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Random inverse packet information and its acquisition

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Abstract

Packet sets and inverse packet sets are two kinds of novel mathematical tools to analyze dynamic information systems. With advances in inverse packet sets, random inverse packet information is proposed by introducing random characteristics into inverse packet sets. Hence, random inverse packet information has dynamic and random characteristics, and is an extended form of inverse packet sets. Furthermore, random feature, dynamic feature, and identification relation about the random inverse packet information are discussed. Finally, based on the above theory, an instance is used to illustrate the applications of intelligent acquisition of investment information.

Keywords: inverse packet set, random inverse packet information, random and dynamic characteristics**AMS 2010 codes:** F18996

1 Introduction

With regard to finite common set theory with static feature, research on some dynamic systems often faced problems because change always exists. It became necessary to construct a new kind of set model with dynamic characteristics. Hence, Refs. [1–4] proposed two types of dynamic set models—packet sets and inverse packet sets (IPSs), by replacing “static” with “dynamic” to improve the finite common set. These dynamic set models provide a better theory foundation for dealing with dynamic applied systems. Later, the mathematical characteristics of the new sets such as quantitative characteristics, algebraic characteristics, geometrical characteristics, genetic characteristics, random characteristics, and theory applications are discussed by more and more scholars [4–22]. Especially, literatures [5–17] developed the latter model by taking information instead of sets to obtain the inverse packet information (IPI) model and provide some applications for information fusion–separation, hidden information discovery, intelligent data digging, and big decomposition–fusion acquisition. However current research on random inverse packet information (RIPI) is rare. Hence, we consider the probabilities of information element migration in IPI and present some concepts about the RIPI and their structures.

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Furthermore, the random feature, dynamic characteristics, and identification relations on RIPI are discussed and applied to intelligent acquisition–separation of investment information.

Convention: $(x) = \{x_1, x_2, \dots, x_s\} \subset U$ is a nonempty finite ordinary information and $\alpha \subset V$ is its nonempty attribute set; F, \bar{F} are information transition function families in which $f \in F, \bar{f} \in \bar{F}$ are transition functions, whose detailed characteristics and occurrence probabilities can be found in Hao et al. [23]. The occurrence probabilities of two events $\{x_i | w_i \bar{\in} (x), f(w_i) = x_i \in (x)\}, \{x | x \in (x), \bar{f}(x) = w \bar{\in} (x)\}$ are simply written as $p_F(f)$ and $p_{\bar{F}}(\bar{f})$ in order.

2 RIPI and its construction

The theory model of IPSs [3, 4] with the inner IPS \bar{X}^F and exterior IPS $\bar{X}^{\bar{F}}$ combined, has the following dynamic characteristics: given a finite common element set $X = \{x_1, x_2, \dots, x_r\}$ with $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_r\}$. I. If some added attributes are transferred by f to α and to get α^F such that $\alpha \subseteq \alpha^F$, then some extra elements are accordingly removed to X to generate a new element set called inner IPS \bar{X}^F , $X \subseteq \bar{X}^F$. II. If some attributes are transferred by \bar{f} from α to generate $\alpha^{\bar{F}}$ such that $\alpha^{\bar{F}} \subseteq \alpha$, then some elements in X are accordingly deleted to generate a new element set called exterior IPS $\bar{X}^{\bar{F}}$, $\bar{X}^{\bar{F}} \subseteq X$. III. If it happens in the same time that some extra attributes are moved into α and some other attributes in α are migrated out, that is, α becomes α^F , and meanwhile α does $\alpha^{\bar{F}}$, $\alpha^{\bar{F}} \subseteq \alpha \subseteq \alpha^F$, then X becomes an IPS $(\bar{X}^F, \bar{X}^{\bar{F}})$, which fulfills $\bar{X}^{\bar{F}} \subseteq X \subseteq \bar{X}^F$ and has dynamic characteristics. All of the IPSs generated by set X constitute a set family called the IPS family $\{(X_i^F, X_j^{\bar{F}}) | i \in I, j \in J\}$ [3]. Especially, if the above process occurs continuously, X would dynamically generate a linked IPS $(\bar{X}_1^F, \bar{X}_1^{\bar{F}}), (\bar{X}_2^F, \bar{X}_2^{\bar{F}}), \dots, (\bar{X}_s^F, \bar{X}_s^{\bar{F}})$, which has the relation $\bar{X}_i^F \subseteq \bar{X}_{i+1}^F$ and $\bar{X}_{i+1}^{\bar{F}} \subseteq \bar{X}_i^{\bar{F}}, i = 1, 2, \dots, s$. Let us treat the sets $\bar{X}^{\bar{F}}, X, \bar{X}^F$ as information indicated orderly by $(\bar{x})^{\bar{F}}, (x)$, and $(\bar{x})^F$. Then we obtain IPI $((\bar{x})^F, (\bar{x})^{\bar{F}})$ with all the characteristics of IPS [18–22].

