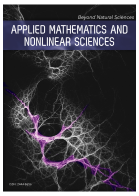
CrossMark

sciendo Applied Mathematics and Nonlinear Sciences 4(2) (2019) 513–522



Applied Mathematics and Nonlinear Sciences

https://www.sciendo.com

The new extended rational SGEEM for construction of optical solitons to the (2+1)–dimensional Kundu–Mukherjee–Naskar model

Tukur Abdulkadir Sulaiman[1,2], Hasan Bulut[2,3]

[1] Department of Mathematics, Federal University Dutse, Jigawa, Nigeria

[2] Department of Mathematics, Firat University, Elazig, Turkey

[3] Department of Mathematics Education, Final International University, Kyrenia, Cyprus

Submission Info

Communicated by Juan Luis García Guirao
Received September 30th 2019
Accepted October 16th 2019
Available online December 24th 2019

Abstract

This work proposes the new extended rational sinh-Gordon equation expansion technique (SGEEM). The computational approach is formulated based on the well-known sinh-Gordon equation. The proposed technique generalizes the sine-Gordon/sinh-Gordon expansion methods in a rational format. The efficiency of the suggested technique is tested on the (2+1)鈥揹imensional Kundu鈥揗ukherjee鈥揘askar (KMN) model. Various of optical soliton solutions have been obtained using this new method. The conditions which guarantee the existence of valid solitons are given.

Keywords: New extended rational SGEEM; KMN model; optical solitons.
AMS 2010 codes: 49K20.

1 Introduction

various complicated nonlinear physical phenomena can be expressed in the form of nonlinear partial dierential equations (NPDEs). Nonlinear Schrödinger's type equations (NLSEs) are unique types of nonlinear partial differential equations which are complex in nature. Such equations can be used to define various nonlinear physical aspects such as plasma physics, fluid dynamics, photonics, quantum electronics, and electromagnetism [1–5]. One of the important and hot subjects for investigating soliton propagation through nonlinear optical fibers is the theory of optical solitons [6]. The spread of ultrashort pulses of electromagnetic radiation in a nonlinear medium is a multidimensional phenomenon. The interaction between various physical elements such as material dispersion, diffraction and nonlinear reaction affect the dynamics of the pulse [6]. This area has drawn the attention of many scientists for more than two decades. Different computational methods have been used to reveal solutions of various type of NLEEs such as the modified $\exp(-\Psi(\eta))$-expansion function method [7–9], the first integral method [10,11], the improved Bernoulli sub-equation function method [12,13], the trial solution

sciendo
ISSN 2444-8656 doi:10.2478/AMNS.2019.2.00048 sciendo

method [14, 15], the new auxiliary equation method [16], the extended simple equation method [17], the solitary wave ansatz method [18], the functional variable method [19] and several others [20–39].

However, a novel extended rational sinh-Gordon equation expansion technique is developed in this research. The new approach is based on the well-known sine-Gordon and sinh-Gordon equations. We employ the new approach to the (2+1)欽揹imensional Kundu欽揗ukherjee欽揘askar model [40] in generating various optical solitons.

The (2+1)欽揹imensional KMN model is given by [40]

$$i\Theta_t + \alpha\Theta_{xy} + i\beta\Theta(\Theta\Theta_x^* - \Theta^*\Theta_x) = 0. \tag{1.1}$$

In Eq. (1.1), the unknown function $\Theta(x,y,t)$ stands for the nonlinear wave envelope. The nonzero constants a and b are the coefficients of the dispersion term and the term that is different from conventional Kerr law nonlinearity or any known non-Kerr law media. The first term Θ_t represents the temporal evolution of the wave. Eq. (1.1) describes the oceanic rogue waves as well as hole waves. It may also be used in describing optical wave propagation through coherently excited resonant wave guides that is doped with Erbium atoms [40, 41].

2 Analysis of the Method

In this section, we give the description of the novel extended rational sinh-Gordon equation expansion technique.

