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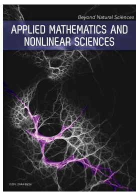


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# Optical solitons to the fractional Schrödinger-Hirota equation

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## Abstract

This study reaches the dark, bright, mixed dark-bright, and singular optical solitons to the fractional Schrödinger-Hirota equation with a truncated $M$-fractional derivative via the extended sinh-Gordon equation expansion method. Dark soliton describes the solitary waves with lower intensity than the background, bright soliton describes the solitary waves whose peak intensity is larger than the background, and the singular soliton solutions is a solitary wave with discontinuous derivatives; examples of such solitary waves include compactions, which have finite (compact) support, and peakons, whose peaks have a discontinuous first derivative. The constraint conditions for the existence of valid solutions are given. We use some suitable values of the parameters in plotting 3-dimensional surfaces to some of the reported solutions.

**Keywords:** M-fractional derivative; sinh-Gordon equation; Schrödinger-Hirota equation; optical soliton
**AMS 2010 codes:** 49K20.

## 1 Introduction

Nonlinear Schrödinger equations (NLSEs) can be used to describe various complex nonlinear physical phenomena arising from the different fields of nonlinear sciences, such as; optical fibers, hydrodynamics, complex acoustics, quantum hall effect, heat pulses in solids and many other nonlinear unstable aspects [1, 2]. The theory of optical solitons is one of the interesting topics for the investigation of soliton propagation through nonlinear optical fibers [3]. Optical solitons are restrained electromagnetic waves that stretch in nonlinear dispersive media and allow the intensity to remain unchanged due to the balance between dispersion and nonlinearity effects [4]. Various analytical approaches for securing optical solitons and other solutions to different kind of NLSEs have been reported to the literature such as the the sine-Gordon expansion method [5–7], the first integral method [8, 9], the improved Bernoulli sub-equation function method [10, 11], the trial solution method [12, 13],

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the new auxiliary equation method [14], the extended simple equation method [15], the solitary wave ansatz method [16], the functional variable method [17], the sub-equation method [18–20] and several others [21–33].

However, in this study, the extended sinh-Gordon equation expansion method (ShGEEM) [34–38] is used in constructing family of optical soliton and other solutions to the fractional Schrödinger-Hirota [39,40] equation with a truncated M-fractional derivative.

The fractional Schrödinger-Hirota equation with a truncated M-fractional derivative is given as

$$\begin{aligned} &i\mathscr{D}_{M,t}^{\alpha,\beta}\psi+\lambda\mathscr{D}_{M,x}^{2\alpha,\beta}\psi+\delta\mathscr{D}_{M,t}^{\alpha,\beta}\mathscr{D}_{M,x}^{\alpha,\beta}\psi+\rho|\psi|^2\psi+i(a\mathscr{D}_{M,x}^{3\alpha,\beta}\psi+b|\psi|^2\mathscr{D}_{M,x}^{\alpha,\beta}\psi) \\ &=ic\mathscr{D}_{M,x}^{\alpha,\beta}\psi+id\mathscr{D}_{M,x}^{\alpha,\beta}(|\psi|^2\psi)+ie\mathscr{D}_{M,x}^{\alpha,\beta}(|\psi|^2)\psi,\ \ 0<\alpha<1,\ \beta>0,\ i=\sqrt{-1}, \end{aligned} \tag{1.1}$$

where $\psi$ is a complex-valued function of $x$ and $t$. The coefficients $\lambda$, $\delta$ and $a$ are the group velocity and spatio-temporal and third-order dispersions terms, respectively. The parameters $b$ and $e$ are the nonlinear dispersion terms. The coefficients $c$ and $d$ are the inter-modal dispersion and the self steepening terms, respectively [39,40].

For the past two decades, the field of fractional calculus has attracted the attention of many researchers. Nonlinear fractional partial differential equations are used to describe various nonlinear phenomena in nonlinear science. There are several definitions of fractional derivatives available in the literature, such as the Riemann-Liouville, Caputo and Grunwald-Letnikov definitions, Atangana-Baleanu derivative in Caputo sense, Atangana-Baleanu fractional derivative in Riemann-Liouville sense [41, 42], the conformable fractional derivative [43]. Atangana *et. al* [44] presented some new properties to the conformable fractional derivative. Recently, Sousa and Oliveira developed the new truncated M-fractional derivative [45]. This new fractional derivative generalizes the conformable derivative proposed by Khalil *et. al* [43].