For inner IPI $(\bar{x})^F$, the dynamic process is shown by adding information elements under the condition that some attributes are migrated into α , as $\exists w_i \notin (x), f(w_i) = x_i \in (x)$, where $(\bar{x})^F$ indicates $(x) \cup \{x_i | w_i \notin (x), f(w_i) = x_i \in (x)\}$. For exterior IPI $(\bar{x})^{\bar{F}}$, the dynamic process is done by some elements in (x) migrated out under the condition that some attributes in α are removed out, as $\exists x_i \in (x), \bar{f}(x_i) = w_i \notin (x)$, where $(\bar{x})^{\bar{F}}$ indicates $(x) - \{x_i \in (x) | \bar{f}(x_i) = w_i \notin (x)\}$. Obviously, all of the inner IPI and exterior IPI generated by (x) can, respectively, form an inner IPI family and an exterior IPI family expressed as $\{(\bar{x})_i^F | i \in I\}, \{(\bar{x})_j^{\bar{F}} | j \in J\}$. Certainly, all of the IPI generated by (x) can also form an IPI family as $\Phi(x) = \{((\bar{x})_i^F, (\bar{x})_j^{\bar{F}}) | i \in I, j \in J\}$. Considering $\{x_i | u_i \bar{\in} (x), f(u_i) = x_i \in (x)\}$ as an event, $(\bar{x})_i^F$ is obtained in the case of the event occurrence probability equal to 1, namely, $(\bar{x})^F$ is obtained with the fact that $\{x_i | u_i \bar{\in} (x), f(u_i) = x_i \in (x)\}$ is bound to happen. As we know, it is stochastic that $w_i \notin (x)$ is transferred in (x) by f . The same goes for exterior IPI and IPI [23].

Definition 1 is called random inner IPI generated by (x) depending on information element migration probability σ , generally written as random inner IPI, such that

$$(\bar{x})^{F\sigma} = (\bar{x})^F \cup (\bar{x})_{\sigma}^+, \quad (1)$$

$(\bar{x})_{\sigma}^+$ is called the added random information with

$$(\bar{x})_{\sigma}^+ = \{x | w \bar{\in} (x), p(\{f(w) = x \in (x)\}) \in [\sigma, 1]\} \quad (2)$$

on condition that α^F fulfills

$$\alpha^F = \alpha \cup \{\alpha' | \delta \in V, \delta \bar{\in} \alpha, f(\delta) = \alpha' \in \alpha\} \quad (3)$$

where $\sigma \in (0, 1)$ and α^F are also thought to be the attribute sets of $(\bar{x})^{F\sigma}$.

Definition 2 $(\bar{x})^{\bar{F}\sigma}$ is called the random exterior IPI obtained by (x) depending on information element migration probability σ , briefly written as random exterior IPI, such that

$$(\bar{x})^{\bar{F}\sigma} = (\bar{x})^{\bar{F}} - \{x | x \in (x), p(\{\bar{f}(x) = w \bar{\in} (x)\}) \in [\sigma, 1]\}, \quad (4)$$

$(\bar{x})_{\sigma}^{-}$ is called deleted random information with

$$(\bar{x})_{\sigma}^{-} = \{x | x \in (x), p(\{\bar{f}(x) = w\bar{\epsilon}(x)\}) \in [\sigma, 1]\} \quad (5)$$

on condition that $\alpha^{\bar{F}}$ fulfills

$$\alpha^{\bar{F}} = \alpha - \{\alpha_i | \alpha_i \in \alpha, \bar{f}(\alpha_i) = \delta_i \bar{\epsilon} \alpha\} \quad (6)$$

where the nonempty attribute set $\alpha^{\bar{F}}$ of $(\bar{x})^{\bar{F}}$ is also that of $(\bar{x})^{\bar{F}\sigma} \neq \emptyset$ and $\sigma \in (0, 1)$.

Definition 3 The information pair formed by the random inner IPI and the random exterior IPI generated by (x) is called a random IPI generated by (x) depending on information element migration probability σ , also called RIPI as

$$((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma}) \quad (7)$$

where $(\alpha^F, \alpha^{\bar{F}})$ is also the attribute set of $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$.