Consider the following sinh-Gordon equation [42]

$$\Theta_{xt} = \gamma \sinh(\Theta). \tag{2.1}$$

One may recall the following as the solutions of Eq. (2.1) [43]:

$$\sinh(\Omega) = \pm\,\mathrm{csch}(\Delta) \quad \text{or} \quad \sinh(\Omega) = \pm\, i\,\mathrm{sech}(\Delta), \tag{2.2}$$

$$\cosh(\Omega) = \pm\coth(\Delta) \quad \text{or} \quad \cosh(\Omega) = \pm\ \tanh(\Delta), \tag{2.3}$$

$$\sinh(\Omega) = \tan(\Delta) \quad \text{or} \quad \sinh(\Omega) = -\cot(\Delta) \tag{2.4}$$

and

$$\cosh(\Omega) = \pm\sec(\Delta) \quad \text{or} \quad \cosh(\Omega) = \pm\ \tan(\Delta), \tag{2.5}$$

where $i = \sqrt{-1}$ and $\Omega' = \sinh(\Omega)$, or $\Omega' = \cosh(\Omega)$. For details, see [43].

Consider the nonlinear partial differential equation

$$P(\Theta_x,\ \Theta^2\Theta_{xx},\ \Theta_{xxx}\ \ldots) = 0, \tag{2.6}$$

where the subscript represents the partial derivative of Θ with respect to x.

Substituting the travelling wave transformation

$$\Theta(x,t) = \Theta(\Delta), \quad \Delta = \omega(x - \mu t) \tag{2.7}$$

into Eq. (2.6), the following nonlinear ordinary differential (NODE) is obtained:

$$D(\Theta,\, \Theta^{'},\, \Theta^{''},\, \Theta^2\Theta^{'},\, \ldots) = 0, \tag{2.8}$$

where the superscript indicates the derivative of the function Θ with respect to Δ.

The general steps of the new generalized rational sinh-Gordon equation expansion method are given as follows:

Step-I: Suppose that Eq. (2.6) adopts the following form of rational solution:

$$\Theta(\Omega) = \frac{\sum_{j=1}^{m} \left[b_j \sinh(\Omega) + a_j \cosh(\Omega)\right]^j + a_0}{\sum_{j=1}^{m} \left[c_j \sinh(\Omega) + d_j \cosh(\Omega)\right]^j + c_0}. \tag{2.9}$$

Step II: The unknown parameters involved are obtained by substituting Eq. (2.9) along with $\Omega' = \sinh(\Omega)$ and/or $\Omega' = \cosh(\Omega)$ into Eq. (2.8). This produces a polynomial in powers of hyperbolic functions. Summing the coefficients of these hyperbolic functions of the same power, provides a group of algebraic equations after equating each summation to zero.

Step III: The solutions of Eq. (2.6) are reached by inserting the values of the unknown parameters into the following rational solutions formed from Eqs. (2.2), (2.3), (2.4) and (2.5), respectively:

$$\Theta(\Delta) = \frac{\sum_{j=1}^{m} \left[\pm b_j i \,\mathrm{sech}(\Delta) \pm a_j \tanh(\Delta)\right]^j + a_0}{\sum_{j=1}^{m} \left[\pm c_j i \,\mathrm{sech}(\Delta) \pm d_j \tanh(\Delta)\right]^j + c_0}. \tag{2.10}$$

$$\Theta(\Delta) = \frac{\sum_{j=1}^{m} \left[\pm b_j \; csch(\Delta) \pm a_j coth(\Delta)\right]^j + a_0}{\sum_{j=1}^{m} \left[\pm c_j \; csch(\Delta) \pm d_j coth(\Delta)\right]^j + c_0}, \tag{2.11}$$

$$\Theta(\Delta) = \frac{\sum_{j=1}^{m} \left[b_j \; tan(\Delta) \pm a_j sec(\Delta)\right]^j + a_0}{\sum_{j=1}^{m} \left[c_j \; tan(\Delta) \pm d_j sec(\Delta)\right]^j + c_0} \tag{2.12}$$

and

$$\Theta(\Delta) = \frac{\sum_{j=1}^{m} \left[-b_j \; cot(\Delta) \pm a_j tan(\Delta)\right]^j + a_0}{\sum_{j=1}^{m} \left[-c_j \; cot(\Delta) \pm d_j tan(\Delta)\right]^j + c_0}. \tag{2.13}$$

3 Applications

In this section, we give the applications of the new extended rational sinh-Gordon equation expansion method.

