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Evaluation of Investment Opportunities With Interval-Valued Fuzzy Topsis Method

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Abstract

The purpose of this study is extended the TOPSIS method based on interval-valued fuzzy set in decision analysis. After the introduction of TOPSIS method by Hwang and Yoon in 1981, this method has been extensively used in decision-making, rankings also in optimal choice. Due to this fact that uncertainty in decision-making and linguistic variables has been caused to develop some new approaches based on fuzzy-logic theory. Indeed, it is difficult to achieve the numerical measures of the relative importance of attributes and the effects of alternatives on the attributes in some cases. In this paper to reduce the estimation error due to any uncertainty, a method has been developed based on interval-valued fuzzy set. In the suggested TOPSIS method, we use Shannon entropy for weighting the criteria and apply the Euclid distance to calculate the separation measures of each alternative from the positive and negative ideal solutions to determine the relative closeness coefficients. According to the values of the closeness coefficients, the alternatives can be ranked and the most desirable one(s) can be selected in the decision-making process.

Keywords: Multi-Criteria decision making, Fuzzy logic theory, Interval- Valued Fuzzy TOPSIS Analysis, Euclid distance, Shannon Entropy
AMS 2010 codes: 03E72

1 Introduction

Decision making is one of the most complicated administrative processes in management. Over the years, various methods have been designed to simplify the process as well as developing new methods. Since, there are many imprecise concepts all around us that routinely expressed in different terms. In fact, the human brain works with considering various factors and based on inferential thinking and value of sentences that modeling of them with mathematical formulas if not impossible would be a complex task.

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Because crisp data are inexpressive to model real life situations Zadeh in 1965, has suggested fuzzy logic that is closer to human thinking and Chen [3] developed the TOPSIS method to fuzzy decision-making situations. The purpose of fuzzy logic as a decision-making technique is to improve decision making process in vague and unclear circumstances. Fuzzy management science, while creating the flexibility in the model, with entering some data such as knowledge, experience and human judgment in the model also offers fully functional responses to it [5]. However, if a decision is not possible for linguistic variables based on fuzzy sets, Interval-valued fuzzy set theory can provide a more detailed modeling. In this paper, interval-valued fuzzy TOPSIS method is proposed to solve MCDM (Multi-Criteria Decision Making) problems, where the weight of the criterias are unequal [2, 6, 7, 10–12].

2 TOPSIS Method

As mentioned, this method was developed by Hwang and Yoon (1981) in which the best alternative should have the shortest distance from an ideal solution and the worst alternative is the furthest from an ideal solution [2, 6].

Assume a multi criteria decision making problem has n alternatives, $A_1,\ A_2,\dots,\ A_n$ and m criterias, $C_1,\ C_2,\dots,\ C_m$. Each alternative is estimated regarding the m criteria. All the values/ratings are determined to alternatives with respect to decision matrix define by $X(x_{ij})_{n\times m}$. The criteria's weight vector is $w = (w_1,\ w_2,\ \dots,\ w_m)$ that $\sum_{j=1}^{m} w_j = 1$. TOPSIS method includes a process consisting of 6-steps as follows:

i Normalize the decision matrix using the following evolution for each r_{ij} .

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \quad i = 1,2,\dots,m \quad j = 1,2,\dots,n \tag{1}$$

ii Multiply the columns of the normalized decision matrix by the connected weights. The weighted and normalized decision matrix is come as:

$$V_{ij} = w_j \times r_{ij};\ i = 1,2,\dots,m \quad j = 1,2,\dots,n \tag{2}$$

Which w_j is the weight of the jth criteria.

iii specify the ideal and negative ideal alternatives respectively as follows:

$$\begin{aligned} A^+ &= \left\{v_1^+, v_2^+, \dots, v_n^+\right\} = \left\{(max_i v_{ij} j \in J_1), (min_i v_{ij} j \in J_2)\, i = 1,2,\dots,m\right\} \\ A^- &= \left\{v_1^-, v_2^-, \dots, v_n^-\right\} = \left\{(min_i v_{ij} j \in J_1),\ (max_i v_{ij} j \in J_2)\, i = 1,2,\dots,m\right\} \end{aligned} \tag{3}$$

Where J_1 is the set of benefit criterias and J_2 is the set of cost criterias.

iv With using of the two Euclidean distances to calculate the distance of the existing alternatives from ideal and negative ideal alternatives as:

$$\begin{aligned} S_i^+ &= \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}\ i = 1,2,\dots,m \\ S_i^- &= \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}\ i = 1,2,\dots,m \end{aligned} \tag{4}$$

v The relevant closeness to the ideal alternatives can be defined as:

$$C_i^+ = \frac{S_i^-}{S_i^- + S_i^+} \quad i = 1,2,\dots,m \tag{5}$$

Where $0 \le C_i^+ \le 1$.

vi According to the relative closeness to the ideal alternatives rank the alternatives the bigger C_i^+ is related to better alternative A_i [1].