## 2 The truncated M-fractional derivative

In this section, some basic definition and theorem about the new truncated M-fractional derivative are given [45].

**Definition 1.** *Let $h:[0,\infty)\to\mathbb{R}$, then the new truncated M-fractional derivative of $h$ of order $\alpha$ is defined as*

$$\mathscr{D}_M^{\alpha,\,\beta}\{(h)(t)\}=\lim_{\varepsilon\to 0}\frac{h(t\mathbb{E}_\beta(\varepsilon t^{1-\alpha}))-h(t)}{\varepsilon},\ \ \forall t>0,\ \ 0<\alpha<1,\ \beta>0, \tag{2.1}$$

*where $\mathbb{E}_\beta(.)$ is a truncated Mittag-Leffler function of one parameter [45].*

**Theorem 1.** *Let $0<\alpha\le 1$, $\beta>0$, $q,\ r\in\mathbb{R}$, and $g,\ h$ $\alpha$-differentiable at a point $t>0$. Then:*

1. $\mathscr{D}_M^{\alpha,\,\beta}\{(qg+rh)(t)\}=q\mathscr{D}_M^{\alpha,\,\beta}\{g(t)\}+r\mathscr{D}_M^{\alpha,\,\beta}\{h(t)\}$.
2. $\mathscr{D}_M^{\alpha,\,\beta}\{(g\,.\,h)(t)\}=g(t)\mathscr{D}_M^{\alpha,\,\beta}\{h(t)\}+h(t)\mathscr{D}_M^{\alpha,\,\beta}\{g(t)\}$.
3. $\mathscr{D}_M^{\alpha,\,\beta}\{\frac{g}{h}(t)\}=\frac{h(t)\mathscr{D}_M^{\alpha,\,\beta}\{g(t)\}-g(t)\mathscr{D}_M^{\alpha,\,\beta}\{h(t)\}}{[h(t)]^2}$.
4. $\mathscr{D}_M^{\alpha,\,\beta}\{c\}=0$, *where $g(t)=c$ is a constant.*
5. *If $g$ is differentiable, then* $\mathscr{D}_M^{\alpha,\,\beta}\{g(t)\}=\frac{t^{1-\alpha}}{\Gamma(\beta+1)}\frac{dg(t)}{dt}$.

## 3 The extended ShGEEM

In this section, the steps of the extended sinh-Gordon equation expansion method are presented.

**Step-1:** Consider the following nonlinear fractional partial differential equation with the new truncated M-fractional derivative:

$$P(\mathscr{D}_{M,x}^{\alpha,\,\beta}\psi,\ \psi^2\mathscr{D}_{M,x}^{2\alpha,\,\beta}\psi,\ \mathscr{D}_{M,t}^{\alpha,\,\beta}\psi,\ \mathscr{D}_{M,t}^{\alpha,\,\beta}\mathscr{D}_{M,x}^{\alpha,\,\beta}\psi,\ \ldots)=0. \tag{3.1}$$

Putting the fractional travelling wave transformation

$$\psi(x,t)=\Phi(\zeta),\quad \zeta=\frac{\Gamma(\beta+1)}{\alpha}\nu(x^{\alpha}-vt^{\alpha}) \tag{3.2}$$

into Eq. (3.1), produces the following nonlinear ordinary differential equation (NODE):

$$D(\Phi,\ \Phi^{'},\ \Phi^{''},\ \Phi^{2}\Phi^{'},\ \ldots)=0, \tag{3.3}$$

**Step-2:** We suppose the trial solution to Eq. (3.3) to be of the form [34]

$$\Phi(\Theta)=\sum_{k=1}^{m}\left[B_k sinh(\Theta)+A_k cosh(\Theta)\right]^{k}+A_0, \tag{3.4}$$

where $A_0,\ A_k,\ B_k\ (k=1,\ 2,\ \ldots,\ m)$ are constants to be determine later and $\Theta$ is a function of $\zeta$ which satisfies the following ordinary differential equations:

$$\Theta^{'}=sinh(\Theta). \tag{3.5}$$

The value of $m$ is determined by using the homogeneous balance principle.