Consider the complex wave transformation

$$\begin{aligned} \Theta(x,y,t) = \Theta(\Delta)e^{i\theta},\; \Theta^*(x,y,t) = \Theta(\Delta)e^{-i\theta}\; \Delta &= \omega(-\sigma_1 x - \sigma_2 y - \mu t), \\ \theta &= -\gamma_1 x - \gamma_2 y + \kappa t + \phi. \end{aligned} \tag{3.1}$$

In Eq. (3.1), γ_1 and γ_2 are the frequencies of the solitons in $x-$ and y-directions, respectively. The constant μ stands for the velocity of the soliton. The parameter κ is the phase constant. The parameters σ_1 and σ_2 stand for the inverse width of the soliton along $x-$ and $y-$directions, respectively [40].

Substituting Eq. (3.1) into (1.1), gives

$$\omega^2\alpha\sigma_1\sigma_2\Theta'' - (\kappa+\alpha\gamma_1\gamma_2)\Theta - 2\beta\gamma_1\Theta^3 = 0 \tag{3.2}$$

from the real part, and

$$\mu = \alpha(\gamma_1\sigma_2+\gamma_2\sigma_1) \tag{3.3}$$

from the real part.

Balancing the terms Θ'' and Θ^3, yields $m = 1$.

With $m = 1$, Eq. (2.9) turns to

$$\Theta(\Omega) = \frac{b_1\sinh(\Omega)+a_1\cosh(\Omega)+a_0}{c_1\sinh(\Omega)+d_1\cosh(\Omega)+c_0}. \tag{3.4}$$

Substituting Eq. (3.4) along with $\Omega' = \sinh(\Omega)$ and/or $\Omega' = \cosh(\Omega)$ into Eq. (3.2), yields a polynomial in powers of hyperbolic functions. Collecting the coefficients of the hyperbolic functions of the same power and equating each summation to zero, yields a system of algebraic equations. Solving the system of algebraic equations, produces the values of a_1, b_1, c_1, d_1, and the other parameters involved. Substituting the values of the parameters into Eqs. (2.10)-(2.13) with fixed value of $m = 1$, gives some new results to Eq. (1.1).

Set-1: When

$$a_0 = d_1\sqrt{-\frac{(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}},\ c_0 = a_1\sqrt{-\frac{2\beta\gamma_1}{(\kappa+\alpha\gamma_1\gamma_2)}},\ \omega = -\sqrt{-\frac{2(\kappa+\alpha\gamma_1\gamma_2)}{\alpha\sigma_1\sigma_2}},$$

$b_1 = -i\sqrt{\frac{(c_1^2-d_1^2)(\kappa+\alpha\gamma_1\gamma_2)-2\beta a_1^2\gamma_1}{2\beta\gamma_1}}$, we have the following mixed dark-bright soliton:

$$\Theta_{1.1}(x,y,t) = \frac{\left(d_1\sqrt{-\frac{(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}}+\sqrt{\frac{(c_1^2-d_1^2)(\kappa+\alpha\gamma_1\gamma_2)-2\beta a_1^2\gamma_1}{2\beta\gamma_1}}\operatorname{sech}[\Delta]+a_1\tanh[\Delta]\right)}{ic_1\operatorname{sech}[\Delta]+d_1\tanh[\Delta]+a_1\sqrt{-\frac{2\beta\gamma_1}{(\kappa+\alpha\gamma_1\gamma_2)}}}e^{i\theta}, \tag{3.5}$$

and the mixed singular soliton

$$\Theta_{1.2}(x,y,t) = \frac{\left(a_1\coth[\Delta]+d_1\sqrt{-\frac{(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}}+\sqrt{\frac{(c_1^2-d_1^2)(\kappa+\alpha\gamma_1\gamma_2)-2\beta a_1^2\gamma_1}{2\beta\gamma_1}}i\operatorname{csch}[\Delta]\right)}{c_1\operatorname{csch}[\Delta]+d_1\coth[\Delta]+a_1\sqrt{-\frac{2\beta\gamma_1}{(\kappa+\alpha\gamma_1\gamma_2)}}}e^{i\theta}, \tag{3.6}$$