3 Interval-Valued Fuzzy Sets

Since the theory of fuzzy sets by Zadeh can be used in vague and imprecise terms, many studies, have developed TOPSIS method in the interval- fuzzy environment.Because of the complexity of the socio-economic environment in many practical decision problems that option often would arrange shady by decision-makers [8,9]. An interval-valued fuzzy set A defined on $(-\infty,+\infty)$ is given by:

$$A = \left\{x, \left[\mu_A^L(x), \mu_A^U(x)\right]\right\} \mu_A^L(x), \mu_A^U(x) \ : X \to [0,1] \ \ \forall x \in X \ , \ \mu_A^L(x) \leq \mu_A^U(x) \tag{6}$$
$$\bar{\mu}_A(x) = \left[\mu_A^L(x), \mu_A^U(x)\right] A = \left\{\left(x, \bar{\mu}_A^{(x)}\right)\right\} \ , x \in (-\infty, +\infty)$$

That $\mu_A^L(x)$ is the lower limit of degree of membership and $\mu_A^U(x)$ is the upper limit of degree of membership.

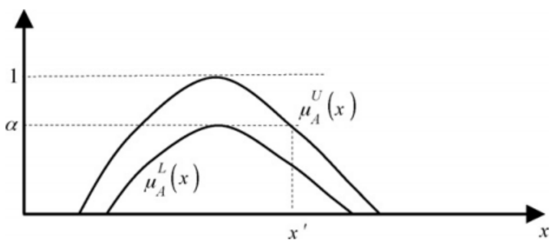

Fig. 1 Interval-valued fuzzy set.

Figure 1 Shows the value of membership at x' of interval-valued fuzzy set A. Thus, the minimum and maximum value of the membership x' are $\mu_A^L(x), \mu_A^U(x)$ respectively.

Here are two interval-valued fuzzy numbers $P_x = [P_x^-; P_x^+]$ and $Q_x = [Q_x^-; Q_x^+]$ due to the [5], we have:

3.1. Definition $P.Q(x.y) = [P_x^-.Q_x^-; P_x^+.Q_x^+]$ if $. \in (+,-,\times,\div)$.

3.2. Definition The Normalized Euclidean distance between $\check{P}$ and $\check{Q}$ is as:

$$D\left(\check{P},\ \check{Q}\right) = \sqrt{\frac{1}{6}\sum_{i=1}^{3}\left[\left(P_{x_i}^- - Q_{x_i}^-\right)^2 + \left(P_{x_i}^+ - Q_{x_i}^+\right)^2\right]} \tag{7}$$

A standard MCDM (Multi-Criteria Decision Making) problem can be briefly demonstrated in a decision matrix that x_{ij} represents value of the ith alternative of A_i with notice to the jth attribute, x_j. In this article, we develop the canonical matrix to interval-valued fuzzy decision matrix. The value and weighing of criteria, have been considered as linguistic variables. By using of Tables 1 and 2, these linguistic variables can be turned to interval-valued fuzzy triangular numbers.

Suppose that $\tilde{X} = [\tilde{x}_{ij}]_{n\times m}$ be a fuzzy decision matrix for a multi criteria decision making problem where $A_1, A_2, \ldots, A_n$ are n possible alternatives and $C_1, C_2, \ldots, C_m$ are m criteria. So $\tilde{x}_{ij}$ is the performance of alternative A_i with notice to criterion C_j. Figure 2 represents $\tilde{x}_{ij}$ and $\tilde{w}_j$ as triangular interval valued fuzzy numbers [10].

$$\tilde{x} = \begin{cases} (x_1, x_2, x_3) \\ (x'_1, x_2, x'_3) \end{cases}$$

Table 1 Definition of linguistic variables for the ratings

Very Poor (VP)	[(0,0);0;(1,1.5)]
Poor (P)	[(0,0.5);1;(2.5,3.5)]
Moderately Poor (MP)	[(0,1.5);3;(4.5,5.5)]
Fair (F)	[(2.5,3.5),5,(6.5,7.5)]
Moderately Good (MG)	[(4.5,5.5),7,(8,9.5)]
Good (G)	[(5.5,7.5),9,(9.5,10)]
Very Good (VG)	[(8.5,9.5),10,(10,10)]

