

## The Numerical Study of a Hybrid Method for Solving Telegraph Equation

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### Abstract

In this study, a robust hybrid method is used as an alternative method, which is a different method from other methods for the approximate of the telegraph equation. The hybrid method is a mixture of the finite difference and differential transformation methods. Three numerical examples are solved to prove the accuracy and efficiency of the hybrid method. The reached results from these samples are shown in tables and graphs.

**Keywords:** 1D telegraph equation, approximate solution, central difference, differential transform method, finite difference method.

**AMS 2010 codes:** 35F16; 65N06; 65N12; 65N15

## 1 Introduction

1D telegraph equation is used for signal investigation at transmission and proliferation of electrical signs. The telegraph equation also shows the mix of dissemination and wave proliferation by the properties of constrained velocity to standard warmth or mass transport condition. 1D telegraph equation is in the following form [1] for  $x \in [a, b], t \geq 0$ :

$$u_{tt} + 2\alpha u_t + \beta^2 u = u_{xx} + f(x, t), \quad (1)$$

with initial conditions

$$u(x, t) = g_1(x), u_t(x, y) = g_2(x), \quad (2)$$

and boundary conditions

$$u(a, t) = \gamma_1, u(b, t) = \gamma_2, x \in [a, b], \quad (3)$$

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where  $\alpha, \beta, a$  and  $b$  are constant and  $g_1, g_2, \gamma_1, \gamma_2$  and  $f$  are known functions.

Several numerical methods were developed for solving 1D telegraph equation, for example, reduced differential transform method, differential quadrature method, new unconditionally stable difference schemes, Chebyshev tau method, dual reciprocity boundary integral equation, cubic B-spline collocation method, modified B-spline differential quadrature method, semi-discretization method, unconditionally stable ADI method, Rothe-wavelet method, collocation method, He's variational iteration method, dual reciprocity boundary integral equation method, etc. [1, 9–19]. The preferred hybrid method is used for solving many types of differential equations. These are differential-algebraic equations [21], partial differential equations [4, 6–8, 22, 23], and fractional differential equations [20]. Also, the other differential equation types can be solved by this hybrid method [25–32].

The hybrid method is given as a blend of the differential transform and the finite difference methods. The differential transform method was proposed by Zhou for the solution of linear and nonlinear differential equations for electrical circuit analysis [24]. This method was developed by Chen and Ho for partial differential equations [5]. Ayaz applied differential transform method for the system of differential equations [3]. According to these studies, the transformation of the  $k$ -th derivative of  $u(x, t)$  based on  $t$ -time variable is as follows:

$$U(i, k) = \frac{1}{k!} \left[ \frac{d^k}{dt^k} u(x, t) \right]_{t=0}, \quad k = 0, 1, 2, \dots, \quad i = 0, 1, 2, \dots, \quad (4)$$

the inverse transformation of  $U(i, k)$  is identified by

$$u(x, t) = \sum_{k=0}^{\infty} U(i, k) t^k = U(i, 0) + U(i, 1)t + U(i, 2)t^2 + \dots, \quad i = 0, 1, 2, 3, \dots, \quad (5)$$

where  $U(i, k) = U(x_i, k)$ ,  $x_i = ih$ ,  $i = 0, 1, 2, 3, \dots$  and  $h$  is the finite difference step interval.

The theorems that can be deduced from Eqs (4) and (5) are as follows.

Let  $G$  be the differential transform of  $g$ , respectively.

**Theorem 1.** If  $g(x, t) = \frac{\partial^2 g}{\partial t^2}$ , then  $G(i, k) = (k+1)(k+2)G(i, k+2)$ .

**Theorem 2.** If  $g(x, t) = \frac{\partial g}{\partial t}$ , then  $G(i, k) = (k+1)G(i, k+1)$ .

**Theorem 3.** If  $g(x, t) = xe^{-t}$ , then  $G(i, k) = x \frac{(-1)^k}{k!}$ .

**Theorem 4.** If  $g(x, t) = \sin x$ , then  $G(i, k) = \sin x$ .

**Theorem 5.** If  $g(x, t) = \sin t$ , then  $G(i, k) = \frac{1^k}{k!} \sin\left(\frac{\pi k}{2}\right)$ .

The proof of Theorems 1–5 are available in [3].