Set-2: When

$$a_0 = d_1\sqrt{-\frac{(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}},\ c_0 = a_1\sqrt{-\frac{2\beta\gamma_1}{(\kappa+\alpha\gamma_1\gamma_2)}},\ b_1 = 0,\ c_1 = 0,\ \omega = \sqrt{-\frac{(\kappa+\alpha\gamma_1\gamma_2)}{2\alpha\sigma_1\sigma_2}},$$

we get the following dark soliton

$$\Theta_{2.1}(x,y,t) = \frac{\left(d_1\sqrt{-\frac{(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}} + a_1\tanh[\Delta]\right)}{a_1\sqrt{-\frac{2\beta\gamma_1}{(\kappa+\alpha\gamma_1\gamma_2)}} + d_1\tanh[\Delta]}e^{i\theta}, \tag{3.7}$$

and the singular soliton

$$\Theta_{2.2}(x,y,t) = \frac{\left(a_1\coth[\Delta] + d_1\sqrt{-\frac{(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}}\right)}{a_1\sqrt{-\frac{2\beta\gamma_1}{(\kappa+\alpha\gamma_1\gamma_2)}} + d_1\coth[\Delta]}e^{i\theta}. \tag{3.8}$$

Set-3: When

$$a_0 = c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}},\ c_0 = -b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}},\ \omega = -\sqrt{\frac{2(\kappa+\alpha\gamma_1\gamma_2)}{\alpha\sigma_1\sigma_2}},$$

$a_1 = -\sqrt{\frac{2\beta b_1^2\gamma_1+(c_1^2-d_1^2)(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}}$, we get the following trigonometric functions solution:

$$\Theta_{3.1}(x,y,t) = \frac{\left(b_1\tan[\Delta] - -\sqrt{\frac{2\beta b_1^2\gamma_1+(c_1^2-d_1^2)(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}}\sec[\Delta] + c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}}\right)}{d_1\sec[\Delta] + c_1\tan[\Delta] - b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}}}e^{i\theta}. \tag{3.9}$$

Set-4: When

$$a_0 = -c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}},\ c_0 = b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}},\ a_1 = 0,\ d_1 = 0,\ \omega = -\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_2}{2\alpha\sigma_1\sigma_2}},$$

we get the following trigonometric function solution:

$$\Theta_{4.1}(x,y,t) = \frac{\left(b_1\tan[\Delta] - c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}}\right)}{c_1\tan[\Delta] + b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}}}e^{i\theta}. \tag{3.10}$$

Set-5: When

$$a_0 = c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}},\ c_0 = -b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}},\ \omega = -\sqrt{\frac{2(\kappa+\alpha\gamma_1\gamma_2)}{\alpha\sigma_1\sigma_2}},$$

$a_1 = -\sqrt{\frac{2\beta b_1^2\gamma_1+(c_1^2-d_1^2)(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}}$, we get the following trigonometric functions solution:

$$\Theta_{5.1}(x,y,t)=\frac{\left(b_1 cot[\Delta]--\sqrt{\frac{2\beta b_1^2\gamma_1+(c_1^2-d_1^2)(\kappa+\alpha\gamma_1\gamma_2)}{2\beta\gamma_1}}\csc[\Delta]+c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}}\right)}{d_1 csc[\Delta]+c_1\cot[\Delta]-b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}}}e^{i\theta}. \tag{3.11}$$

Set-6: When

$$a_0=-c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}},\ c_0=b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}},\ a_1=0,\ d_1=0,\ \omega=-\sqrt{\frac{2(\kappa+\alpha\gamma_1\gamma_2)}{\alpha\sigma_1\sigma_2}},$$

we get the following trigonometric functions solution:

$$\Theta_{6.1}(x,y,t)=-\frac{\left(b_1\cot[\Delta]+c_1\sqrt{\frac{\kappa+\alpha\gamma_1\gamma_1}{2\beta\gamma_1}}\right)}{b_1\sqrt{\frac{2\beta\gamma_1}{\kappa+\alpha\gamma_1\gamma_2}}-c_1\cot[\Delta]}e^{i\theta}. \tag{3.12}$$

Remarks: Solutions (3.5)-(3.8) are valid only for $\alpha\sigma_1\sigma_2(\kappa+\alpha\gamma_1\gamma_2)<0$, and solutions (3.9)-(3.12) are valid only for $\alpha\sigma_1\sigma_2(\kappa+\alpha\gamma_1\gamma_2)>0$.

4 Results and Discussion

In this study, We succeeded in formulating the extended rational sinh-Gordon equation expansion technique. The developed method is employed to the (2+1)鈥揹imensional Kundu鈥揗ukherjee鈥揘askar model to test its efficiency. Mixed dark-bright, singular solitons and trigonometric functions solutions are successfully constructed.

