertising costs by network externalities, which generates the demand to match the appropriate online celebrities. In the analysis model, we classify these two types of manufactures into the same group, and the corresponding utility function is defined as follows:

$$u_c = m\alpha_1\lambda n_s - p_1 - tx_i \quad (1)$$

Where m refers to the number of times that offline manufactures of goods and services look for proper online celebrity endorsements on the MCN platform during a trading cycle; α_1 is expressed as the marginal utility of finding the appropriate online celebrity candidates or can also be interpreted as the cross network effect of offline manufactures of goods and services; λ refers to the probability of finding the appropriate online celebrity endorsement on the platform; t means the opportunity cost of offline manufactures of goods and services looking for the appropriate online promotion window by themselves; n_s is expressed as the number of the active online celebrities on the Internet and $x_i \in U(0, 1)$, which is defined as the difficulty of the offline manufactures of goods and services to find online advertising window by themselves.

(2) The group of online celebrity

The group of online celebrity refers to these workers who equip with their own idolization images and influence power in the network society can convert their recommendations by their own characters on the Internet to economic benefits. MCN platform agencies must first provide enough resources to develop or find sufficient groups of online celebrity to form the ability to attract clients in the market, when the platform attracts enough offline manufactures of goods and service, there is a positive feedback to the group of online celebrity, enabling online celebrities to meet the requirements of offline business, which creates a positive circulation of network cross effect. The utility function of the group of online celebrity is expressed as follows:

$$u_s = v + \alpha_2\lambda mn_c - p_2 - fy_j \quad (2)$$

Where v refers to the value that the MCN platform provides for the group of online celebrity by themselves; α_2 is expressed as the marginal utility of finding the appropriate offline manufactures of goods and services; λ refers to the probability of finding the appropriate offline manufactures of goods and services on the platform; p_2 refers to the registration fees or deposits that the platform charges the group of online celebrity; $f y_j$ means the opportunity cost of the group of online celebrity looking for the appropriate offline manufactures of goods and services by themselves and $y_j \in U(0, 1)$, which is defined as the difficulty of the group of online celebrity to find potential customers by themselves.

(3) MCN platform

MCN platform relies on its port access to network society and the advantage of network users' recognition. So offline manufactures of goods and services and online celebrities can be attracted to the largest extent. At the same time, cross network effect is formed in this process to provide information services for bilateral users and improve the matching efficiency of sales and network publicity, though a certain fee is charged by the MCN platform. In order to simplify our analysis, the profit function is defined as follows:

$$\pi_p = p_1 n_c + (p_2 + \theta) n_s \quad (3)$$

Where p_1 refers to the charges to offline manufactures of goods and services; p_2 is expressed as the charges to the group of the online celebrity; θ refers to the bonus taken by the MCN platform from the benefits due to the success that the group of online celebrity matches with offline manufactures of goods and services. In order to simplify analysis, the marginal and fixed costs are assumed 0.

3.2 Bilateral market demand of the platform

It is assumed that both offline manufactures of goods and services and online celebrities are rational. Only when their respective participation constraints are satisfied, they choose to participate in the platform transaction, that is, the utility function of both sides is greater than 0. For offline manufactures of goods and services, this critical value is x_i^* where they decided whether join the platform. On the other hand, for the group of online celebrity, this critical value is y_j^* . Assuming that the expected number of active offline manufactures of goods and services and active online celebrities is equal to the actual number of these two groups. The number of offline manufactures of goods and services and the group of online celebrity individually is:

$$\begin{cases} n_c(\text{probability}) = x_i^* = \frac{\alpha_1 \lambda m n_s - p_1}{f} \\ n_s(\text{probability}) = y_j^* = \frac{v + \alpha_2 \lambda m n_c - p_2}{f} \end{cases} \quad (4)$$