The central difference derivative is given by the finite difference method, see Amirali and Amirali [2],

$$\frac{\partial^2 u}{\partial x^2} = \frac{U(i+1, k) - 2U(i, k) + U(i-1, k)}{h^2}$$

The truncation error constituted by the finite difference method is bigger than that constituted by the differential transform method. Hence, the aim of this study gives a hybrid method which is a mixture of these methods. Using the hybrid method, the approximate solution of the telegraph equation as different from the other methods is obtained from a very powerful iterative scheme.

## 2 Numerical Experiments of the Telegraph Equation with Hybrid Method

Here, we give three experiments of both linear and nonlinear 1D telegraph equations to approve the power and reliability of the hybrid method.

**Example 1.** We will consider the following linear telegraph equation with initial and boundary conditions [18]:

$$u_{tt} + 2u_t + 2u = u_{xx} + xe^{-t}, \quad x \in [0, 1], \quad (6)$$

$$u(x, 0) = x, u_t(x, 0) = -x, \tag{7}$$

$$u(0, t) = 0, u(1, t) = e^{-t}, \tag{8}$$

where  $f(x, t) = xe^{-t}$ .

The exact solution to our test problem (6)–(8) is  $u(x, t) = xe^{-t}$ .

Applying the hybrid method for Eqs (6)–(8), first, we have differential transforms of terms dependent on  $t$ -time variable in the telegraph Eq. (6), and then the central difference of derivative terms dependent on the  $x$ -position variable is obtained. The  $x$ -position variable is replaced with mesh positions in other  $x$ -terms in the telegraph equation.

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &\rightarrow (k+1)(k+2)U(i, k+2), \\ \frac{\partial u}{\partial t} &\rightarrow (k+1)U(i, k+1), \\ u(x, t) &\rightarrow U(i, k), \\ \frac{\partial^2 u}{\partial x^2} &\rightarrow \frac{U(i+1, k) - 2U(i, k) + U(i-1, k)}{h^2}, \\ xe^{-t} &\rightarrow x_i \frac{(-1)^k}{k!}, \\ u(x, 0) &\rightarrow U(i, 0) = x_i, \\ u_t(x, 0) &\rightarrow U(i, 0) = -x_i, \\ u(0, t) &\rightarrow U(0, k) = 0, \\ u(1, t) &\rightarrow U(1, k) = \frac{(-1)^k}{k!}, \\ x_i &= ih, h = 0.1, i = 0, 1, 2, \dots, k = 0, 1, 2, \dots \end{aligned}$$

If these differential transforms and the central difference equivalents are written in Eq. (6), we have recurrence correlation and the approximate solution is reached on mesh points  $x_i$  as

$$U(i, k+2) = \frac{1}{(k+1)(k+2)} \left[ \frac{U(i+1, k) - 2U(i, k) + U(i-1, k)}{h^2} - 2(k+1)U(i, k+1) - 2U(i, k) + x_i \frac{(-1)^k}{k!} \right],$$

$$\begin{aligned} x_i=0, u(0, t) &= \sum_{k=0}^{\infty} U(0, k)t^k = U(0, 0) + U(0, 1)t + U(0, 2)t^2 + \dots \\ &= 0 + t + 0t^2 + \dots + 0t^{10} + \dots \end{aligned}$$

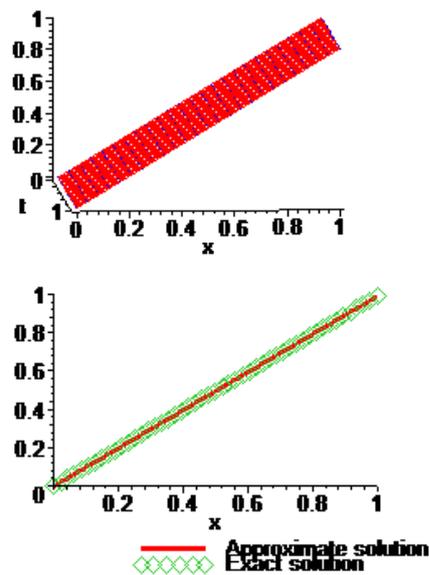
$$\begin{aligned} x_i=0.1, u(0.1, t) &= \sum_{k=0}^{\infty} U(0.1, k)t^k = U(0.1, 0) + U(0.1, 1)t + \dots \\ &= 0.1 - 0.1t + 0.05t^2 + \dots + 0.2755731922 \cdot 10^{-7}t^{10} \end{aligned}$$