Considering Definitions 1–3 and the above assumption, it is easily noted that Formulas (1) and (4) can, respectively, be represented with other forms as the following:

$$(\bar{x})^{F\sigma} = (\bar{x}) \cup (\bar{x})_{\sigma}^{+} = (x) \cup \{x | w\bar{\epsilon}(x), p_F(f) \in [\sigma, 1]\} \quad (8)$$

$$(\bar{x})^{\bar{F}\sigma} = (x) - (\bar{x})_{\sigma}^{-} = (x) - \{x | x \in (x), p_{\bar{F}}(\bar{f}) \in [\sigma, 1]\} \quad (9)$$

Formulas (8) and (9) show RIPI to be as one information pair generated by not only the corresponding IPI, but also the ordinary information (x) , as shown in Fig. 1.

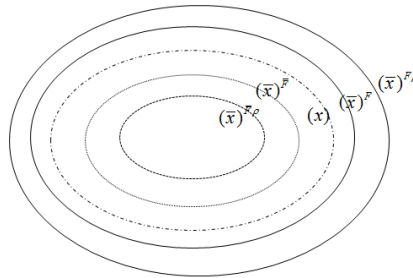


Fig. 1 The relationship among (x) , $((\bar{x})^F)$, $(\bar{x})^{\bar{F}}$, and $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$

All of the RIPI generated by information (x) constitute an RIPI family as

$$R\Phi(x) = \{((\bar{x})_i^{F\sigma}, (\bar{x})_j^{\bar{F}\sigma}) | \sigma \in (0, 1), i \in I, j \in J\}. \quad (10)$$

According to Definitions 1–3, Propositions 1–4 are simply derived as follows.

Proposition 1 Let $p_F(f) \equiv 0$ for $\forall f \in F$, then $(\bar{x})^F, (\bar{x})^{F\sigma}$ are not indented expressed as $\text{UNI}((\bar{x})^F, (\bar{x})^{F\sigma})$.

Proposition 2 Let $p_{\bar{F}}(\bar{f}) \equiv 0$ for $\forall \bar{f} \in \bar{F}$, then $\text{UNI}((\bar{x})^{\bar{F}}, (\bar{x})^{\bar{F}\sigma})$.

Proposition 3 Let $p_{\bar{F}}(\bar{f}) = p_F(f) \equiv 0$ for $\forall \bar{f} \in \bar{F}$ and $\forall f \in F$, then $\text{UNI}((\bar{x})^F, (\bar{x})^{\bar{F}}), ((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma}))$.

Proposition 4 For $\forall \sigma \in (0, 1)$, there is $\text{UNI}(\Phi(x), R\Phi(x))$.

Propositions 1–4 state that RIPI $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$ is the extension of $((\bar{x})^F, (\bar{x})^{\bar{F}})$, and $((\bar{x})^F, (\bar{x})^{\bar{F}})$ is the particular case of $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$. Under certain conditions, RIPI could restore to homologous IPI, and to information (x) .

Theorem 1 (Relation theorem between RIPI and IPI) Assume $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma}) \in R\Phi(x)$ and $((\bar{x})^F, (\bar{x})^{\bar{F}}) \in \Phi(x)$, then for $\forall \sigma \in (0, 1)$ we have

$$((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma}) \quad (11)$$

where Formula (11) represents the relation of Fig. 1.

Proof The assumption condition and Formulas (1) and (4) guarantee that $(\bar{x})^F \subseteq (\bar{x})^{F\sigma}$ and $(\bar{x})^{\bar{F}\sigma} \subseteq (\bar{x})^{\bar{F}}$ are fulfilled. According to the definition of IPI derived by common information (x) in Refs [3, 4, and 18], we can obtain $(\bar{x})^{\bar{F}} \subseteq (\bar{x})^F$. Hence, we get Formula (11) by set hereditary property.

Theorem 2 (Generation theorem of RIPI) Assume that $(\alpha^F, \alpha^{\bar{F}})$ is the attribute sets of RIPI $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$. There exists the nonempty pair $(\Delta\alpha, \nabla\alpha) \neq \emptyset$ such that $\alpha^F - (\alpha \cup \Delta\alpha) = \emptyset$ and $\alpha^{\bar{F}} - (\alpha - \nabla\alpha) = \emptyset$, where $(\Delta\alpha, \nabla\alpha) \neq \emptyset$ is equal to $\Delta\alpha \neq \emptyset$ and $\nabla\alpha \neq \emptyset$.

Proof Let $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$ be different from (x) , namely, $(\bar{x})^{F\sigma} = (\bar{x})^{\bar{F}\sigma} = (x)$ fails the Theorem 2 condition, then there exists one working between $(\bar{x})^{F\rho} \neq (x)$ and $(\bar{x})^{\bar{F}\rho} \neq (x)$ at least. Let us assume that $(\bar{x})^{F\sigma} \neq (x)$, the generating process of random inner IPI $(\bar{x})^{F\rho}$ depending on its attribute set α^F points out that α^F meets $\alpha \subset \alpha^F$. Assuming $\Delta\alpha = \alpha^F - \alpha$, we obtained $\Delta\alpha \neq \emptyset$ and $\alpha^F - (\alpha \cup \Delta\alpha) = \emptyset$ according to Definition 1. In the same way, it is proved that there exists $\nabla\alpha \neq \emptyset$ such that $\alpha^{\bar{F}} - (\alpha - \nabla\alpha) = \emptyset$.