Because of (4), following outcome could be gotten:

$$\begin{cases} n_c = \frac{\lambda m \alpha_1 (v - p_2) - f p_1}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} \\ n_s = \frac{t(v - p_2) - \lambda m \alpha_2 p_1}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} \end{cases} \quad (5)$$

3.3 Optimal pricing of the MCN platform

The game among MCN platform and offline manufactures of goods and services and online celebrities is a two-stage dynamic game of complete information. The order of the game is that in the 1st phase, MCN platform take price to offline manufactures of goods and services and the group of online celebrity in bilateral markets at the same time; In the 2nd phase, both sides decide whether participate the platform. According to the reverse solution method, the number of offline manufactures of goods and services and the group of online celebrity participating into the platform in the 2nd phase should be calculated firstly, and then the optimal pricing of the platform is solved according to the number of producers and online celebrity groups. The outcome could be calculated by (4) and (5).

$$\pi_p(p_1, p_2) = p_1 n_c + (p_2 + \theta) n_s = \frac{p_1 [\lambda m \alpha_1 (v - p_2) - f p_1] + (p_2 + \theta) [t(v - p_2) - \lambda m \alpha_2 p_1]}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} \quad (6)$$

Hypothesis:

(1) The opportunity cost differences between producers and web celebrity groups are large enough, $f t - \lambda^2 m^2 \alpha_1 \alpha_2 > 0$, and the platform profit function is the joint concave function about p_1, p_2 .

(2) All offline manufactures of goods and services and online celebrities cannot be fully attracted by the platform.

(3) $\alpha_1, \alpha_2, \lambda, m, f, t, v, \theta > 0$.

For the equation (6), the outcome of the first partial derivative about p_1, p_2 equaling to 0 is:

$$\begin{cases} \frac{\partial \pi_p}{\partial p_1} = \frac{-2f p_1 - \lambda m (\alpha_1 + \alpha_2) p_2 + \lambda m \alpha_1 v - \lambda m \alpha_2 \theta}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} = 0 \\ \frac{\partial \pi_p}{\partial p_2} = \frac{-2t p_2 - \lambda m (\alpha_1 + \alpha_2) p_1 + t v - t \theta}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} = 0 \end{cases} \quad (7)$$

According to the hypothesis, all the second partial derivatives about p_1, p_2 are less than 0, which means that Hessian matrix is negative definitely $|\Delta| > 0$.

$$\Delta = \begin{pmatrix} \frac{-2f}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} & \frac{-\lambda m (\alpha_1 + \alpha_2)}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} \\ \frac{-\lambda m (\alpha_1 + \alpha_2)}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} & \frac{-2t}{f t - \lambda^2 m^2 \alpha_1 \alpha_2} \end{pmatrix} \quad (8)$$

Solving the equation system (7), the following outcome could be obtained:

$$\begin{cases} p_1^* = \frac{\lambda m t (\alpha_1 - \alpha_2) (v + \theta)}{4 f t - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2} \\ p_2^* = \frac{2 f t (v - \theta) - \lambda^2 m^2 (\alpha_1 + \alpha_2) (\alpha_1 v - \alpha_2 \theta)}{4 f t - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2} \end{cases} \quad (9)$$

According to the equation (5), (6) and (9), the outcome could be solved as:

$$\begin{cases} n_c^* = \frac{\lambda m t (\alpha_1 + \alpha_2) (v + \theta)}{4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2} \\ n_s^* = \frac{2t(v + \theta)}{4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2} \end{cases} \quad (10)$$

$$\pi_p^*(p_1, p_2) = p_1 n_c + (p_2 + \theta) n_s = \frac{t(v + \theta)^2}{4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2} \quad (11)$$

Proposition 1:

(1) There is a positive correlation between the price that should pay for the MCN platform and their cross-network effects. When $\alpha_1 > \alpha_2$, $p_1^* > 0$. When $\alpha_1 \leq \alpha_2$, $p_1^* \leq 0$.