$$\begin{aligned} x_i=1, u(1, t) &= \sum_{k=0}^{\infty} U(1, k)t^k = U(1, 0) + U(1, 1)t + U(1, 2)t^2 + \dots \\ &= 1 - 1t + 0.5t^2 + \dots + 0.2755731922 \cdot 10^{-6}t^{10} + \dots \end{aligned}$$

After we find these approximate solutions for  $x$  and  $t = 0.01$ , we give Table 1. Furthermore, in Figure 1 as 2D and 3D, we compare the results of exact and numerical solution of Example 1 for  $t = 0.01$  and  $x = 0.1$ .

**Table 1** Exact solution, approximate solution, and error values for  $t = 0.01$

$x$	Exact solution	Approximate solution	Error
0.0	0.0000000000	0.0000000000	0
0.1	0.0990049833	0.0990049833	0
0.2	0.1980099668	0.1980099667	$0.1 \times 10^{-9}$
0.3	0.2970149501	0.2970149501	0
0.4	0.3960199335	0.3960199335	0
0.5	0.4950249169	0.4950249168	$0.1 \times 10^{-9}$
0.6	0.5940299002	0.5940299002	0
0.7	0.6930348836	0.6930348836	0
0.8	0.7920398670	0.7920398670	0
0.9	0.8910448504	0.8910448503	$0.1 \times 10^{-9}$
1.0	0.9900498337	0.9900498337	0



**Fig. 1** Comparison of exact and approximate solutions for Example 1.

**Example 2.** Let us consider the following linear telegraph equation [18]:

$$u_{tt} + u_t - u = u_{xx}, \quad x \in [0, 1], \tag{9}$$

$$u(x, 0) = \sin x, u_t(x, 0) = -\sin x, t \geq 0, \tag{10}$$

$$u(0, t) = 0, u(1, t) = \sin(1)e^{-t}, \tag{11}$$

where  $f(x, t) = 0$ .

Eqs (9) and (10) have the following exact solution:

$$u(x, t) = e^{-t} \sin x.$$

If Eq. (9) is separated by hybrid method, we obtain differential transforms of terms dependent on  $t$ -time variable. Then, the central difference of derivative terms dependent on the  $x$ -position variable is obtained. The other  $x$ -position variables in the problem are replaced with  $x$ :

$$\begin{aligned} \frac{\partial u}{\partial t} &\rightarrow U(i, k) = (k + 1)U(i, k + 1), \\ \frac{\partial^2 u}{\partial x^2} &\rightarrow \frac{U(i + 1, k) - 2U(i, k) + U(i - 1, k)}{h^2}, \\ u(x, t) &\rightarrow U(i, k), \\ u(x, 0) &\rightarrow U(i, 0) = \sin x_i, \\ u_t(x, 0) &\rightarrow U(i, 0) = -\sin x_i, \\ u(0, t) &\rightarrow U(0, k) = 0, \\ u(1, t) &\rightarrow U(1, k) = \sin(1) \frac{(-1)^k}{k!}, \\ x_i &= ih, h = 0.1, i = 0, 1, 2, \dots, k = 0, 1, 2, \dots \end{aligned}$$

If these differential transforms and the central difference equivalents are written in Eq. (9), then the following recurrence relation is obtained:

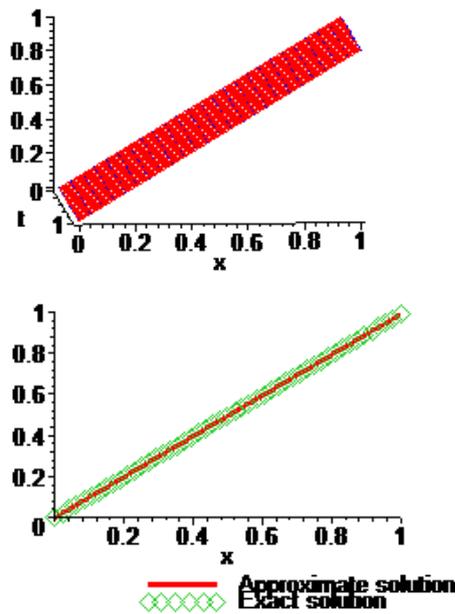
$$U(i, k + 2) = \frac{1}{(k + 1)(k + 2)} \left[ \frac{U(i + 1, k) - 2U(i, k) + U(i - 1, k)}{h^2} - (k + 1)U(i, k + 1) + U(i, k) \right],$$