Recently, Yamgoue *et al.* [44] introduced the rational sine-Gordon expansion method. The authors introduced the following trial solution which was generated from the sine-Gordon equation [45, 46]:

$$\Theta(\Delta)=\frac{\sum_{j=1}^m tanh^{j-1}(\Delta)\left[b_j\ sech(\Delta)+a_j tanh(\Delta)\right]+a_0}{\sum_{j=1}^m tanh^{j-1}(\Delta)\left[c_j\ sech(\Delta)+d_j tanh(\Delta)\right]+c_0}. \tag{4.1}$$

In this study, we come up with the following sets of trial solutions that were generated from the sinh-Gordon equation [42]:

$$\Theta(\Delta)=\frac{\sum_{j=1}^m\left[\pm b_j\ i\mathrm{sech}(\Delta)\pm a_j\tanh(\Delta)\right]^j+a_0}{\sum_{j=1}^m\left[\pm c_j\ i\mathrm{sech}(\Delta)\pm d_j\tanh(\Delta)\right]^j+c_0}, \tag{4.2}$$

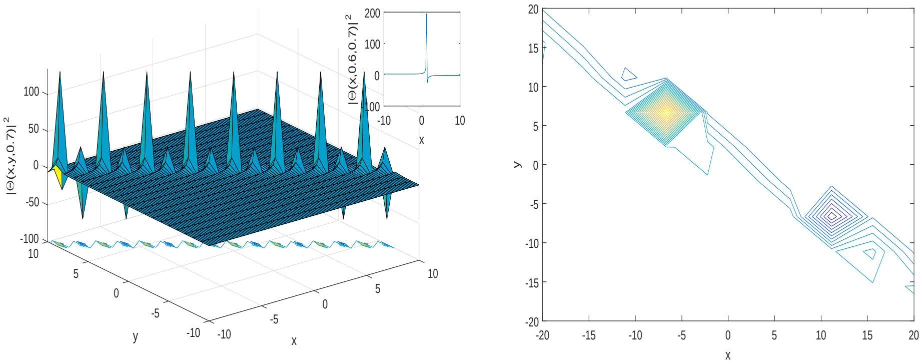
$$\Theta(\Delta)=\frac{\sum_{j=1}^m\left[\pm b_j\ \mathrm{csch}(\Delta)\pm a_j\coth(\Delta)\right]^j+a_0}{\sum_{j=1}^m\left[\pm c_j\ \mathrm{csch}(\Delta)\pm d_j\coth(\Delta)\right]^j+c_0}, \tag{4.3}$$

$$\Theta(\Delta)=\frac{\sum_{j=1}^m\left[b_j\ \tan(\Delta)\pm a_j\sec(\Delta)\right]^j+a_0}{\sum_{j=1}^m\left[c_j\ \tan(\Delta)\pm d_j\sec(\Delta)\right]^j+c_0} \tag{4.4}$$

and

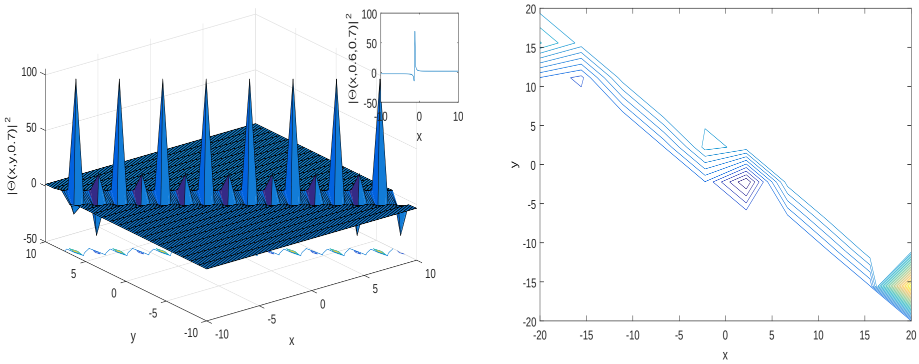
$$\Theta(\Delta)=\frac{\sum_{j=1}^m\left[-b_j\ \cot(\Delta)\pm a_j\tan(\Delta)\right]^j+a_0}{\sum_{j=1}^m\left[-c_j\ \cot(\Delta)\pm d_j\tan(\Delta)\right]^j+c_0}. \tag{4.5}$$

These newly introduced trial solutions generalize all the kind of solutions that may be obtained by using the sine- and sinh-Gordon expansion methods in rational format.



(a) (b)

Figure 1 The (a) 2-, 3-dimensional and (b) contour surfaces of Eq. (3.5).



