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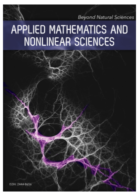

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Investigation of A Fuzzy Problem by the Fuzzy Laplace Transform

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Abstract

This paper is on the solutions of a fuzzy problem with triangular fuzzy number initial values by fuzzy Laplace transform. In this paper, the properties of fuzzy Laplace transform, generalized differentiability and fuzzy arithmetic are used. The example is solved in relation to the studied problem. Conclusions are given.

Keywords: Fuzzy logic, Fuzzy differential equation, Fuzzy Laplace Transform.
AMS 2010 codes: 03E72, 44A10.

1 Introduction

Many researchers study fuzzy logic [1,2]. Zadeh [3] and Dubois and Prade [4] introduced fuzzy number and fuzzy arithmetic. Also, Kandel and Byatt [5] introduced the term ”fuzzy differential equation”. Firstly, Chang and Zadeh introduced the concept of fuzzy derivative [6]. Dubois and Prade [7] followed up their approach. Other methods were studied in several papers [8–12].

Fuzzy differential equations are important topic many fields. For example, population models [13] , civil engineering [14], population dynamics model [15], growth model [16].

To solve fuzzy differential equation is useful by fuzzy Laplace transform. Firstly, Allahviranloo and Ahmadi introduced fuzzy Laplace transform [17]. To solve problems in many areas of fuzzy differential equation, fuzzy Laplace transform was used in many papers [18–22].

In this paper, the solutions of a fuzzy problem with triangular fuzzy number initial values are investigated by fuzzy Laplace transform. Generalized differentiability, fuzzy arithmetic are used. Purpose of this study is to investigate solutions using fuzzy Laplace transform for the studied problem.

It is given in section 2 preliminaries, in section 3 findings and main results, in section 4 conclusions.

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2 Preliminaries

Definition 1. *[23] A fuzzy number is a mapping $u : \mathbb{R} \to [0,1]$ satisfying the properties $\overline{\{x \in \mathbb{R} \mid u(x) > 0\}}$ is compact, u is normal, u is convex fuzzy set, u is upper semi-continuous on $\mathbb{R}$.*

Let $\mathbb{R}_F$ show the set of all fuzzy numbers.

Definition 2. *[24] Let be $u \in \mathbb{R}_F$. $[u]^\alpha = [\underline{u}_\alpha, \overline{u}_\alpha] = \{x \in \mathbb{R} \mid u(x) \geq \alpha\}, 0 < \alpha \leq 1$ is α-level set of u. If $\alpha = 0$, $[u]^0 = cl\{suppu\} = cl\{x \in \mathbb{R} \mid u(x) > 0\}$.*

Remark 1. *[24] The parametric form $[\underline{u}_\alpha, \overline{u}_\alpha]$ of a fuzzy number satisfying the following requirements is a valid α-level set.*
$\underline{u}_\alpha$ is left-continuous monotonic increasing (nondecreasing) bounded on $(0,1]$,
$\overline{u}_\alpha$ is left-continuous monotonic decreasing (nonincreasing) bounded on $(0,1]$,
$\underline{u}_\alpha$ and $\overline{u}_\alpha$ are right-continuous for $\alpha = 0$,
$\underline{u}_\alpha \leq \overline{u}_\alpha$, $0 \leq \alpha \leq 1$.

Definition 3. *[23] The α- level set of A, $[A]^\alpha = [\underline{A}_\alpha, \overline{A}_\alpha] = \left[\underline{a} + \left(\frac{\overline{a}-\underline{a}}{2}\right)\alpha, \overline{a} - \left(\frac{\overline{a}-\underline{a}}{2}\right)\alpha\right]$ $(\underline{A}_1 = \overline{A}_1, \underline{A}_1 - \underline{A}_\alpha = \overline{A}_\alpha - \overline{A}_1)$ is a symmetric triangular fuzzy number with support $[\underline{a}, \overline{a}]$.*

Definition 4. *[8, 24, 25] Let be $u, v \in \mathbb{R}_F$. If $u = v + w$ such that there exists $w \in \mathbb{R}_F$, w is the Hukuhara difference of u and v, $w = u \ominus v$.*

Definition 5. *[24–26] Let be $f : [a,b] \to \mathbb{R}_F$ and $x_0 \in [a,b]$. If there exists $f'(x_0) \in \mathbb{R}_F$ such that for all $h > 0$ sufficiently small, $\exists f(x_0+h) \ominus f(x_0)$, $f(x_0) \ominus f(x_0-h)$ and the limits hold*

$$\lim_{h\to 0} \frac{f(x_0+h) \ominus f(x_0)}{h} = \lim_{h\to 0} \frac{f(x_0) \ominus f(x_0-h)}{h} = f'(x_0),$$

f is Hukuhara differentiable at x_0.