Now, we find approximate solutions on  $x_i$  mesh points for  $t = 0.01$ , Numerical values are demonstrated in Table 2. To compare the results of exact and approximate solution of Example 2, Figure 2 is plotted.

$$\begin{aligned} x_i = 0, u(0, t) &= \sum_{k=0}^{\infty} U(0, k)t^k = U(0, 0) + U(0, 1)t + U(0, 2)t^2 + \dots \\ &= 0 + t + 0t^2 + \dots + 0t^{10} + \dots \\ x_i = 0.1, u(0.1, t) &= \sum_{k=0}^{\infty} U(0.1, k)t^k = U(0.1, 0) + U(0.1, 1)t + \dots \\ &= 0.09983341665 - 0.09983341665t + 0.09933466549t^2 + \dots + 0.2751141332 \cdot 10^{-7}t^{10} + \dots \\ x_i = 1, u(1, t) &= \sum_{k=0}^{\infty} U(1, k)t^k = U(1, 0) + U(1, 1)t + U(1, 2)t^2 + \dots \\ &= 0.8414709848 - 0.8414709848t + 0.4207354924t^2 + \dots + 0.2318868455 \cdot 10^{-6}t^{10} + \dots \end{aligned}$$

**Table 2** Exact solution, approximate solution, and error values for  $t = 0.01$

$x$	Exact solution	Approximate solution	Error
0.0	0.0000000000	0.0000000000	0
0.1	0.9884005755	0.9884005755	0
0.2	0.1966925379	0.1980099667	$0.1 \times 10^{-9}$
0.3	0.2925797315	0.2925797314	$0.1 \times 10^{-9}$
0.4	0.3855435650	0.3855435651	$0.1 \times 10^{-9}$
0.5	0.4746551748	0.4746551748	0
0.6	0.5590241869	0.5590241869	0
0.7	0.6378076141	0.6378076141	0
0.8	0.7102182785	0.7102182785	0
0.9	0.7755326766	0.7755326765	$0.1 \times 10^{-9}$
1.0	0.8330982086	0.8330982087	$0.1 \times 10^{-9}$



**Fig. 2** Comparison of exact and approximate solutions for Example 2.

In the form of 2D and 3D, the results of exact and approximate solutions of Example 2 are compared for  $t = 0.01$  and  $x = 0.1$  as in Figure 2.

**Example 3.** Considering the following nonlinear telegraph equation [19]:

$$u_{tt} + 2u_t = u_{xx} - u^2 + e^{2x+4t} - e^{x-2t}, x \in [0, 1], \tag{12}$$

which has the following initial conditions, boundary conditions, and exact solution, respectively:

$$u(x, 0) = e^x, u_t(x, 0) = -2e^x, t \geq 0, \tag{13}$$

$$u(0, t) = e^{-2t}, u(1, t) = e^{1-2t}, x \in [0, 1], \tag{14}$$

$$u(x, t) = e^{x-2t},$$

where  $f(x, t) = e^{2x+4t} - e^{x-2t}$ .

Using the hybrid method for the solution of the problem (12)–(14), we have

$$\frac{\partial^2 u}{\partial t^2} \rightarrow U(i, k+2) = (k+1)(k+2)U(i, k+2),$$

$$\frac{\partial u}{\partial t} \rightarrow U(i, k) = (k+1)U(i, k+1),$$

$$\frac{\partial^2 u}{\partial x^2} \rightarrow \frac{U(u+1, k) - 2U(i, k) + U(i-1, k)}{h^2}$$