Table 2 Definition of linguistic variables for the importance of each criterion

Very low (VL)	[(0,0);0;(0.1,0.15)]
Low (L)	[(0,0.05);0.1;(0.25,0.35)]
Medium low (ML)	[(0,0.15);0.3;(0.45,0.55)]
Medium (M)	[(0.25,0.35),0.5,(0.65,0.75)]
Medium high (MH)	[(0.45,0.55),0.7,(0.8,0.95)]
High (H)	[(0.55,0.75),0.9,(0.95,1)]
Very high (VH)	[(0.85,0.95),1,(1,1)]

Here $\tilde{x}$ can be indicated by $\tilde{x} = \left[\left(x_1, x'_1\right); x_2; (x'_3; x_3)\right]$. The normalized performance of rating as an expansion to Chen [3] for $\tilde{x} = \left[\left(a_{ij}, a'_{ij}\right); b_{ij}; (c'_{ij}, c_{ij})\right]$ can be calculated as:

$$\begin{aligned}
\tilde{r}_{ij} &= \left[\left(\frac{a_{ij}}{c_j^+}, \frac{a'_{ij}}{c_j^+}\right); \frac{b_{ij}}{c_j^+}; \left(\frac{c'_{ij}}{c_j^+}; \frac{c_{ij}}{c_j^+}\right)\right], \qquad i = 1,2,\ldots,n \quad j \in \Omega_b \\
\tilde{r}_{ij} &= \left[\left(\frac{a_j^-}{a'_{ij}}, \frac{a_j^-}{a_{ij}}\right); \frac{a_j^-}{b_{ij}}; \left(\frac{a_j^-}{c_{ij}}; \frac{a_j^-}{c'_{ij}}\right)\right], \qquad i = 1,2,\ \ldots,n \quad j \in \Omega_c \\
c_j^+ &= \max_i c_{ij}\ ,\ j \in \Omega_b \\
a_j^- &= \min_i a'_{ij}\ ,\ j \in \Omega_c
\end{aligned} \tag{8}$$

Therefore, the normalized matrix $\tilde{R} = [\tilde{r}_{ij}]_{n\times m}$ can be obtained.

Here the suggested technique for building up the TOPSIS to interval- valued fuzzy TOPSIS can be described as follows:

i Make the weighted normalized fuzzy decree matrix with notice that each criterias has own importance as: $\tilde{V} = [\tilde{v}_{ij}]_{n\times m}$ that $\tilde{v}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j$. Now from Defintion 3.1:

$$\tilde{v}_{ij} = \left[\left(\tilde{r}_{1_{ij}} \times \tilde{w}_{1_j}, \tilde{r}'_{1_{ij}} \times \tilde{w}'_{1_j}\right); \tilde{r}_{2_{ij}} \times \tilde{w}_{2_j}; (\tilde{r}'_{3_{ij}} \times \tilde{w}'_{3_j}, \tilde{r}_{3_{ij}} \times \tilde{w}_{3_j})\right] = \left[\left(g_{ij}, g'_{ij}\right); h_{ij}; (l'_{ij}, l_{ij})\right] \tag{9}$$

ii Defined the optimal and negative optimal solution as:

$$A^+ = [(1,1);1;(1,1)]\ , \qquad j \in \Omega_b\ A^- = [(0,0);0;(0,0)], \qquad j \in \Omega_c \tag{10}$$

iii Normalized Euclidean distance can be figured out using Definition 3.2 as follows:

$$\begin{aligned}
D^-\left(\tilde{N}, \tilde{M}\right) &= \sqrt{\frac{1}{3}\sum_{i=1}^{3}\left[\left(N_{x_i}^- - M_{y_i}^-\right)^2\right]} \\
D^+\left(\tilde{N}, \tilde{M}\right) &= \sqrt{\frac{1}{3}\sum_{i=1}^{3}\left[\left(N_{x_i}^+ - M_{y_i}^+\right)^2\right]}
\end{aligned} \tag{11}$$

Where D^-, D^+ are the initial and secondary distance measure, respectively.