(a) (b)

Figure 2 The (a) 2-, 3-dimensional and (b) contour surfaces of Eq. (3.7).

5 Conclusion

In this research, the extended rational sinh-Gordon equation expansion method is developed. The newly developed technique gives variety of wave solutions when tested on the (2+1)鈥揹imensional Kundu鈥揗ukherjee鈥揘askar model. Dark, mixed dark-bright, singular, mixed singular solitons and trigonometric functions solutions are successfully constructed. The conditions which guarantee the existence of the valid solutions to this model are given. The 2- , 3-dimensional and contour graphs to this model are plotted. It is known that dark soliton describes the solitary waves with lower intensity than the background, bright soliton describes the solitary waves whose peak intensity is larger than the background [**?**]. The singular soliton solutions is a solitary wave with discontinuous derivatives; instances of such solitary waves are compactions, which have finite (compact) assistance, and peakons, whose peaks have a discontinuous first derivative [48, 49]. The generalized sinh-Gordon equation expansion method is efficient and powerful mathematical tool which may be used in generating varieties of wave solutions to different kind of nonlinear wave equations.

References

[1] H. Bulut, T.A. Sulaiman and B. Demirdag, *Dynamics of soliton solutions in the chiral nonlinear Schr***ö***dinger equations*, Nonlinear Dynamics, **91(3)** 1985-1991 (2018)
[2] A. Biswas, M. Ekici, A. Sonmezoglu, Q. Zhou, S.P. Moshokoa and M. Belic, *Optical soliton perturbation with full nonlinearity for Kundu-Eckhaus equation by extended trial function scheme*, Optik-Int. J. Light and Electron Optics, **160** 17-23 (2018)
[3] T.A. Sulaiman, T. Akturk, H. Bulut and H.M. Baskonus, *Investigation of various soliton solutions to the Heisenberg ferromagnetic spin chain equation*, Journal of Electromagnetic Waves and Applications, https://doi.org/10.1080/09205071.2017.1417919 1-13 (2017)
[4] A. Biswas, Q. Zhou, M.Z. Ullah, H. Triki, S.P. Moshokoa and M. Belic, *Optical soliton perturbation with anti-cubic nonlinearity by semi-inverse variational principle*, Optik-International Journal for Light and Electron Optics, **143** 131-134 (2017)
[5] M. Eslami and A. Neirameh, *New exact solutions for higher order nonlinear Schr***ö***dinger equation in optical fibers*, Optical and Quantum Electronics, **50(1)** 47 (2018)
[6] K. Ali, S.T.R.a Rizvi, B. Nawaz and M. Younis, *Optical solitons for paraxial wave equation in Kerr media*,Modern Physics Letters B, 1950020 (2018)
[7] S. Duran, M. Askin and T.A. Sulaiman, *New soliton properties to the ill-posed Boussinesq equation arising in nonlinear physical science*, IJOCTA, **7(3)** (2017) 240-247
[8] A. Yokus, H.M. Baskonus, T.A. Sulaiman and H. Bulut, *Numerical simulations and solutions of the two component second order KdV evolutionary system*, Numerical Methods of Partial Differential Equations, **34(1)** 211-227 (2018)
[9] H.M. Baskonus, H. Bulut and A. Atangana, *On the Complex and Hyperbolic Structures of Longitudinal Wave Equation in a Magneto-Electro-Elastic Circular Rod*, Smart Materials and Structures, **25(3)** 035022 (2016)
[10] M. Eslami, F.S. Khodadad, F. Nazari and H. Rezazadeh, *The first integral method applied to the Bogoyavlenskii equations by means of conformable fractional derivative*, Optical and Quantum Electronics, **49(12)** 391 (2017)