$$e^{2x+4t} \rightarrow e^{2x_i} \left( \frac{4^k}{k!} \right),$$

$$e^{x-2t} \rightarrow e^{x_i} \left( \frac{(-2)^k}{k!} \right),$$

$$(U(x, t))^2 \rightarrow (U(i, k))^2,$$

$$u(x, 0) \rightarrow U(i, 0) = e^{x_i}$$

$$u_t(x, 0) \rightarrow U(i, 1) = -2e^{x_i},$$

$$u(0, t) \rightarrow U(0, k) = 0,$$

$$u(0, t) \rightarrow U(0, k) = \frac{(-2)^k}{k!},$$

$$u(1, t) \rightarrow U(1, k) = e^{x_i} \left( \frac{(-2)^k}{k!} \right),$$

$$x_i = ih, h = 0.1, i = 0, 1, 2, \dots, k = 0, 1, 2, \dots$$

If these differential transforms and the central difference equivalents are written in the Eq. (12), then the following recurrence relation is obtained:

$$U(i, k+2) = \frac{1}{(k+1)(k+2)} \left[ \frac{U(i+1, k) - 2U(i, k) + U(i-1, k)}{h^2} - 2(k+1)U(i, k+1) - (U(i, k))^2 + e^{2x_i} \left( \frac{4^k}{k!} \right) - e^{x_i} \left( \frac{(-2)^k}{k!} \right) \right]$$

and then we have the approximate solutions on  $x_i$  mesh points for  $t = 0.01$  as:

$$x_i = 0, u(0, t) = \sum_{k=0}^{\infty} U(0, k)t^k = U(0, 0) + U(0, 1)t + U(0, 2)t^2 + \dots$$

$$= 1 - 2t + 2t + \dots + 0.0002821869489t^{10} + \dots$$

$$x_i = 0.1, u(0.1, t) = \sum_{k=0}^{\infty} U(0.1, k)t^k = U(0.1, 0) + U(0.1, 1)t + \dots$$

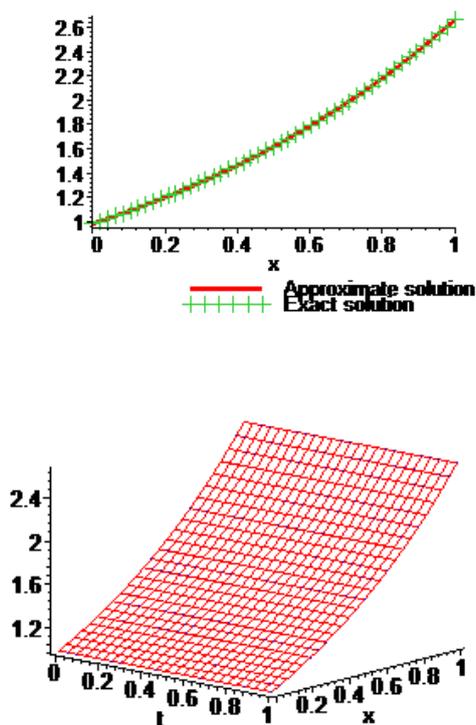
$$= 1.105170918 - 2.210341836t + 2.210341836t^2 + \dots + 0.0003118648093t^{10} + \dots$$

$$x_i = 1, u(1, t) = \sum_{k=0}^{\infty} U(1, k)t^k = U(1, 0) + U(1, 1)t + U(1, 2)t^2 + \dots$$

$$= 2.718281828 - 5.436563656t + 5.436563655t^2 + \dots + 0.0007670636553t^{10} + \dots$$

**Table 3** Exact solution, approximate solution, and error values for  $t = 0.01$

$x$	Exact solution	Approximate solution	Error
0.0	0.980198673	0.980198673	0
0.1	1.083287067	1.083287068	$0.1 \times 10^{-8}$
0.2	1.197217363	1.197217363	0
0.3	1.323129812	1.323129813	$0.1 \times 10^{-8}$
0.4	1.462284589	1.462284590	$0.1 \times 10^{-8}$
0.5	1.616074402	1.616074403	$0.1 \times 10^{-8}$
0.6	1.786038431	1.786038431	0
0.7	1.973877732	1.973877732	0
0.8	2.181472265	2.181472265	0
0.9	2.410899706	2.410899707	$0.1 \times 10^{-8}$
1.0	2.664456242	2.664456241	$0.1 \times 10^{-8}$



**Fig. 3** Comparison of exact and approximate solutions for Example 3.

The obtained data of Example 3 are given in Table 3, and the plot of corresponding exact and approximate solutions are demonstrated in Figure 3 for  $t = 0.01$  and  $x = 0.1$ .