Hence, we can calculate distance from the ideal alternative for each alternative as follows:

$$D_{i1}^{+} = \sum_{j=1}^{m} \sqrt{\frac{1}{3}\left[(g_{ij}-1)^2 + (h_{ij}-1)^2 + (l_{ij}-1)^2\right]} \tag{12}$$

$$D_{i2}^{+} = \sum_{j=1}^{m} \sqrt{\frac{1}{3}\left[(g'_{ij}-1)^2 + (h_{ij}-1)^2 + (l'_{ij}-1)^2\right]}$$

As the same way, calculate gap of the negative ideal solution by:

$$D_{i1}^{-} = \sum_{j=1}^{m} \sqrt{\frac{1}{3}\left[(g_{ij}-0)^2 + (h_{ij}-0)^2 + (l_{ij}-0)^2\right]} \tag{13}$$

$$D_{i2}^{-} = \sum_{j=1}^{m} \sqrt{\frac{1}{3}\left[(g'_{ij}-0)^2 + (h_{ij}-0)^2 + (l'_{ij}-0)^2\right]}$$

Eqs. (12) and (13) are used to specify the distance from ideal and negative ideal alternatives in interval values.

iv The involved sepreation can be calculated by:

$$RC_1 = \frac{D_{i2}^{-}}{D_{i2}^{+} + D_{i2}^{-}} \tag{14}$$

The latest worths of RC_i^* are identified as:

$$Rc_i^* = \frac{RC_1 + RC_2}{2} \tag{15}$$

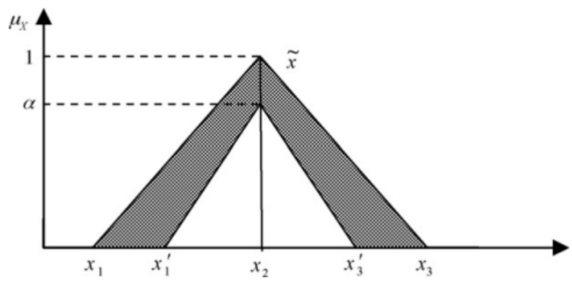

Fig. 2 Interval-valued triangular fuzzy number

4 The Implementation ofthe Extended Technique toSolveProblems

Suppose aninvestment corporation plans to allocate its limited resources toinvest on four projectsaccording toimportance and profitability of each project respectively.In thispaper, a model is presented for prioritizing investments in various industrial fields.In this case, committee of company's decision makers intend to evaluate and ultimately rank the possible company's investment options. Desired options for investment are given in the following table.

Firstly, criteria and sub-criteria were determined by applying the strategic documents of company. There are three main criteria as: "Industrial efficiency", "Compliance with company's strategy" and "The campany's industrial experience". The hierarchy of criteria and sub-criteria shows in Table 4.

Table 3 Desired options for investment

Code	Title
A1	Project 1
A2	Project 2
A3	Project 3
A4	Project 4

Table 4 The hierarchy of criteria and sub-criteria

	C1-Industrial efficiency		C2-Compliance withcompany'sstrategy		C3-Industrial experience
C11	Increasing demand for industrial products (P)	C21	Ability to attract foreign investors (P)	C31	Receivables in the subordinate (C)
C12	Alternative Products (C)	C22	Entrepreneurship (P)	C32	Implementation process of industrial projects (P)
C13	government intervention in product pricing (P)	C23	Technology transfer Capacity (P)		
C14	Current value of the industry on the exchange (P)	C24	Ability to reduce dependency on foreign products (P)		
C15	Average process for the delivery of industrial projects (C)	C25	Amount of dependency on foreign raw material (C)		
		C26	Exportamount (P)		

4.1 Solution Steps

i After weighing the basic criteria by decision makers separately and unaware each other based on the target, then decision matrix is created by specified linguistic variables.

As already mentioned, each linguistic variable has an interval fuzzy value. Table 6. gives these values as. So, the final decision matrix is given in Tables 7 with interval fuzzy numbers.

ii In this step, decision matrix is normalized by the equation (8) and the results are expressed in Tables 8.

iii As stated earlier, the weight of each criteria was previously determined by the decision makers (Shannon entropy) as given in Tables 9.