Definition 6. *[24] Let be $f : [a,b] \to \mathbb{R}_F$ and $x_0 \in [a,b]$. If there exists $f'(x_0) \in \mathbb{R}_F$ such that for all $h > 0$ sufficiently small, $\exists f(x_0+h) \ominus f(x_0)$, $f(x_0) \ominus f(x_0-h)$ and the limits hold*

$$\lim_{h\to 0} \frac{f(x_0+h) \ominus f(x_0)}{h} = \lim_{h\to 0} \frac{f(x_0) \ominus f(x_0-h)}{h} = f'(x_0),$$

f is (1)-differentiable at x_0. If there exists $f'(x_0) \in \mathbb{R}_F$ such that for all $h > 0$ sufficiently small, $\exists f(x_0) \ominus f(x_0+h)$, $f(x_0-h) \ominus f(x_0)$ and the limits hold

$$\lim_{h\to 0} \frac{f(x_0) \ominus f(x_0+h)}{-h} = \lim_{h\to 0} \frac{f(x_0-h) \ominus f(x_0)}{-h} = f'(x_0),$$

f is (2)-differentiable.

Theorem 1. *[27]Let $f : [a,b] \to \mathbb{R}_F$ be fuzzy function and denote $[f(x)]^\alpha = \left[\underline{f}_\alpha(x), \overline{f}_\alpha(x)\right]$, for each $\alpha \in [0,1]$.*

(i) If the function f is (1)-differentiable, the lower function $\underline{f}_\alpha$ and the upper function $\overline{f}_\alpha$ are differentiable, $\left[f'(x)\right]^\alpha = \left[\underline{f}'_\alpha(x), \overline{f}'_\alpha(x)\right]$,

(ii) If the function f is (2)-differentiable, the lower function $\underline{f}_\alpha$ and the upper function $\overline{f}_\alpha$ are differentiable, $\left[f'(x)\right]^\alpha = \left[\overline{f}'_\alpha(x), \underline{f}'_\alpha(x)\right]$.

Theorem 2. *[27]Let $f^{'}:[a,b]\to\mathbb{R}_F$ be fuzzy function and denote $[f(x)]^{\alpha}=\left[\underline{f}_{\alpha}(x),\overline{f}_{\alpha}(x)\right]$, for each $\alpha\in[0,1]$, the function f is (1)-differentiable or (2)-differentiable.*

(i) If the functions f and $f^{'}$ are (1)-differentiable, the functions $\underline{f}^{'}_{\alpha}$ and $\overline{f}^{'}_{\alpha}$ are differentiable, $\left[f^{''}(x)\right]^{\alpha}=\left[\underline{f}^{''}_{\alpha}(x),\overline{f}^{''}_{\alpha}(x)\right]$,

(ii) If the function f is (1)-differentiable and the function $f^{'}$ is (2)-differentiable, the functions $\underline{f}^{'}_{\alpha}$ and $\overline{f}^{'}_{\alpha}$ are differentiable, $\left[f^{''}(x)\right]^{\alpha}=\left[\overline{f}^{''}_{\alpha}(x),\underline{f}^{''}_{\alpha}(x)\right]$,

(iii) If the function f is (2)-differentiable and the function $f^{'}$ is (1)-differentiable, the functions $\underline{f}^{'}_{\alpha}$ and $\overline{f}^{'}_{\alpha}$ are differentiable, $\left[f^{''}(x)\right]^{\alpha}=\left[\overline{f}^{''}_{\alpha}(x),\underline{f}^{''}_{\alpha}(x)\right]$,

(iv) If the functions f and $f^{'}$ are (2)-differentiable, the functions $\underline{f}^{'}_{\alpha}$ and $\overline{f}^{'}_{\alpha}$ are differentiable, $\left[f^{''}(x)\right]^{\alpha}=\left[\underline{f}^{''}_{\alpha}(x),\overline{f}^{''}_{\alpha}(x)\right]$.