Eq. (3.5) is obtained from the fractional sinh-Gordon equation given by [34]

$$\mathscr{D}_{M,t}^{\alpha,\,\beta}\mathscr{D}_{M,x}^{\alpha,\,\beta}\psi=\gamma sinh(\psi). \tag{3.6}$$

Eq. (3.5) possesses the following solutions [34]:

$$sinh(\Theta)=\pm csch(\zeta)\quad\text{or}\quad sinh(\Theta)=\pm\, i\, sech(\zeta), \tag{3.7}$$

$$cosh(\Theta)=\pm coth(\zeta)\quad\text{or}\quad cosh(\Theta)=\pm\ tanh(\zeta), \tag{3.8}$$

respectively, where $i=\sqrt{-1}$.

**Step-3:** Putting Eq. (3.4), its possible derivatives with the fixed value of $m$ along with Eq. (3.5) into Eq. (3.3), yields an equation in powers of hyperbolic functions; $\Theta^{'l}sinh^{i}(\Theta)cosh^{j}(\Theta)$ $(l=0,1$ and $i,\ j=0,\ 1,\ 2,\ \ldots)$. We collect a set of over-determined nonlinear algebraic equations in $A_0,\ A_k,\ B_k,\ \nu,\ v$ by setting the coefficients of $\Theta^{'l}sinh^{i}(\Theta)cosh^{j}(\Theta)$ to zero.

**Step-4:** The collected set of over-determined nonlinear algebraic equations is then solved with aid of computational software to determine the values of the parameters $A_0,\ A_k,\ B_k,\ \nu,\ v$.

**Step-5:** Based on Eqs. (3.7) and (3.8), Eq. (3.1) possesses the following forms of solutions:

$$\Phi(\zeta)=\sum_{k=1}^{m}\left[\pm iB_k\ sech(\zeta)\pm A_k tanh(\zeta)\right]^k+A_0, \tag{3.9}$$

$$\Phi(\zeta)=\sum_{k=1}^{m}\left[\pm B_k\, csch(\zeta)\pm A_k coth(\zeta)\right]^k+A_0. \tag{3.10}$$

## 4 Application

In this section, we present the application of the ShGEEM to the the Schrödinger-Hirota equation.

Consider equation (Eq. (1.1)) given in section 1.

Substituting the fractional complex wave transformation

$$\psi(x,t)=\Phi(\zeta)e^{i\theta},\ \zeta=\frac{\Gamma(\beta+1)}{\alpha}v(x^{\alpha}-vt^{\alpha}),\ \theta=\frac{\Gamma(\beta+1)}{\alpha}(-\kappa x^{\alpha}+\omega t^{\alpha}+\Omega) \tag{4.1}$$

into Eq. (1.1), we get the following NODE and the constraint conditions:

$$v^2(\lambda-\delta v+3a\kappa)\Phi''-(\omega+\lambda\kappa^2-\delta\kappa\omega+a\kappa^3+c\kappa)\Phi+(\rho+\kappa b-\kappa d)\Phi^3=0, \tag{4.2}$$

$$e=\frac{(b-3d)(\lambda-v\delta)-3a(2kd+\rho)}{2(3ak+\lambda-v\delta)} \tag{4.3}$$

and

$$c=\frac{a(\omega+\kappa(-8\kappa\lambda+v(-3+6\kappa\delta)+2\delta\omega))-8a^2\kappa^3-(\lambda-v\delta)(v+2\kappa\lambda-v\kappa\lambda-\delta\omega)}{2a\kappa+\lambda-v\delta}. \tag{4.4}$$

Balancing the terms $\Phi^3$ and $\Phi''$, yields $m=1$.