Table 5 Decision matrix according to linguistic variables

	C11	C12	C13	C14	C15	C21	C22	C23	C24	C25	C26	C31	C32
A1	MG	P	MP	G	MP	MG	MG	M	M	MP	M	MP	MG
A2	VG	VP	P	MG	VP	G	G	G	G	P	MG	P	VG
A3	P	M	M	MP	P	M	MP	P	VP	MP	MP	M	M
A4	M	MP	MP	MG	P	MG	M	P	MG	MP	MG	MP	M

Table 6 Interval fuzzy value of linguistic variables

[(3.83,4.83);6.33;(7.5,8.83)]	VP
[(4.5,5.5);6.67;(7.67,8.33)]	P
[(5.17,6.17);7.33;(8.17,9)]	MP
[(6.17,7.5);8.67;(,9.179.83)]	M
[(7.17,8.17);9;(9.33,9.83)]	MG
[(7.5,8.83);9.67;(9.83,10)]	G
[(8.5,9.5);10;(10,10)]	VG

iv Now we can make the weighted normalized fuzzy decision matrix by using the Eq. (9) given that each criterion has different importance. As in Table 10.

v By using Eq. (11,12,13) the distance of each alternative is calculated from the ideal alternative $\left[D_{i1}^{+}, D_{i2}^{+}\right]$, given in Table 11.

vi At this step, the fuzzy relative closeness of each alternative is calculated by using the respective distinctions of each pair and the results are given in Table (11).

vii In the last step alternatives are listed in Table (12) according to their relative closeness.

Now calculate c_j^+ and a_j^- as followes:

$$\tilde{x}_{ij} = [(a_{ij}, \acute{a}_{ij}), b_{ij}, (\acute{c}_{ij}, c_{ij})]$$

$$c_j^+ = max_i c_{ij},\ j \in \Omega_b,\ \ a_j^- = min_i a'_{ij}\ , j \in \Omega_c$$

10	4.83	5.5	10	4.83	10	9.83	10	10	5.5	9.83	5.5	10
c_1^+	a_2^-	a_3^-	C_4^+	a_5^-	C_6^+	C_7^+	C_8^+	C_9^+	a_{10}^-	C_{11}^+	a_{12}^-	C_{13}^+

Now with using of:

$$\tilde{r}_{ij} = \left[\left(\frac{a_{ij}}{c_j^+}, \frac{a'_{ij}}{c_j^+}\right), \frac{b_{ij}}{c_j^+}, \left(\frac{c'_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right)\right],\ \ i = 1,2,\ldots,n\ \ ,\ j \in \Omega_b$$

$$\tilde{r}_{ij} = \left[\left(\frac{a_j^-}{a'_{ij}}, \frac{a_j^-}{a_{ij}}\right), \frac{a_j^-}{b_{ij}}, \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{c'_{ij}}\right)\right],\ \ i = 1,2,\ldots,n\ \ ,\ j \in \Omega_c$$

Make the $\tilde{R} = [\tilde{r}_{ij}]_{n \times m}$.

5 Conclusions

The increasing complexity of socio-economic communities causes the intricacy and ambiguity in the priorities of decision-makers; because decision-making is often done in some circumstances such as lack of information and knowledge, lack of decision-makers consensus, time limits... So, in such situation, Decision-making in an interval-valued fuzzy environment would be convenient. The main characteristic of using interval-valued fuzzy environment is that the membership functions would be an interval rather than an exact number. In fuzzy set theory, it is difficult to express a thought or linguistic variables entirely by an integer number in [0, 1]. Thus, expressing degree of certainty by an interval of [0, 1] would be more appropriate. It's worth paying attention, the use of interval valuation numbers gives an occasion to proficients to define lower and upper bounds values as an interval for matrix elements and weights of criteria.