(2) There is a negative correlation between the price that the group of online celebrity should pay for the MCN platform and their cross-network effects. When $\theta \geq \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2)}$, $p_2^* \leq 0$. Otherwise, $p_2^* > 0$.

(3) In the information platform for online celebrities, the charging prices of offline manufactures of goods and services and the group of online celebrity are satisfied as follows:

When $\theta \geq \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2) - \lambda m t (\alpha_1 - \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2) + \lambda m t (\alpha_1 - \alpha_2)}$, $p_1^* \geq p_2^*$. Otherwise, $p_1^* < p_2^*$.

3.4 Sensitivity analysis

According to the equation (8), cross network effects, value-added service revenues, and the number of times about offline manufactures of goods and services participating in transactions could affect equilibrium prices. Proposition 2 could be gotten by sensitivity analysis for $\alpha_1, \alpha_2, \theta, m$.

Proposition 2:

When $\alpha_1 \geq \alpha_2$, p_1^* increases as α_1, θ, m increases. p_2^* decreases as α_1, θ, m increases. When $\alpha_1 \leq \alpha_2$, p_1^* decreases as α_2, θ, m increases. p_2^* increases as α_2, θ, m increases.

Proof:

$$\frac{\partial p_1^*}{\partial \alpha_1} = \frac{\lambda m t (v + \theta) [4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2 + 2\lambda^2 m^2 t (\alpha_1^2 - \alpha_2^2)]}{[4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]^2} \quad (12)$$

$$\frac{\partial p_1^*}{\partial \alpha_2} = \frac{-\lambda m t (v + \theta) [4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2 - 2\lambda^2 m^2 t (\alpha_1^2 - \alpha_2^2)]}{[4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]^2} \quad (13)$$

$$\frac{\partial p_1^*}{\partial \theta} = \frac{\lambda m t (\alpha_1 - \alpha_2)}{4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2} \quad (14)$$

$$\frac{\partial p_1^*}{\partial m} = \frac{\lambda t (\alpha_1 - \alpha_2) (v + \theta) [4ft + \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]}{[4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]^2} \quad (15)$$

$$\frac{\partial p_2^*}{\partial \alpha_1} = \frac{-\alpha_2 \lambda^2 m^2 (v + \theta) [4ft \frac{\alpha_1}{\alpha_2} - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]}{[4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]^2} \quad (16)$$

$$\frac{\partial p_2^*}{\partial \alpha_2} = \frac{\alpha^2 \lambda^2 m^2 (v + \theta) [4ft - \lambda^2 m^2 \frac{\alpha_1}{\alpha_2} (\alpha_1 + \alpha_2)^2]}{[4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]^2} \quad (17)$$

$$\frac{\partial p_2^*}{\partial \theta} = \frac{-2ft + \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2)}{4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2} < 0 \quad (18)$$

$$\frac{\partial p_2^*}{\partial m} = \frac{-4ft\lambda^2 m(v + \theta)(\alpha_1^2 - \alpha_2^2)}{[4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]^2} \quad (19)$$

When $\alpha_1 \geq \alpha_2$, $\frac{\partial p_1^*}{\partial \alpha_1} \geq 0$, $\frac{\partial p_1^*}{\partial \theta} \geq 0$, $\frac{\partial p_1^*}{\partial m} \geq 0$, $\frac{\partial p_2^*}{\partial \alpha_1} \leq 0$, $\frac{\partial p_2^*}{\partial m} \leq 0$.

When $\alpha_1 \leq \alpha_2$, $\frac{\partial p_1^*}{\partial \alpha_2} \leq 0$, $\frac{\partial p_1^*}{\partial \theta} \leq 0$, $\frac{\partial p_1^*}{\partial m} \leq 0$, $\frac{\partial p_2^*}{\partial \alpha_2} \geq 0$, $\frac{\partial p_2^*}{\partial m} \geq 0$.

Whether $\alpha_1 \geq \alpha_2$ or $\alpha_1 \leq \alpha_2$, $\frac{\partial p_2^*}{\partial \theta} < 0$.