With $m=1$, Eq. (3.4) takes the form

$$\Phi(\Theta)=B_1 sinh(\Theta)+A_1 cosh(\Theta)+A_0. \tag{4.5}$$

Substituting Eq. (4.5) and it is second derivative along with Eq. (3.5), gives and equation in powers of hyperbolic functions. We collect the set of over-determined nonlinear algebraic equations as explained in the description of the method. We further simplify the set of algebraic equations to obtained the values of the parameters. To get the solutions of Eq. (1.1), we substitute the values of the parameters into Eqs. (3.9) and (3.10).

**Case-1:** When

$$A_0=0,\ A_1=-\sqrt{\frac{\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega)}{b\kappa-\kappa d+\rho}},\ B_1=A_1,$$

$v=-\sqrt{-\frac{(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega))}{3ak-v\delta+\lambda}}$,

we have the mixed dark-bright optical soliton

$$\psi_1(x,t)=\pm\sqrt{\frac{\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega)}{b\kappa-\kappa d+\rho}}\Bigg(i\,sech\left[\frac{\Gamma(\beta+1)}{\alpha}\nu(x^{\alpha}-vt^{\alpha})\right] +tanh\left[\frac{\Gamma(\beta+1)}{\alpha}\nu(x^{\alpha}-vt^{\alpha})\right]\Bigg)e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\kappa x^{\alpha}+\omega t^{\alpha}+\Omega))}, \tag{4.6}$$

where $(b\kappa-\kappa d+\rho)(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega))>0$ and $(3ak-v\delta+\lambda)(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega))<0$ for valid soliton.

**Case-2:** When

$$A_0=0,\ A_1=-\sqrt{\frac{\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega)}{b\kappa-\kappa d+\rho}},\ B_1=0,$$

$\nu=-\sqrt{\frac{\omega(\kappa\delta-1)-\kappa(c+\kappa(a\kappa+\lambda))}{2(3ak-v\delta+\lambda)}}$,

we have the dark and singular solitons

$$\psi_2(x,t)=\pm\sqrt{\frac{\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega)}{b\kappa-\kappa d+\rho}} \times tanh\left[\frac{\Gamma(\beta+1)}{\alpha}\nu(x^{\alpha}-vt^{\alpha})\right]e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\kappa x^{\alpha}+\omega t^{\alpha}+\Omega))} \tag{4.7}$$

and

$$\psi_3(x,t)=\pm\sqrt{\frac{\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega)}{b\kappa-\kappa d+\rho}} \times coth\left[\frac{\Gamma(\beta+1)}{\alpha}\nu(x^{\alpha}-vt^{\alpha})\right]e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\kappa x^{\alpha}+\omega t^{\alpha}+\Omega))}, \tag{4.8}$$

respectively, where $(b\kappa-\kappa d+\rho)(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega))>0$ and $(3ak-v\delta+\lambda)(\omega(\kappa\delta-1)-\kappa(c+\kappa(a\kappa+\lambda)))>0$ for valid solitons.

**Case-3:** When

$$A_0=0,\ A_1=0,\ B_1=-\sqrt{\frac{-2(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega))}{b\kappa-\kappa d+\rho}},$$

$\nu=-\sqrt{\frac{\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega)}{3ak-v\delta+\lambda}}$,

we have the bright and singular solitons

$$\psi_4(x,t)=\pm\sqrt{\frac{2(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega))}{b\kappa-\kappa d+\rho}} \times sech\left[\frac{\Gamma(\beta+1)}{\alpha}\nu(x^{\alpha}-vt^{\alpha})\right]e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\kappa x^{\alpha}+\omega t^{\alpha}+\Omega))} \tag{4.9}$$