3 RIPI characteristics

Supplementing some additional attributes into α , some information elements would be migrated into information (x) depending on certain probability in succession and form a chain of random inner IPI showing the following dynamic process:

$$(\bar{x})_1^{F\sigma} \subseteq (\bar{x})_2^{F\sigma} \subseteq \dots \subseteq (\bar{x})_s^{F\sigma} \quad (12)$$

Deleting some attributes out from α continuously, some information elements in (x) are migrated successively, depending on certain probability and form a chain of random exterior IPI showing the dynamic process as follows:

$$(\bar{x})_s^{\bar{F}\sigma} \subseteq (\bar{x})_{s-1}^{\bar{F}\sigma} \subseteq \dots \subseteq (\bar{x})_1^{\bar{F}\sigma} \quad (13)$$

If the above change processes take place at the same time, we obtain a chain of the RIPI implying the dynamic process as

$$((\bar{x})_i^{F\sigma}, (\bar{x})_i^{\bar{F}\sigma}) \quad (14)$$

According to Formulas (12)–(14), we get the dynamic characteristics depending on the attribute sets indicated by Theorems 3–5.

Theorem 3 (Depending attribute theorem of RIPI) Let $(\bar{x})_i^{F\rho}, (\bar{x})_j^{F\rho}$ be the random inner IPI and α_i^F, α_j^F expressing their attribute sets in order. Then $(\bar{x})_i^{F\sigma} \subseteq (\bar{x})_j^{F\sigma}$ iff $\alpha_i^F \subseteq \alpha_j^F$.

Theorem 4 (Depending attribute theorem of RIPI) Let $(\bar{x})_i^{\bar{F}\rho}, (\bar{x})_j^{\bar{F}\rho}$ be random exterior IPI and $\alpha_i^{\bar{F}}, \alpha_j^{\bar{F}}$ expressing their attribute sets in order. Then $(\bar{x})_i^{\bar{F}\sigma} \subseteq (\bar{x})_j^{\bar{F}\sigma}$ iff $\alpha_i^{\bar{F}} \subseteq \alpha_j^{\bar{F}}$.

Theorem 5 (Depending attribute theorem of RIPI) Let $((\bar{x})_i^{F\sigma}, (\bar{x})_i^{\bar{F}\sigma}), ((\bar{x})_j^{F\sigma}, (\bar{x})_j^{\bar{F}\sigma}) \in R\Phi(x)$ and $(\alpha_i^F, \alpha_i^{\bar{F}}), (\alpha_j^F, \alpha_j^{\bar{F}})$ be their attribute sets, respectively. Then $((\bar{x})_i^{F\sigma}, (\bar{x})_i^{\bar{F}\sigma}) \subseteq ((\bar{x})_j^{F\sigma}, (\bar{x})_j^{\bar{F}\sigma})$ iff $(\alpha_i^F, \alpha_i^{\bar{F}}) \subseteq (\alpha_j^F, \alpha_j^{\bar{F}})$.

Inference 1 Let $\text{card}(V - \alpha) = t$. Then information (x) can generate $t!$ dynamic chains of RIPI.

Inference 2 Let $\text{card}(\alpha) = m$. Then information (x) can generate $m!$ dynamic chains of random exterior IPI.

Inference 3 Let $\text{card}(\alpha) = m$ and $\text{card}(V - \alpha) = t$. Then information (x) can generate $t! \times m!$ dynamic chains of RIPI.

According to the dynamic characteristics of RIPI, the measurement of dynamic change degree is proposed in Definitions 4–6.

Definition 4 Let $(\bar{x})^{F\sigma}$ be a random inner IPI derived by (x) . Then call the real number $\gamma(\bar{x})^{F\sigma}$ to be F -measure degree of $(\bar{x})^{F\sigma}$ relative to (x) as

$$\gamma(\bar{x})^{F\sigma} = \|x^{(F\sigma)} - x^{(0)}\| / \|x^{(0)}\| \quad (15)$$

where $(x) = \{x_1, x_2, \dots, x_s\}$, $(x)^{F\sigma} = \{x_1, x_2, \dots, x_s, x_{s+1}, \dots, x_{s+t}\}$; the sequence of information value is expressed as

$$x_i = (x_{i1}, x_{i2}, \dots, x_{im}), i = 1, 2, \dots, s+t, x_{ik} \in [0, 1], \text{ and}$$