Briefly, throughout the study, each figure (Figures 1–3) consists of two images. The first picture is two dimensional and the second picture is three dimensional. They compare exact and approximate solutions for  $t = 0.01$  and  $x = 0.1$ .

### 3 Conclusion

A robust hybrid numerical method was presented for the numerical solution of the linear and nonlinear telegraph equations. The hybrid method was introduced. Three test problems were solved by using this method.

The exact solution, approximate solution, and error values were computed for  $t = 0.01$  and different  $x$  values. The results were demonstrated in Tables 1–3. 2D and 3D forms of graphics were plotted in Figures 1–3. It was seen that the curves of the exact and approximate solutions are almost identical. As a result, the numerical results proved that the proposed method was working very well. Therefore, we propose that the hybrid method can be used for the other partial differential equations as an alternative method.

## References

- [1] G. Arora and V. Josh, (2016), Comparison of numerical solution of 1D hyperbolic telegraph equation using B-Spline and trigonometric B-Spline by differential quadrature method, *Indian Journal of Science and Technology*, vol. 9, no. 45.
- [2] G. Amirali and İ. Amirali, (2018), Nümerik Analiz, *Seçkin Yayıncılık*, Turkey, September.
- [3] F. Ayaz, (2004), Solutions of the system of differential equations by differential transform method, *Appl. Math. Comput.*, vol. 147, pp. 547-567.
- [4] C.K. Chen, H.Y. Lai and C.C. Liu, (2009), Nonlinear micro circular plate analysis using hybrid differential transformation/finite difference method, *CMES* 40, pp. 155-174.
- [5] C.K. Chen and S.H. Ho, (1999), Solving partial differential equations by two dimensional differential transform method, *Applied Mathematics and Computation*, vol. 106, pp. 171-179.
- [6] H.P. Chu and C.L. Chen, (2014), Hybrid differential transform and finite difference method to solve the nonlinear heat conduction problem, *Communication in Nonlinear Science and Numerical Simulation*, vol. 13, pp.1605-1614.
- [7] S.P. Chu, (2014), Hybrid differential transform and finite difference method to solve the nonlinear heat conduction problems, *WHAMPOA- An Interdisciplinary Journal*, vol. 66, pp. 15-26.
- [8] İ. Çilingir Süngü and H. Demir, (2018), New Algorithm for the lid-driven cavity flow problem with Boussinesq-stokes suspension, *Karalmas Science and Engineering Journal*, vol. 8, no. 2, pp. 462-472.
- [9] M. Dehghan and A. Ghesmati, (2010), Solution of the second-order one dimensional hyperbolic telegraph equation by using the dual reciprocity boundary integral equation (DRBIE) method, *Eng. Anal. Bound. Elem.*, vol. 34, no.1, pp. 51-59.
- [10] M. Dehghan, S.A. Yousefi and A. Lotfi, (2011), The use of He's variational iteration method for solving the telegraph and fractional telegraph equations, *Int. J. Numer. Methods Biomed. Eng.*, vol. 27, pp. 219–231.
- [11] M. Dosti and A. Nazemi, (2011), Solving one-dimensional hyperbolic telegraph equation using cubic b-spline quasi-interpolation, *World Academy of Science, Engineering and Technology*, vol. 5, no. 4, pp. 935-40.
- [12] M. Esmailbeigi, M.M. Hosseini and S.T. Mohyud-Din, (2011), A new approach of the radial basis functions method for telegraph equations, *Int. J. Phys. Sci.*, vol. 6, no. 6, pp. 1517–1527.
- [13] M. Lakestani and B.N. Saray, (2010), Numerical solution of telegraph equation using interpolating scaling functions, *Comput. Math. Appl.*, vol. 60, no. 7, pp. 1964–1972.
- [14] L.B. Liu and H.W. Liu, (2013), Compact difference schemes for solving telegraphic equations with Neumann boundary conditions, *Applied Mathematics and Computation*, vol. 219, pp. 10112–10121.
- [15] R.C. Mittal and R. Bhatia, (2014), A collocation method for numerical solution of hyperbolic telegraph equation with Neumann boundary conditions, *International Journal of Computational Mathematics*, vol. 2014, Article ID 526814, 9 pages.
- [16] J. Rashidinia, S. Jamalzadeh and F. Esfahani, (2014), Numerical solution of one dimensional telegraph equation using cubic b-spline collocation method, *Journal of Interpolation Approximation in Scientific Computing*, vol. 2014, pp. 1-8.
- [17] A. Saadatmandi and M. Dehghan, (2010), Numerical solution of hyperbolic telegraph equation using the chebyshev tau method, *Numerical Methods Partial Differential Equations*, vol. 26, no. 1, pp. 239-52.
- [18] B. Soltanalizadeh, (2011), Differential transformation method for solving one-space-dimensional telegraph equation, *Comp. Appl. Math.*, vol. 30, no 3, pp. 639–653.
- [19] V.K. Srivastava, V.K. Awasthi, R.K. Chaurasia, and M. Tamsir, (2013), The telegraph equation and its solution by reduced differential transform method, *Hindawi Publishing Corporation Modelling and Simulation in Engineering*, vol. 2013, 6 pages.
- [20] İ. Süngü and H. Demir, (2015), A new approach and solution technique to solve time fractional nonlinear reaction-diffusion equations, *Hindawi Publishing Corporation Mathematical Problems in Engineering*, 13 pages.
- [21] İ. Süngü and H. Demir, (2012), Application of the hybrid differential transform method to the nonlinear equations, *Applied Mathematics*, vol. 3, pp. 246-250.
- [22] İ. Süngü and H. Demir, (2012), Solutions of the system of differential equations by differential transform/finite difference method, *Nwsa-Physical Sciences*, vol. 7, pp. 1308-7304.
- [23] Y.L. Yeh, C.C. Wang and M.J. Jang, (2007), Using finite difference and differential transformation method to analyze of large deflections of orthotropic rectangular plate problem, *Applied Mathematics and Computation*, vol. 190, pp.