Table 7 Interval valued fuzzy decision matrix

	C11	C12	C13	C14	C15
A1	[(7.17,8.17);9;(9.33,9.83)]	[(4.5,5.5);6.67;(7.67,8.33)]	[(5.17,6.17);7.33;(8.17,9)]	[(7.5,8.83);9.67;(9.83,10)]	[(5.17,6.17);7.33;(8.17,9)]
A2	[(8.5,9.5);10;(10,10)]	[(3.83,4.83);6.33;(7.5,8.83)]	[(4.5,5.5);6.67;(7.67,8.33)]	[(7.17,8.17);9;(9.33,9.83)]	[(3.83,4.83);6.33;(7.5,8.83)]
A3	[(4.5,5.5);6.67;(7.67,8.33)]	[(6.17,7.5);8.67;(,9.179.83)]	[(6.17,7.5);8.67;(,9.179.83)]	[(5.17,6.17);7.33;(8.17,9)]	[(4.5,5.5);6.67;(7.67,8.33)]
A4	[(6.17,7.5);8.67;(,9.179.83)]	[(5.17,6.17);7.33;(8.17,9)]	[(5.17,6.17);7.33;(8.17,9)]	[(7.17,8.17);9;(9.33,9.83)]	[(4.5,5.5);6.67;(7.67,8.33)]
	C21	C22	C23	C24	C25
A1	[(7.17,8.17);9;(9.33,9.83)]	[(7.17,8.17);9;(9.33,9.83)]	[(6.17,7.5);8.67;(,9.179.83)]	[(6.17,7.5);8.67;(,9.179.83)]	[(5.17,6.17);7.33;(8.17,9)]
A2	[(8.5,9.5);10;(10,10)]	[(6.17,7.5);8.67;(,9.179.83)]	[(8.5,9.5);10;(10,10)]	[(7.5,8.83);9.67;(9.83,10)]	[(4.5,5.5);6.67;(7.67,8.33)]
A3	[(6.17,7.5);8.67;(,9.179.83)]	[(5.17,6.17);7.33;(8.17,9)]	[(4.5,5.5);6.67;(7.67,8.33)]	[(3.83,4.83);6.33;(7.5,8.83)]	[(5.17,6.17);7.33;(8.17,9)]
A4	[(7.17,8.17);9;(9.33,9.83)]	[(6.17,7.5);8.67;(,9.179.83)]	[(4.5,5.5);6.67;(7.67,8.33)]	[(7.17,8.17);9;(9.33,9.83)]	[(5.17,6.17);7.33;(8.17,9)]
	C26	C31	C32		
A1	[(6.17,7.5);8.67;(,9.179.83)]	[(5.17,6.17);7.33;(8.17,9)]	[(7.17,8.17);9;(9.33,9.83)]		
A2	[(7.17,8.17);9;(9.33,9.83)]	[(4.5,5.5);6.67;(7.67,8.33)]	[(8.5,9.5);10;(10,10)]		
A3	[(5.17,6.17);7.33;(8.17,9)]	[(6.17,7.5);8.67;(,9.179.83)]	[(6.17,7.5);8.67;(,9.179.83)]		
A4	[(7.17,8.17);9;(9.33,9.83)]	[(5.17,6.17);7.33;(8.17,9)]	[(6.17,7.5);8.67;(,9.179.83)]		

Table 8 Normalize Decision Matrix

	C11	C12	C13	C14	C15
A1	[(0.72,0.82);0.9;(0.93,0.98)]	[(0.88,1.07);0.72;(0.58,0.63)]	[(0.89,1.06);0.75;(0.61,0.67)]	[(0.75,0.88);0.97;(0.98,1)]	[(0.78,0.93);0.66;(0.54,0.59)]
A2	[(0.85,0.95);1;(1,1)]	[(1,1.3);0.76;(0.55,0.64)]	[(1,1.22);0.82;(0.62,0.72)]	[(0.72,0.82);0.9;(0.93,0.98)]	[(1,1.3);0.76;(0.55,0.64)]
A3	[(0.45,0.55);0.67;(0.77,0.83)]	[(0.64,0.78);0.56;(0.49,0.53)]	[(0.73,0.89);0.63;(0.56,0.6)]	[(0.52,0.62);0.7;(0.82,0.9)]	[(0.88,1.07);0.72;(0.58,0.63)]
A4	[(0.62,0.75);0.73;(0.82,0.9)]	[(0.78,0.93);0.66;(0.54,0.59)]	[(0.89,1.06);0.75;(0.61,0.67)]	[(0.72,0.82);0.9;(0.93,0.98)]	[(0.88,1.07);0.72;(0.58,0.63)]

	C21	C22	C23	C24	C25
A1	[(0.72,0.82);0.9;(0.93,0.98)]	[(0.73,0.83);0.92;(0.95,0.95)]	[(0.62,0.75);0.73;(0.82,0.9)]	[(0.62,0.75);0.73;(0.82,0.9)]	[(0.89,1.06);0.75;(0.61,0.67)]
A2	[(0.85,0.95);1;(1,1)]	[(0.63,0.76);0.88;(0.93,1)]	[(0.85,0.95);1;(1,1)]	[(0.75,0.88);0.97;(0.98,1)]	[(1,1.22);0.82;(0.62,0.72)]
A3	[(0.62,0.75);0.73;(0.82,0.9)]	[(0.53,0.63);0.75;(0.83,0.92)]	[(0.45,0.55);0.67;(0.77,0.83)]	[(0.38,0.48);0.63;(0.75,0.88)]	[(0.89,1.06);0.75;(0.61,0.67)]
A4	[(0.72,0.82);0.9;(0.93,0.98)]	[(0.63,0.76);0.88;(0.93,1)]	[(0.45,0.55);0.67;(0.77,0.83)]	[(0.72,0.82);0.9;(0.93,0.98)]	[(0.89,1.06);0.75;(0.61,0.67)]