$$x^{(0)} = (\sum_{i=1}^s x_{i1}, \sum_{i=1}^s x_{i2}, \dots, \sum_{i=1}^s x_{im}),$$

$$x^{(F\sigma)} = (\sum_{i=1}^{s+t} x_{i1}, \sum_{i=1}^{s+t} x_{i2}, \dots, \sum_{i=1}^{s+t} x_{im}),$$

$$\|x^{(F\sigma)} - x^{(0)}\| = [\sum_{k=1}^m (\sum_{i=1}^{s+t} x_{ik} - \sum_{i=1}^s x_{ik})^\rho]^{1/\rho},$$

$$\|x^{(0)}\| = [\sum_{k=1}^m (\sum_{i=1}^s x_{ik})^\rho]^{1/\rho}, k = 1, 2, \dots, m, \rho \in \mathbb{Z}^+.$$

Definition 5 Let $(\bar{x})^{\bar{F}\sigma}$ be a random exterior IPI derived by (x) . Then call $\gamma(\bar{x})^{\bar{F}\sigma}$ \bar{F} -measure degree of $(\bar{x})^{\bar{F}\sigma}$ relative to (x) as

$$\gamma(\bar{x})^{\bar{F}\sigma} = \|x^{(0)} - x^{(\bar{F}\sigma)}\| / \|x^{(0)}\| \quad (16)$$

where $(x) = \{x_1, x_2, \dots, x_s\}$, $(x)^{\bar{F}\sigma} = \{x_1, x_2, \dots, x_{s-p}\}$, $0 \leq p < s$, $p \in \mathbb{Z}^+$; $x^{(0)}$ and $\|x^{(0)}\|$ are the same as Definition 4, and the sequence of information value is written as

$$x_i = (x_{i1}, x_{i2}, \dots, x_{im}), x_{ik} \in [0, 1], i = 1, 2, \dots, s, \text{ and}$$

$$x^{(\bar{F}\sigma)} = (\sum_{i=1}^{s-p} x_{i1}, \sum_{i=1}^{s-p} x_{i2}, \dots, \sum_{i=1}^{s-p} x_{im}),$$

$$\|x^{(0)} - x^{(\bar{F}\sigma)}\| = [\sum_{k=1}^m (\sum_{i=1}^s x_{ik} - \sum_{i=1}^{s-p} x_{ik})^\rho]^{1/\rho}.$$

Definition 6 Let $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$ be an RIPI generated by (x) . Then call the real number pair composed by Formulas (15) and (16) to be (F, \bar{F}) -measure degree of $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$ relative to (x) , and

$$(\gamma(\bar{x})^{F\sigma}, \gamma(\bar{x})^{\bar{F}\sigma}) \quad (17)$$

Because $((\bar{x})^F, (\bar{x})^{\bar{F}})$ is a special case of $((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma})$, $(\gamma(\bar{x})^F, \gamma(\bar{x})^{\bar{F}})$ is chosen to express (F, \bar{F}) -measure degree of $((\bar{x})^F, (\bar{x})^{\bar{F}})$ when $\text{UNI}((\bar{x})^F, (\bar{x})^{F\sigma})$, $\text{UNI}((\bar{x})^{\bar{F}}, (\bar{x})^{\bar{F}\sigma})$ in Definitions 4 and 5.

Formula (15) notes the change measurement between $(\bar{x})^{F_p}$ and (x) caused by attribute supplementing set $\Delta\alpha$; the same goes for Formulas (16) and (17). Thus Propositions 5–7 can be obtained.

Proposition 5 Given F -measure degree $\gamma(\bar{x})^{F\sigma}$, $\gamma(\bar{x})^{F\sigma} \neq 0$ iff $\text{IDE}((\bar{x})^{F\sigma}, (x))$ or $\text{IDE}(\alpha^F, \alpha)$.

Proposition 6 Given \bar{F} -measure degree $\gamma(\bar{x})^{\bar{F}\sigma}$, $\gamma(\bar{x})^{\bar{F}\sigma} \neq 0$ iff $\text{IDE}((\bar{x})^{\bar{F}\sigma}, (x))$ or $\text{IDE}(\alpha^{\bar{F}}, \alpha)$.

Proposition 7 Given (F, \bar{F}) -measure degree $(\gamma(\bar{x})^{F\sigma}, \gamma(\bar{x})^{\bar{F}\sigma})$, $(\gamma(\bar{x})^{F\sigma}, \gamma(\bar{x})^{\bar{F}\sigma}) \neq 0$ iff $\text{IDE}((\bar{x})^{F\sigma}, (\bar{x})^{\bar{F}\sigma}, (x))$ or $\text{IDE}((\alpha^F, \alpha^{\bar{F}}), \alpha)$.