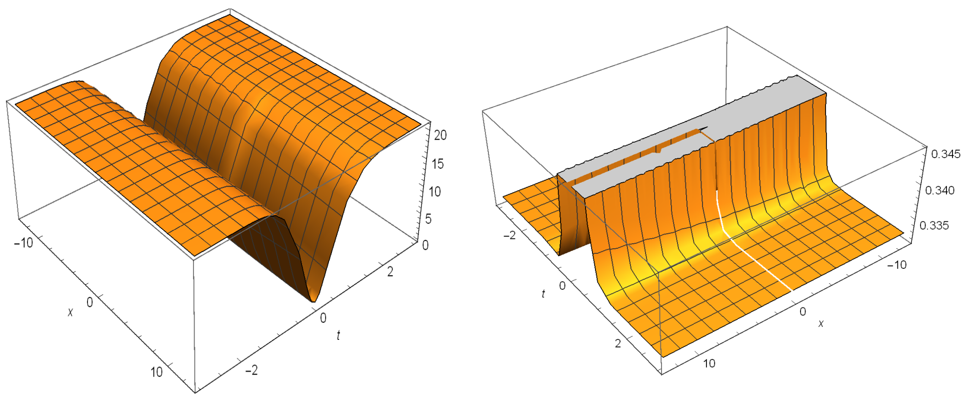
and

$$\psi_5(x,t) = \pm\sqrt{-\frac{2(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega))}{b\kappa-\kappa d+\rho}} \times csch\left[\frac{\Gamma(\beta+1)}{\alpha}v(x^{\alpha}-vt^{\alpha})\right]e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\kappa x^{\alpha}+\omega t^{\alpha}+\Omega))}, \tag{4.10}$$

respectively, where $(3ak-v\delta+\lambda)(\omega+\kappa(c+\kappa(a\kappa+\lambda)-\delta\omega)) > 0$ for valid solitons.

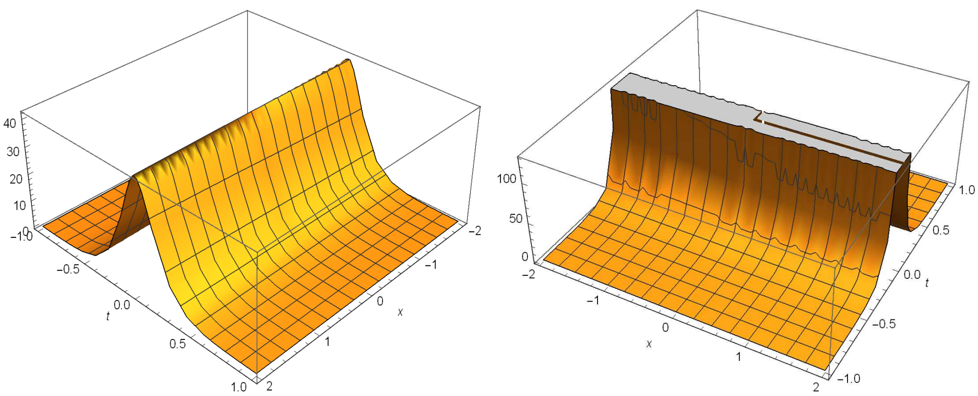
## 5 Physical representation of the reported results

In order to have clear and good understanding of the physical properties of the constructed dark, bright and singular soliton solutions, under the choice of the suitable values of parameters and good choice of the fractional value of $\alpha$, the 3-dimensional graphs are plotted. The perspective view of the dark, bright and singular solitons can be seen from the (a) part of figs. 1, 2, and (a), (b) parts of figs. 1 and 2, respectively.



(a) (b)

**Fig. 1** The 3D surfaces of (a) Eq. (4.7) and (b) Eq. (4.8) with fractional value $\alpha = 0.9$.



(a) (b)

**Fig. 2** The 3D surfaces of (a) Eq. (4.9) and (b) Eq. (4.10) with fractional value $\alpha = 0.9$.

## 6 Conclusions

In this study, the dark, bright, mixed dark-bright and singular optical solitons to the fractional Schrödinger-Hirota equation with a truncated M-fractional derivative are successfully revealed by using the extended sinh-Gordon equation expansion method. The truncated M-fractional derivative is a generalized form of the conformable fractional derivative. The definition of the new fractional derivative is smoothly used in transforming the fractional Schrödinger-Hirota equation to nonlinear ordinary differential equation. The reported results may be useful in explaining the physical meaning of the studied nonlinear model. The extended sinh-Gordon equation expansion method is powerful technique in obtaining wave solutions to various complex fractional nonlinear model.

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