3.5 Extension: only charge to one side

It is assumed that, due to internal or external reasons of the platform, the platform can only charge to one side of bilateral users and be free for another side. The purpose of the platform is to attract the offline manufactures of goods and services or online celebrities, especially in the initial stage of information platform.

(1) only charge to manufactures of goods and services

Due to the Proposition 1, when $\theta \geq \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2) - \lambda mt(\alpha_1 - \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2) + \lambda mt(\alpha_1 - \alpha_2)}$, $p_1^* \geq p_2^*$

When $\alpha_1 > \alpha_2$, $p_1^* > 0$. Because of $p_2 = 0$, the profit function is expressed as following:

$$\pi_{pc}(p_1) = p_1 n_c + \theta n_s = \frac{p_1 (\lambda m \alpha_1 v - f p_1) + \theta (tv - \lambda m \alpha_2 p_1)}{ft - \lambda^2 m^2 \alpha_1 \alpha_2} \quad (20)$$

The outcome of its first derivative about p_1 is:

$$\frac{\partial \pi_{pc}(p_1)}{\partial p_1} = \frac{\lambda m \alpha_1 v - 2f p_1 - \lambda m \alpha_2 \theta}{ft - \lambda^2 m^2 \alpha_1 \alpha_2} = 0 \quad (21)$$

$$p_1^{pc} = \frac{\lambda m (\alpha_1 v - \alpha_2 \theta)}{2f} \quad (22)$$

$$\pi_{pc}(p_1) = \frac{4ftv\theta + \lambda^2 m^2 \alpha_2 (\alpha_1 v - \alpha_2 \theta)^2}{4f(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \quad (23)$$

Proposition 3: When $\theta \geq \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2) - \lambda mt(\alpha_1 - \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2) + \lambda mt(\alpha_1 - \alpha_2)}$, $p_2^* \leq 0$. When only charging fees to the offline manufactures of goods and services separately, the equilibrium price, number of producers and quantity of the online celebrities are less than or equal to charging bilateral fees'. When $\theta < \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2) - \lambda mt(\alpha_1 - \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2) + \lambda mt(\alpha_1 - \alpha_2)}$, $p_2^* > 0$. When only charging fees to the offline manufactures of goods and services separately, the equilibrium price, number of producers and quantity of the online celebrities are larger than charging bilateral fees'.

Proof:

$$\Delta \pi = \pi_{pc}(p_1) - \pi_p^*(p_1, p_2) = \frac{-(p_2^*)^2 [4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]}{4(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \leq 0 \quad (24)$$

$$\Delta p_1 = p_1^{pc} - p_1^* = \frac{\lambda m (\alpha_1 + \alpha_2) p_2^*}{2f} \quad (25)$$

$$\Delta n_c = n_c^{pc} - n_c^* = \frac{\lambda m (\alpha_1 - \alpha_2) p_2^*}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \quad (26)$$

$$\Delta n_s = n_s^{pc} - n_s^* = \frac{[2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2)] p_2^*}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \quad (27)$$

Therefore, when $\theta \geq \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2) - \lambda mt(\alpha_1 - \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2) + \lambda mt(\alpha_1 - \alpha_2)}$, $p_2^* \leq 0, \Delta p_1 \leq 0, \Delta n_c \leq 0, \Delta n_s \leq 0$.

When $\theta < \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2) - \lambda mt(\alpha_1 - \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2) + \lambda mt(\alpha_1 - \alpha_2)}$, $p_2^* > 0, \Delta p_1 > 0, \Delta n_c > 0, \Delta n_s > 0$.