[11] M. Eslami, M. Mirzazadeh, B.F. Vajargah and A. Biswas, *Optical solitons for the resonant nonlinear Schr***ö***dinger's equation with time-dependent coefficients by the first integral method*, Optik, **125(13)** 3107-3116 (2014)
[12] H.M. Baskonus and H. Bulut, *Exponential Prototype Structure for (2+1)-Dimensional Boiti-Leon-Pempinelli systems in Mathematical Physics*, Waves in Random and Complex Media, **26(2)** 189-196 (2016)
[13] H. Bulut, H.A. Isik and T.A. Sulaiman, *On Some Complex Aspects of the (2+1)-dimensional Broer-Kaup-Kupershmidt System*, ITM Web of Conferences, **13** 01019 (2017)
[14] M. Eslami, *Trial solution technique to chiral nonlinear Schrodinger's equation in (1+2)-dimensions*, Nonlinear Dynamics, **85(2)** 813-816 (2016)
[15] A. Biswas, M. Mirzazadeh, M. Eslami, Q. Zhou, A. Bhrawy and M. Belic, *Optical solitons in nano-fibers with spatio-temporal dispersion by trial solution method*, Optik, **127(18)** 7250-7257 (2016)
[16] M.M.A. Khater, A.R. Seadawy and D. Lu, *Optical soliton and rogue wave solutions of the ultra-short femto-second pulses in an optical fiber via two different methods and its applications*, Optik, **158** 434-450 (2018)
[17] D. Lu, A.R. Seadawy and M.M.A. Khater, *Dispersive optical soliton solutions of the generalized Radhakrishnan-Kundu-Lakshmanan dynamical equation with power law nonlinearity and its applications*, Optik, **164** 54-64 (2018)
[18] A.H. Bhrawy, A.A. Alshaery, E.M. Hilal, Z. Jovanoski and A. Biswas, *Bright and dark solitons in a cascaded system*, Optik, **125** 6162-6165 (2014)
[19] H. Aminikhah, A.H. Sheikhani and H. Rezazadeh, *Travelling wave solutions of nonlinear systems of PDEs by using the functional variable method*, Boletim da Sociedade Paranaense de Matematica, **34(2)** 213-229 (2015)
[20] A.R. Seadawy, *Modulation instability analysis for the generalized derivative higher order nonlinear Shr***ö***dinger equation and its the bright and dark soliton solutions*, Journal of Electromagnetic Waves and Applications, **31** 1353-1362 (2017)
[21] Q. Zhou, M. Ekici, M. Mirzazadeh and A. Sonmezoglu, *The investigation of soliton solutions of the coupled sine-Gordon equation in nonlinear optics*, J. Mod. Opt., **64(16)** 1677-1682 (2017)
[22] Q. Zhou, C. Wei, H. Zhang, J. Lu, H. Yu, P. Yao and Q. Zhu, *Exact Solutions to the Resonant Nonlinear Schr***ö***dinger Equation with Both Spatio-Temporal and Inter-Modal Dispersions*, Proceedings of the Romanian Academy, Series A, **17(4)** 307-313 (2016)
[23] H. Bulut, T.A. Sulaiman, H.M. Baskonus and T. Akturk, *On the bright and singular optical solitons to the (2+1)-dimensional NLS and the Hirota equations*, Opt Quant Electron, **50** 134 (2018)
[24] Q. Zhou and A. Biswas, *Optical solitons in parity-time-symmetric mixed linear and nonlinear lattice with non-Kerr law nonlinearity*, Superlattices Microstruct., **109** 588-598 (2017)
[25] H.M. Baskonus, T.A. Sulaiman and H. Bulut, *Novel complex and hyperbolic forms to the strain wave equation in microstructured solids*, Optical and Quantum Electronics, **50(1)** 14 (2018)
[26] H.M. Baskonus, T.A. Sulaiman and H. Bulut, *Bright, dark optical and other solitons to the generalized higher-order NLSE in optical fibers*, Optical and Quantum Electronics, **50(6)** 253 (2018)