	C26	C31	C32
A1	[(0.63,0.76);0.88;(0.93,1)]	[(0.89,1.06);0.75;(0.61,0.67)]	[(0.72,0.82);0.9;(0.93,0.98)]
A2	[(0.73,0.83);0.92;(0.95,0.95)]	[(1,1.22);0.82;(0.62,0.72)]	[(0.85,0.95);1;(1,1)]
A3	[(0.53,0.63);0.75;(0.83,0.92)]	[(0.73,0.89);0.63;(0.56,0.6)]	[(0.62,0.75);0.73;(0.82,0.9)]
A4	[(0.73,0.83);0.92;(0.95,0.95)]	[(0.89,1.06);0.75;(0.61,0.67)]	[(0.62,0.75);0.73;(0.82,0.9)]

Table 9 Weight values of criteria

[(0.85,0.95);1;(1,1)]	VH
[(0.55,0.75);0.9;(0.95,1)]	H
[(0.45,0.55);0.7;(0.8,0.95)]	MH
[(0.25,0.35);0.5;(0.65,0.75)]	M
[(0,0.15);0.3;(0.45,0.55)]	ML
[(0,0.05);0.1;(0.25,0.35)]	L
[(0,0);0;(0.1,0.15)]	VL

Table 9 Weight of criterias

C11	VH	C21	L	C31	M
C12	H	C22	ML	C32	ML
C13	H	C23	M		
C14	L	C24	ML		
C15	MH	C25	VL		
		C26	M		

Table 10 Weighted normalize fuzzy decision matrix

	C11	C12	C13	C14	C15
A1	[(0.61,0.78);0.9;(0.93,0.98)]	[(0.48,0.80);0.65;(0.55,0.63)]	[(0.49,0.8);0.68;(0.58,0.67)]	[(0,0.04);0.09;(0.25,0.35)]	[(0.35,0.51);0.46;(0.43,0.56)]
A2	[(0.72,0.9);1;(1,1)]	[(0.55,0.98);0.68;(0.52,0.64)]	[(0.55,0.92);0.74;(0.59,0.72)]	[(0,0.04);0.09;(0.23,0.34)]	[(0.45,0.72);0.53;(0.44,0.61)]
A3	[(0.38,0.52);0.67;(0.77,0.83)]	[(0.35,0.59);0.50;(0.47,0.53)]	[(0.40,0.67);0.57;(0.53,0.6)]	[(0,0.03);0.07;(0.21,0.32)]	[(0.4,0.6);0.5;(0.46,0.6)]
A4	[(0.53,0.71);0.73;(0.82,0.9)]	[(0.43,0.70);0.59;(0.51,0.59)]	[(0.49,0.8);0.68;(0.58,0.67)]	[(0,0.04);0.09;(0.23,0.34)]	[(0.4,0.6);0.5;(0.46,0.6)]

	C21	C22	C23	C24	C25
A1	[(0,0.04);0.09;(0.23,0.35)]	[(0,0.12);0.28;(0.43,0.52)]	[(0.16,0.26);0.37;(0.53,0.68)]	[(0,0.11);0.22;(0.37,0.5)]	[(0,0);0;(0.06,0.1)]
A2	[(0,0.05);0.1;(0.25,0.35)]	[(0,0.11);0.26;(0.42,0.55)]	[(0.21,0.33);0.5;(0.65,0.75)]	[(0,0.13);0.29;(0.44,0.55)]	[(0,0);0;(0.06,0.11)]
A3	[(0,0.04);0.07;(0.21,0.32)]	[(0,0.08);0.23;(0.37,0.51)]	[(0.11,0.19);0.34;(0.5,0.62)]	[(0,0.07);0.19;(0.34,0.48)]	[(0,0);0;(0.06,0.1)]
A4	[(0,0.04);0.09;(0.23,0.34)]	[(0,0.11);0.26;(0.42,0.55)]	[(0.11,0.19);0.34;(0.5,0.62)]	[(0,0.12);0.27;(0.42,0.54)]	[(0,0);0;(0.06,0.1)]