(2) only charge to the group of online celebrity

Because of the Proposition 1, when $\theta \leq \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2) - \lambda mt(\alpha_1 - \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2) + \lambda mt(\alpha_1 - \alpha_2)}$, $p_1^* \leq p_2^*$. When $\theta \leq \frac{v[2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2)]}{2ft - \lambda^2 m^2 \alpha_2 (\alpha_1 + \alpha_2)}$, $p_2^* \geq 0$. Due to $p_1 = 1$, the profit function is expresses as following:

$$\pi_{ps}(p_2) = (p_2 + \theta) n_s = \frac{(p_2 + \theta) t(v - p_2)}{ft - \lambda^2 m^2 \alpha_1 \alpha_2} \quad (28)$$

$$\frac{\partial \pi_{ps}(p_2)}{\partial p_2} = \frac{t(v - p_2 - p_1 - \theta)}{ft - \lambda^2 m^2 \alpha_1 \alpha_2} = 0 \quad (29)$$

$$p_2^{ps} = \frac{v - \theta}{2} \quad (30)$$

$$\begin{cases} n_c^{ps} = \frac{\lambda m \alpha_1 (v + \theta)}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \\ n_s^{ps} = \frac{t(v + \theta)}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \end{cases} \quad (31)$$

$$\pi_{ps}(p_2) = \frac{t(v + \theta)^2}{4(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \quad (32)$$

Proposition 4: When $\alpha_1 > \alpha_2$, due to only charging fees to the group of online celebrity separately, the equilibrium price is less than charging bilateral fees' and the quantity of offline manufactures of goods and services is larger than charging bilateral fees'. When $\alpha_1 \leq \alpha_2$, due to only charging fees to the group of online celebrity separately, the equilibrium price is larger than or equal to charging bilateral fees' and the quantity of offline manufactures of goods and services is less than charging bilateral fees'.

Proof:

$$\Delta \pi = \pi_{ps}(p_1) - \pi_p^*(p_1, p_2) = \frac{-\lambda^2 m^2 t(v + \theta)^2 (\alpha_1 - \alpha_2)^2}{4(ft - \lambda^2 m^2 \alpha_1 \alpha_2) [4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]} < 0 \quad (33)$$

$$\Delta p_2 = p_2^{ps} - p_2^* = \frac{\lambda^2 m^2 (v + \theta) (\alpha_1^2 - \alpha_2^2)}{2 [4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]} \quad (34)$$

$$\Delta n_c = n_c^{ps} - n_c^* = \frac{\lambda m (\alpha_1 - \alpha_2) (v + \theta) [2ft - \lambda^2 m^2 \alpha_1 (\alpha_1 + \alpha_2)]}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2) [4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]} \quad (35)$$

$$\Delta n_s = n_s^{ps} - n_s^* = \frac{-\lambda^2 m^2 t(v - \theta) (\alpha_1 - \alpha_2)^2}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2) [4ft - \lambda^2 m^2 (\alpha_1 + \alpha_2)^2]} \quad (36)$$

Therefore, when $\alpha_1 > \alpha_2$, $\Delta p_2 > 0, \Delta n_c > 0, \Delta n_s < 0$. When $\alpha_1 \leq \alpha_2$, $\Delta p_1 \leq 0, \Delta n_c \leq 0, \Delta n_s \leq 0$.

4 Competition bidding of duopoly platform

Assuming that the offline manufactures of goods and services and the group of online celebrity can only join one platform, and the platform charges for both sides. Under the condition of the charges of platform, if the group of online celebrity could find the appropriate offline manufactures of goods and services, it is not necessary to pay for two platform fees at the same time. On the other hand, offline manufactures of goods and services demand to join two platforms for finding appropriate online celebrities but this thought is restricted by platform exclusivity agreements.