Theorem 5 Let $(\bar{x})_i^{F\sigma}, (\bar{x})_j^{F\sigma}$ be random inner IPI. Then $(\bar{x})_i^{F\sigma} \subseteq (\bar{x})_j^{F\sigma}$ iff $\gamma_i(x)^{F\sigma} \leq \gamma_j(x)^{F\sigma}$.

Proof Considering $(\bar{x})_j^{F\sigma} = \{x_1, x_2, x_3, \dots, x_q\}$ and $(\bar{x})_i^{F\sigma} = \{x_1, x_2, x_3, \dots, x_q, x_{q+1}, \dots, x_{q+t}\}$, $(\bar{x})_i^{F\sigma} \subseteq (\bar{x})_j^{F\sigma}$ implies that $(\bar{x})_j^{F\sigma}$ is a sub-information of $(\bar{x})_i^{F\sigma}$, so $\sum_{i=1}^q x_{ik} \leq \sum_{i=1}^{q+t} x_{ik}, k = 1, 2, \dots, m$ iff

$$\gamma_i(\bar{x})^{F\sigma} = \|x_i^{(F\sigma)} - x^{(0)}\| / \|x^{(0)}\| \leq \gamma_j(\bar{x})^{F\sigma} = \|x_j^{(F\sigma)} - x^{(0)}\| / \|x^{(0)}\|.$$

Accordingly, we have the following result.

Table 1 The profit discrete distributions $x_i^{(0)}, x_i^{(0)}$ of $W, W_i, i = 1, 2, 3, 4$ from June to December in 2019

k	1	2	3	4	5	6
$x_1^{(0)}$	0.51	0.30	0.52	0.22	0.50	0.44
$x_2^{(0)}$	0.21	0.28	0.45	0.36	0.66	0.53
$x_3^{(0)}$	0.43	0.29	0.60	0.40	0.28	0.24
$x_4^{(0)}$	0.19	0.35	0.48	0.66	0.67	0.26

Theorem 6 Suppose that $(\bar{x})_i^{\bar{F}\sigma}$ and $(\bar{x})_j^{\bar{F}\sigma}$ are random exterior IPI. Then $(\bar{x})_i^{\bar{F}\sigma} \subseteq (\bar{x})_j^{\bar{F}\sigma}$ iff $\gamma_i(x)^{\bar{F}\sigma} \leq \gamma_j(x)^{\bar{F}\sigma}$.

Inference 4 Suppose $((\bar{x})_i^{F\sigma}, (\bar{x})_i^{\bar{F}\sigma}), ((\bar{x})_j^{F\sigma}, (\bar{x})_j^{\bar{F}\sigma})$ are RIPI.

$$((\bar{x})_i^{F\sigma}, (\bar{x})_i^{\bar{F}\sigma}) \subseteq ((\bar{x})_j^{F\sigma}, (\bar{x})_j^{\bar{F}\sigma}) \text{ iff } (\gamma_i(\bar{x})^{F\sigma}, \gamma_i(\bar{x})^{\bar{F}\sigma}) \leq (\gamma_j(\bar{x})^{F\sigma}, \gamma_j(\bar{x})^{\bar{F}\sigma}).$$

where $(\gamma_i(\bar{x})^{F\sigma}, \gamma_i(\bar{x})^{\bar{F}\sigma}) \leq (\gamma_j(\bar{x})^{F\sigma}, \gamma_j(\bar{x})^{\bar{F}\sigma})$ implies that $\gamma_i(\bar{x})^{F\sigma} \leq \gamma_j(\bar{x})^{F\sigma}$ and $\gamma_i(\bar{x})^{\bar{F}\sigma} \geq \gamma_j(\bar{x})^{\bar{F}\sigma}$.

4 Applications of RIPI model in intelligent acquisition–separation of investment information

For convenience, call $x^{(0)}$ the information value and $x^{(F\sigma)}$ the inner IPI value in Definition 4; call $x^{(\bar{F}\sigma)}$ the exterior IPI value based on which Definition 7 is given.

Definition 7 Call

$$\text{if } \alpha \Rightarrow \alpha^F, \text{ then } x^{(0)} \Rightarrow x^{(F\sigma)} \quad (18)$$

an inner RIPI reasoning in which $\alpha \Rightarrow \alpha^F, x^{(0)} \Rightarrow x^{(F\sigma)}$ are equivalent to $\alpha \subseteq \alpha^F, x^{(0)} \subseteq x^{(F\sigma)}$, respectively; call

$$\text{if } \alpha \Rightarrow \alpha^{\bar{F}}, \text{ then } x^{(\bar{F}\sigma)} \Rightarrow x^{(0)} \quad (19)$$

an exterior RIPI reasoning in which $\alpha^{\bar{F}} \Rightarrow \alpha$ and $x^{(\bar{F}\sigma)} \Rightarrow x^{(0)}$ are equivalent so that $\alpha^{\bar{F}}, x^{(\bar{F}\sigma)}$ are subsets of $\alpha, x^{(0)}$, respectively.