1146-1156.

- [24] J.K. Zhou, (1986), *Differential transformation and its applications for electrical circuits*, Huazhong University Press, Wuhan, P.R. China, In Chinese.
- [25] F. Düşünceli and E. Çelik, (2017), Numerical solution for high-order linear complex differential equations with variable coefficients, *Numerical Methods for Partial Differential Equations*, vol. 34, no. 5, pp. 1645-1658.
- [26] F. Düşünceli and E. Çelik, (2017), Fibonacci matrix polynomial method for linear complex differential equations, *Asian Journal of Mathematics and Computer Research*, vol. 15, no. 3, pp. 229-238.
- [27] A. Yokuş, (2018), Comparison of Caputo and conformable derivatives for time-fractional Korteweg-de Vries equation via the finite difference method, *International Journal of Modern Physics B*, vol. 32, no. 29, 1850365.
- [28] A. Yokuş, (2017), Numerical solution for space and time fractional order Burger type equation, *Alexandria Engineering Journal*, vol. 57, no. 3, 2085-2091.
- [29] A. Yokuş and S. Gülbahar, (2019), Numerical solutions with linearization techniques of the fractional Harry Dym equation, *Applied Mathematics and Nonlinear Sciences*, vol. 4, no.1, 35-42.
- [30] A. Yokuş and M. Tuz, (2017), An application of a new version of  $(G'/G)$ -expansion method, *AIP Conference Proceedings*, vol. 1798, no. 1.
- [31] A. Yokuş, H.M. Baskonuş, T.A. Sulaiman and H. Bulut, (2018), Numerical simulation and solutions of the two-component second order KdV evolutionary system, *Numer Methods Partial Differential Eq.*, vol. 34, no. 1, pp. 211–227.
- [32] A. Yokuş, T.A. Sulaiman, H.M. Baskonuş and S.P. Atmaca, (2018), On the exact and numerical solutions to a nonlinear model arising in mathematical biology, *The Third International Conference on Computational Mathematics and Engineering Sciences (CMES2018)*, vol. 22, 01061, doi.org/10.1051/itmconf/20182201061.