4.1 Quantity of bilateral users on the platform

Assuming that two platforms k_1, k_2 are located in $x = 0$ and $x = 1$ respectively, utility function for manufacturers of offline goods and services in the boundary line is:

$$u_c^{k_1} = m\alpha_1 \lambda n_s^{k_1} - p_1^{k_1} - tx_i = u_c^{k_2} = m\alpha_1 \lambda n_s^{k_2} - p_1^{k_2} - t(1 - x_i) \quad (37)$$

Assuming the whole market is covered, the outcome is expressed as following:

$$\begin{cases} n_c^{k_1} = \frac{1}{2} + \frac{\lambda m \alpha_1 (n_s^{k_1} - n_s^{k_2}) - (p_1^{k_1} - p_1^{k_2})}{2t} \\ n_c^{k_2} = 1 - n_c^{k_1} \end{cases} \quad (38)$$

Similarly, if two platforms are located in $y = 0$ and $y = 1$, utility function for the group of online celebrity is:

$$u_s^{k_1} = v + \alpha_2 \lambda m n_c^{k_1} - p_2^{k_1} - fy_j = u_s^{k_2} = v + \alpha_2 \lambda m n_c^{k_2} - p_2^{k_2} - f(1 - y_j) \quad (39)$$

Assuming the whole market is covered, the outcome is expressed as following:

$$\begin{cases} n_s^{k_1} = \frac{1}{2} + \frac{\lambda m \alpha_2 (n_c^{k_1} - n_c^{k_2}) - (p_2^{k_1} - p_2^{k_2})}{2f} \\ n_s^{k_2} = 1 - n_s^{k_1} \end{cases} \quad (40)$$

Solving the equation (38) and (40), the outcome is expressed as following:

$$\begin{cases} n_c^{k_1} = \frac{1}{2} + \frac{\lambda m \alpha_1 (p_2^{k_2} - p_2^{k_1}) + f(p_1^{k_2} - p_1^{k_1})}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \\ n_s^{k_1} = \frac{1}{2} + \frac{\lambda m \alpha_2 (p_1^{k_2} - p_1^{k_1}) + t(p_2^{k_2} - p_2^{k_1})}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \\ n_c^{k_2} = \frac{1}{2} - \frac{\lambda m \alpha_1 (p_2^{k_2} - p_2^{k_1}) + f(p_1^{k_2} - p_1^{k_1})}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \\ n_s^{k_2} = \frac{1}{2} - \frac{\lambda m \alpha_2 (p_1^{k_2} - p_1^{k_1}) + t(p_2^{k_2} - p_2^{k_1})}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \end{cases} \quad (41)$$

4.2 The optimal pricing

Solving the demand function of equation (41) and profit function of equation (3), the profit functions of these two platforms could be gotten.

$$\begin{cases} \pi_p^{k_1} = p_1^{k_1} \left[\frac{1}{2} + \frac{\lambda m \alpha_1 (p_2^{k_2} - p_2^{k_1}) + f(p_1^{k_2} - p_1^{k_1})}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \right] + (p_2^{k_1} + \theta) \left[\frac{1}{2} + \frac{\lambda m \alpha_2 (p_1^{k_2} - p_1^{k_1}) + t(p_2^{k_2} - p_2^{k_1})}{2(\frac{1}{2}(k_1 - \lambda^2 \alpha_1 \alpha_2))} \right] \\ \pi_p^{k_2} = p_1^{k_2} = p_1^{k_2} \left[\frac{1}{2} + \frac{\lambda m \alpha_1 (p_2^{k_2} - p_2^{k_1}) + f(p_1^{k_2} - p_1^{k_1})}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \right] + (p_2^{k_2} + \theta) \left[\frac{1}{2} + \frac{\lambda m \alpha_2 (p_1^{k_2} - p_1^{k_1}) + t(p_2^{k_2} - p_2^{k_1})}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \right] \end{cases} \quad (42)$$

$$\Delta(\text{hessian}) = \begin{pmatrix} \frac{-f}{ft - \lambda^2 m^2 \alpha_1 \alpha_2} & \frac{-\lambda m(\alpha_1 + \alpha_2)}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} \\ \frac{-\lambda m(\alpha_1 + \alpha_2)}{2(ft - \lambda^2 m^2 \alpha_1 \alpha_2)} & \frac{-t}{ft - \lambda^2 m^2 \alpha_1 \alpha_2} \end{pmatrix} \quad (43)$$