Proposition 8 If $\alpha_i^F \Rightarrow \alpha_{i+1}^F$, then $\gamma(\bar{x})_i^{F\sigma} \leq \gamma(\bar{x})_{i+1}^{F\sigma}$ in which the attribute sets of $(\bar{x})_i^F, (\bar{x})_{i+1}^F$ are $\alpha_i^F, \alpha_{i+1}^F$, respectively, and they satisfy $\alpha_{i+1}^F = \alpha_i^F \cup \{\alpha' | \delta \in V, \delta \notin \alpha, f(\delta) = \alpha' \in \alpha\}$.

Proposition 9 If $\alpha_{i+1}^{\bar{F}} \Rightarrow \alpha_i^{\bar{F}}$, then $\gamma(\bar{x})_{i+1}^{\bar{F}\sigma} \leq \gamma(\bar{x})_i^{\bar{F}\sigma}$ in which the attribute sets of $(\bar{x})_i^{\bar{F}}, (\bar{x})_{i+1}^{\bar{F}}$ are $\alpha_i^{\bar{F}}, \alpha_{i+1}^{\bar{F}}$, respectively, and they satisfy $\alpha_{i+1}^{\bar{F}} = \alpha_i^{\bar{F}} - \{\alpha_k | \alpha_k \in \alpha, \bar{f}(\alpha_k) = \delta_k \notin \alpha\}$.

Assumption For simplicity, this section only proposes the applications of random inner IPI in intelligent separation–acquisition of investment information. Suppose that W is a group company that produces petroleum and chemical products $W = \{W_1, W_2, W_3, W_4, W_5\}$, where $W_i \in W, i = 1, 2, 3, 4, 5$ are subsidiary corporations of W . $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ is the attribute set of W (product market characteristics of W). Information form of W is $(x) = \{x_1, x_2, x_3, x_4, x_5\}$. Due to trade secret, the group company and its subsidiary corporations and attributes (market characteristics) are expressed as W, W_i , and $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$, respectively. $x^{(0)}, x_i^{(0)}$ are profit discrete value distributions of W, W_i from January to June in 2019 as

$$x^{(0)} = (x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, x_4^{(0)}), \\ x_i^{(0)} = (x_{i1}^{(0)}, x_{i2}^{(0)}, x_{i3}^{(0)}, \dots, x_{i6}^{(0)}), i = 1, 2, 3, 4.$$

Values in $x^{(0)}, x_i^{(0)}$ are derived from dealing with the real profit value, and the results do not influence the analysis of the case. The profit discrete distributions $x^{(0)}, x_i^{(0)}$ of W, W_i are listed in Table 1.

By profit discrete distribution in Table 1, the profit information value of W is

$$x^{(0)} = (x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, x_4^{(0)}) = (\sum_{i=1}^4 x_{i1}^{(0)}, \sum_{i=1}^4 x_{i2}^{(0)}, \sum_{i=1}^4 x_{i3}^{(0)}, \sum_{i=1}^4 x_{i4}^{(0)}, \sum_{i=1}^4 x_{i5}^{(0)}, \sum_{i=1}^4 x_{i6}^{(0)})$$

Table 2 The profit discrete distributions $x^{(F0.8)}, x_i^{(F0.8)}$ of group company $W^{F0.8}$ and sub-company $W_i, i = 1, 2, 3, 4, 5, 6$ from January to June in 2020

k	1	2	3	4	5	6
$x_1^{(F0.8)}$	0.51	0.30	0.52	0.22	0.50	0.44
$x_2^{(F0.8)}$	0.21	0.28	0.45	0.36	0.66	0.53
$x_3^{(F0.8)}$	0.43	0.28	0.60	0.40	0.28	0.24
$x_4^{(F0.8)}$	0.19	0.35	0.48	0.66	0.67	0.26
$x_5^{(F0.8)}$	0.61	0.60	0.72	0.69	0.65	0.70
$x_6^{(F0.8)}$	0.71	0.69	0.82	0.72	0.66	0.69

$$= (1.34, 1.22, 2.05, 1.64, 2.11, 1.47).$$

A global disease COVID-19 broke out during preliminary stage in 2020 and caused a series of economic changes such as some manufacturing industry profits reduced in different probabilities. In contrast, products relating to protective apparatus, therapeutic apparatus, their appurtenance, and so on, have great market potential and earn better profit in big probabilities. This random dynamic change suited the RIPI model in this paper. For simplicity, this section only considers the latter.