[27] T.A. Sulaiman, G. Yel and H. Bulut, *M-fractional solitons and periodic wave solutions to the Hirota-Maccari system*, Modern Physics Letters B, **33(05)** 1950052 (2019)
[28] I. Bendahmane, H. Triki, A. Biswas, A.S. Alshomrani, Q. Zhou, S.P. Moshokoa and M. Belic, *Bright, dark and W-shaped solitons with extended nonlinear Schr***ö***dinger's equation for odd and even higher-order terms*, Superlattices Microstruct., **114** 53-61 (2018)
[29] A. Sonmezoglu, M. Yao, M. Ekici, M. Mirzazadeh and Q. Zhou, *Explicit solitons in the parabolic law nonlinear negative-index materials*, Nonlinear Dynamics, **88(1)** 595-607 (2017)
[30] R. Yilmazer and E. Bas, *Explicit Solutions of Fractional Schr***ö***dinger Equation via Fractional Calculus Operators*, Int. J. Open Problems Compt. Math., **5(2)** 133-141 (2012)
[31] R. Yilmazer and E. Bas, *Explicit Solutions of Fractional Schr***ö***dinger Equation via Fractional Calculus Operators*, Int. J. Open Problems Compt. Math., **5(2)** 133-141 (2012)
[32] F.S. Khodadad, F. Nazari, M.Eslami and H. Rezazadeh, *Soliton solutions of the conformable fractional Zakharov-Kuznetsov equation with dual-power law nonlinearity*, Optical and Quantum Electronics, **49(11)** 384 (2017)
[33] H. Rezazadeh, M.S. Osman, M. Eslami, M. Ekici, A. Sonmezoglu, M. Asma and A. Biswas, *Mitigating Internet bottleneck with fractional temporal evolution of optical solitons having quadratic-cubic nonlinearity*, Optik, **164** 84-92 (2018)
[34] R. Yilmazer, *N-Fractional Calculus Operator* N^{μ} *Method to a Modified Hydrogen Atom Equation*, Math Commun., **15** 489-501 (2010)
[35] K.A. Touchent, Z. Hammouch, T. Mekkaoui and C. Unlu, *A Boiti-Leon Pimpinelli equations with time-conformable derivative*, IJOCTA, **9(3)** 95-101 (2019)
[36] D. Bienvenue, B. Gambo, J. Mibaille, Z. Hammouch, and A. Houwe, *Chirped Solitons with Fractional Temporal Evolution in Optical Metamaterials*, Methods of Mathematical Modelling: Fractional Differential Equations, 205 (2019)
[37] Z. Hammouch, T. Mekkaoui and P. Agarwal, *Optical solitons for the Calogero-Bogoyavlenskii-Schiff equation in (2+1) dimensions with time-fractional conformable derivative*, The European Physical Journal Plus, **133** 258 (2018)
[38] T.A. Sulaiman, H.n Bulut, A. Yokus and H.M. Baskonus, *On the exact and numerical solutions to the coupled Boussinesq equation arising in ocean engineering*, Indian Journal of Physics, **93(5)** 647-656 (2019)
[39] O.A. Ilhan, H. Bulut, T.A. Sulaiman and H.M. Baskonus, *Dynamic of solitary wave solutions in some nonlinear pseudoparabolic models and Dodd-Bullough-Mikhailov equation*, Indian Journal of Physics, **93(8)** 999-1007 (2018)
[40] M. Ekici, A. Sonmezoglua, A. Biswas and M.R. Belic, *Optical solitons in (2+1)-Dimensions with Kundu-Mukherjee-Naskar equation by extended trial function scheme*, Chinese Journal of Physics, **57** 72-77 (2019)
[41] A. Biswas, Y. Yildirim, E. Yasar, Q. Zhou, S.P. Moshokoa and M. Belic, *Optical soliton perturbation with quadratic-cubic nonlinearity using a couple of strategic algorithms*, Chinese Journal of Physics, **56** 1990-1998 (2018)
[42] Z. Yan, *A sinh-Gordon equation expansion method to construct doubly periodic solutions for nonlinear differential equations*, Chaos, Solitons and Fractals, 16(2) (2003), 291-297.
[43] X. Xian-Lin and T. Jia-Shi, *Travelling Wave Solutions for Konopelchenko-Dubrovsky Equation Using an Extended sinh-Gordon Equation Expansion Method*, Commun. Theor. Phys., 50 (2008), 1047.
[44] S.B. Yamgoue, G.R. Deffo and F.B. Pelap, *A new rational sine-Gordon expansion method and its application to nonlinear wave equations arising in mathematical physics*, Eur. Phys. J. Plus, 134 (2019), 380.
[45] C. Yan, *A Simple Transformation for Nonlinear Waves*, Physics Letters A, 22(4) (1996), 77-84.
[46] H. Bulut, T.A. Sulaiman and H.M. Baskonus, *New solitary and optical wave structures to the Korteweg-de Vries equation with dual-power law nonlinearity*, Optical and Quantum Electronics, 48 (2016), 564.
[47] A.C. Scott, *Encyclopedia of Nonlinear Science, Routledge*, Taylor and Francis Group, New York, (2005).
[48] P. Rosenau, *What is a Compacton?*, Notices of the American Mathematical Society, 52(7), 738-739 (2005).
[49] M. Onorato, S. Residori and F. Baronio, *Rogue and Shock Waves in Nonlinear Dispersive Media*, Springer, Switzerland, (2016).

This page is intentionally left blank