Because Δ is negative definitely, the maximum of these profit functions could be solved by setting the first derivation equal to 0.

$$\begin{cases} p_1^{k_1} = p_1^{k_2} = t - \lambda m \alpha_2 \\ p_2^{k_1} = p_2^{k_2} = f - \theta - \lambda m \alpha_1 \end{cases} \quad (44)$$

$$\pi_p^{k_1} = \pi_p^{k_2} = \frac{f + t - \lambda m(\alpha_1 + \alpha_2)}{2} \quad (45)$$

The equation (44) shows that when there are two competing platforms, the optimal pricing of the two MCN platforms for the manufactures of goods and services and the group of online celebrity are equal, and the price increases as the enlarge of degree of differentiation between different offline manufactures of goods and services and between different online celebrities. But the price goes down with the development of platform technology and with improvement of the quantity about offline manufactures of goods and services.

On the other hand, the optimal price decreases as the increase of the cross network effect of both sides. Meanwhile, the price of online celebrities decreases as the increase of value-added service benefits to the platform. It can be seen from (45) that the equilibrium profit of the two platforms is equal, which increases as the rise of the differentiation degree of bilateral users and decreases with the rise of network effect.

5 Conclusion

With the increasingly development of the Internet technology, the MCN information platform based on the Internet plays a more and more important role in the development for offline commodities and services, especially in widening the publicity window and attracting online consumers. There are obvious two-sided market characters for this type of MCN platform. Through the analysis of the pricing mechanism in the profit model of the platform and solution of the pricing decision model about the MCN platform, there are some following conclusions.

First of all, the optimal price decision method based on the cross network externalities and value-added service fees of MCN platform is proposed. At the same time, the conditions of collecting fees and free charges for offline manufactures of goods and services and the group of online celebrity are respectively pointed out by sensitivity analysis. Under the market structure of the monopoly platform and when the strength of cross network effect about offline manufactures of goods and services is larger than online celebrities', the manufacturer should be charged. Otherwise, offline manufactures of goods and services should be free or subsidized. When online celebrity group's value to the platform is greater than a certain critical value, the group of online celebrity should be free or subsidized; otherwise, online celebrity group will be charged. In value-added service platform mainly for online celebrity group (offline manufactures of goods and services) and when the group of online celebrity (offline manufactures of goods and services) to the platform creates more value than a certain critical value, the price of these manufactures (the group of online celebrity) should be higher than the group of online celebrity (offline manufactures of goods and services). Otherwise, the price for offline manufactures of goods and services (the group of online celebrity) should be less than the group of online celebrity (offline manufactures of goods and services).

Secondly, comparing with unilateral charge and bilateral charge, the rule that the price of unilateral charge is higher than bilateral charge is found, but the platform profit of bilateral charge is higher than unilateral charge. In the actual operation of MCN platform, if the market concentration degree is relatively large (the situation of forming a near-monopoly) and the number of bilateral users is basically equaling, then the reasonable allocation of fees to bilateral users will obtain greater benefits than collecting fees only from the unilateral user.

Finally, if the monopoly platform is extended to the competition platform, the method of symmetrical equilibrium optimal price is proposed. Under the market structure of monopolistic competition between the two platforms, the balanced profits of the two platforms and the balanced pricing of bilateral users are equal respectively. These balanced profits and prices increase with the rise of the degree of differentiation between offline manufactures and the group of online celebrity; these balanced profits and balanced prices decrease as the development of platform technology and the increase of the quantity of offline manufactures; these profits and prices reduce with the improvement of cross network effects, at the same time, the price of online celebrity group goes down as the rise of revenue that the group of online celebrity creates for the MCN platform.

Regarding for the future prospects of the model about the Internet celebrity economy, researchers can further study the vertical integration model and vertical non-integration model of the entire industry chain from the perspective of game theory respectively.

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