Suppose that $\alpha_7 = \text{outbreak of COVID-19}$ and the attribute set of $f(x)$ is $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$, then α^F is derived through transferring α_7 into α as

$$\begin{aligned}\alpha^F &= \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\} \cup \{\alpha_7\} \\ &= \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\}.\end{aligned}\quad (20)$$

Under the condition, sub-companies $W_6, W_7, W_8, W_9 \in W$ (specific products of W_5, W_6, W_7, W_8 omit) would bring much profit differently with probabilities 1, 0.8, 0.75, 0.67 after overall consideration. If probability $\rho = 0.8$ is chosen, then W_5, W_6 turn into W to form $W^{F0.8}$ whose information is $(x)^{F\sigma} = \{x_1, x_2, x_3, \dots, x_6\}$,

$$\begin{aligned}x^{(F0.8)} &= (x_1^{(F0.8)}, x_2^{(F0.8)}, x_3^{(F0.8)}, x_4^{(F0.8)}, x_5^{(F0.8)}, x_6^{(F0.8)}), \\ x_i^{(F0.8)} &= (x_{i1}^{(F0.8)}, x_{i2}^{(F0.8)}, x_{i3}^{(F0.8)}, x_{i4}^{(F0.8)}, x_{i5}^{(F0.8)}, x_{i6}^{(F0.8)}), i = 1, 2, 3, 4, 5, 6.\end{aligned}\quad (21)$$

The detailed profit discrete distribution of $W^{F0.8}$ is shown in Table 2.

By profit discrete distribution in Table 2, the profit information value of $W^{F0.8}$ is

$$\begin{aligned}x^{(F0.8)} &= (x_1^{(F0.8)}, x_2^{(F0.8)}, x_3^{(F0.8)}, x_4^{(F0.8)}, x_5^{(F0.8)}, x_6^{(F0.8)}) \\ &= (\sum_{i=1}^6 x_{i1}^{(F0.8)}, \sum_{i=1}^6 x_{i2}^{(F0.8)}, \sum_{i=1}^6 x_{i3}^{(F0.8)}, \sum_{i=1}^6 x_{i4}^{(F0.8)}, \sum_{i=1}^6 x_{i5}^{(F0.8)}, \sum_{i=1}^6 x_{i6}^{(F0.8)}) \\ &= (2.66, 2.51, 3.59, 3.05, 3.42, 2.86), \quad (22) \\ \gamma(x)^{F0.8} &= \|x^{(F0.8)} - x^{(0)}\| / \|x^{(0)}\| = 0.857,\end{aligned}\quad (22)$$

Analysis on intelligent acquisition of RIPI(x)^{Fp}

On the condition that α^F is generated by supplementing attributes α_7 into α , one can obtain the random inner inverse packet information $(x)^{F0.8} = \{x_1, x_2, x_3, \dots, x_6\}$ based on information $(x) = \{x_1, x_2, x_3, x_4\}$ through using Definition 1 and fulfill Formula (18).

$x^{(F0.8)}$ is intelligently separated out and acquired. If α_7 does not occur, $x^{(F0.8)}$ would never have been gained, or $(x)^{F\sigma}$ would never have been known depending on the probability 0.8. The example simply tells us that the following:

I. When α and α^F satisfy $\alpha \subseteq \alpha^F$, information $(x)^{F\sigma}$ is intelligently discovered randomly out of information (x) by using the random inner inverse packet information generation model. While W_5, W_6 are found out of W due to $(x)^{F\sigma}$.

II. When α_7 is thought to be a chance attribute and it invades the attribute set α . Random inner IPI $(x)^{F\sigma}$ is generated by (x) in Definition 1.

III. When the chance attributes α_7 invades α , the profit discrete distribution data $x^{(0)}$ of group company is turned to $x^{(F\sigma)}$, which makes the profit of W increase. This conclusion has been proved in the financial statement published by W .

IV. Formula (23) means that W_5, W_6 will bring the extra profit 85.7% with a probability of 0.8 or greater.

5 Discussion

In Refs. [3, 4], dynamic feature was brought into common set X and proposed the structure of IPS. Based on IPS, IPI, and its applications in resolving practical problems with dynamic characteristics and heredity are discussed in [8, 13, 17, 20, and 21]. The randomness of element transfer is considered in this paper according to the dynamic characteristics of IPI [23]. By integrating the possibility knowledge into IPI, this paper proposes the concepts and structures of RIPI and their applications. RIPI theory enriches IPI and enlarges its application category. It also provides a new theory tool for studying the information system.

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