	C26	C31	C32
A1	[(0.16,0.27);0.44;(0.6,0.75)]	[(0.22,0.37);0.38;(0.4,0.5)]	[(0,0.12);0.27;(0.42,0.54)]
A2	[(0.18,0.29);0.46;(0.62,0.71)]	[(1,0.43);0.41;(0.4,0.54)]	[(0,0.14);0.3;(0.45,0.55)]
A3	[(0.13,0.22);0.38;(0.54,0.69)]	[(0.18,0.31);0.32;(0.36,0.45)]	[(0,0.11);0.22;(0.37,0.5)]
A4	[(0.18,0.29);0.46;(0.62,0.71)]	[(0.22,0.37);0.38;(0.4,0.5)]	[(0,0.11);0.22;(0.37,0.5)]

Table 11 Distance of alternatives from ideal alternatives

A1	D_{11}^{+}	D_{12}^{+}	D_{11}^{-}	D_{12}^{-}	A2	D_{11}^{+}	D_{12}^{+}	D_{11}^{-}	D_{12}^{-}
C11	0.234521	0.14	0.826035	0.890468	C11	0.161658	0.057735	0.916224	0.967815
C12	0.443471	0.31459	0.564329	0.697472	C12	0.422414	0.278328	0.587452	0.781537
C13	0.424264	0.288039	0.588473	0.719097	C13	0.382187	0.225389	0.631981	0.798415
C14	0.892562	0.851293	0.153406	0.209921	C14	0.898332	0.853483	0.142595	0.204369
C15	0.585947	0.491155	0.415933	0.511631	C15	0.528205	0.3879	0.475044	0.624873
C21	0.898146	0.851293	0.142595	0.209921	C21	0.889288	0.843603	0.155456	0.212132
C22	0.783156	0.712928	0.296254	0.312463	C22	0.792465	0.716984	0.28519	0.356931
C23	0.665833	0.591608	0.384448	0.471487	C23	0.576368	0.503786	0.48874	0.554196
C24	0.816497	0.742092	0.248529	0.321714	C24	0.778396	0.698451	0.304248	0.366742
C25	0.791623	0.776745	0.034641	0.057735	C25	0.980408	0.964728	0.034641	0.063509
C26	0.627163	0.549363	0.439394	0.525674	C26	0.607838	0.541541	0.457675	0.516333
C31	0.67082	0.585064	0.342929	0.420833	C31	0.485833	0.543016	0.665357	0.463537
C32	0.789515	0.710868	0.288271	0.355387	C32	0.772981	0.690917	0.31225	0.37063
Sum	8.623517	7.605039	4.725234	5.703802	Sum	8.276371	7.305862	5.456853	6.281021
A3	D_{11}^{+}	D_{12}^{+}	D_{11}^{-}	D_{12}^{-}	A4	D_{11}^{+}	D_{12}^{+}	D_{11}^{-}	D_{12}^{-}
C11	0.426693	0.350333	0.628808	0.685128	C11	0.329747	0.235938	0.703847	0.784644
C12	0.563738	0.461519	0.444747	0.541295	C12	0.494335	0.376917	0.514166	0.628808
C13	0.505239	0.38893	0.505239	0.614763	C13	0.423832	0.289425	0.588473	0.719097
C14	0.91086	0.869521	0.127802	0.189912	C14	0.898332	0.853483	0.142595	0.204369
C15	0.548209	0.43589	0.455192	0.568624	C15	0.528205	0.3879	0.475044	0.624873
C21	0.91086	0.865814	0.127802	0.190526	C21	0.898332	0.853483	0.142595	0.204369
C22	0.814412	0.748198	0.251529	0.326292	C22	0.792465	0.716984	0.28519	0.356931
C23	0.701831	0.641898	0.354824	0.422729	C23	0.701831	0.641898	0.354824	0.422729
C24	0.835005	0.772744	0.22487	0.300777	C24	0.789367	0.711548	0.288271	0.355387
C25	0.980408	0.967815	0.034641	0.057735	C25	0.848528	0.967815	0.34641	0.057735
C26	0.671541	0.602467	0.388544	0.472193	C26	0.607838	0.541541	0.457675	0.516333
C31	0.717496	0.643169	0.296873	0.365605	C31	0.671516	0.586316	0.342929	0.420833
C32	0.817578	0.741732	0.248529	0.321714	C32	0.817578	0.741732	0.248529	0.321714
Sum	9.403869	8.490031	4.089401	5.057293	Sum	8.801903	7.904981	4.890547	5.617823

Table 12 The final ranking of Options

	RC1	RC2	RC*	RANK
A1	0.428572	0.353983	0.391278	2
A2	0.462286	0.602653	0.532469	1
A3	0.373306	0.30307	0.338188	4
A4	0.415433	0.357171	0.386302	3

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