Definition 7. *[18, 19] Let $f:[a,b]\to\mathbb{R}_F$ be fuzzy function. The fuzzy Laplace transform of f is*

$$F(s)=L(f(t))=\int_0^{\infty}e^{-st}f(t)\,dt=\left[\lim_{\tau\to\infty}\int_0^{\tau}e^{-st}\underline{f}(t)\,dt,\lim_{\tau\to\infty}\int_0^{\tau}e^{-st}\overline{f}(t)\,dt\right],$$

$$F(s,\alpha)=L\left((f(t))^{\alpha}\right)=\left[L\left(\underline{f}_{\alpha}(t)\right),L\left(\overline{f}_{\alpha}(t)\right)\right],$$

$$L\left(\underline{f}_{\alpha}(t)\right)=\int_0^{\infty}e^{-st}\underline{f}_{\alpha}(t)\,dt=\lim_{\tau\to\infty}\int_0^{\tau}e^{-st}\underline{f}_{\alpha}(t)\,dt,$$

$$L\left(\overline{f}_{\alpha}(t)\right)=\int_0^{\infty}e^{-st}\overline{f}_{\alpha}(t)\,dt=\lim_{\tau\to\infty}\int_0^{\tau}e^{-st}\overline{f}_{\alpha}(t)\,dt.$$

Theorem 3. *[18, 19]Suppose that f is continuous fuzzy-valued function on $[0,\infty)$ and exponential order α and that $f^{'}$ is piecewise continuous fuzzy-valued function on $[0,\infty)$.*

If the function f is (1) differentiable,

$$L\left(f^{'}(t)\right)=sL(f(t))\ominus f(0),$$

if the function f is (2) differentiable,

$$L\left(f^{'}(t)\right)=(-f(0))\ominus(-sL(f(t))).$$

Theorem 4. *[18, 19]Suppose that f and $f^{'}$ are continuous fuzzy-valued functions on $[0,\infty)$ and exponential order α and that $f^{''}$ is piecewise continuous fuzzy-valued function on $[0,\infty)$.*

If the functions f and $f^{'}$ are (1) differentiable,

$$L\left(f^{''}(t)\right)=s^2L(f(t))\ominus sf(0)\ominus f^{'}(0),$$

if the function f is (1) differentiable and the function $f^{'}$ is (2) differentiable,

$$L\left(f^{''}(t)\right)=-f^{'}(0)\ominus\left(-s^2\right)L(f(t))-sf(0),$$

if the function f is (2) differentiable and the function f' is (1) differentiable,

$$L\left(f^{''}(t)\right) = -sf(0) \ominus \left(-s^2\right) L(f(t)) \ominus f^{'}(0),$$

if the functions f and f' are (2) differentiable,

$$L\left(f^{''}(t)\right) = s^2 L(f(t)) \ominus sf(0) - f^{'}(0).$$

Theorem 5. *[17, 19]Let be $f(t)$, $g(t)$ continuous fuzzy-valued functions and c_1 and c_2 constants, then*

$$L(c_1 f(t) + c_2 g(t)) = (c_1 L(f(t))) + (c_2 L(g(t))).$$

3 Findings and Main Results

We study the problem

$$u''(t) + u(t) = [A]^\alpha, t > 0 \tag{3.1}$$

$$u(0) = [B]^\alpha, \;\; u^{'}(0) = [C]^\alpha \tag{3.2}$$

by the fuzzy Laplace transform, where A, B and C are symmetric triangular fuzzy numbers with supports $[\underline{a}, \overline{a}]$, $[\underline{b}, \overline{b}]$ and $[\underline{c}, \overline{c}]$, respectively. Also, the $\alpha-$level sets of A, B, C are

$$[A]^\alpha = \left[\underline{A}_\alpha, \overline{A}_\alpha\right] = \left[\underline{a} + \left(\frac{\overline{a} - \underline{a}}{2}\right)\alpha, \overline{a} - \left(\frac{\overline{a} - \underline{a}}{2}\right)\alpha\right],$$

$$[B]^\alpha = \left[\underline{B}_\alpha, \overline{B}_\alpha\right] = \left[\underline{b} + \left(\frac{\overline{b} - \underline{b}}{2}\right)\alpha, \overline{b} - \left(\frac{\overline{b} - \underline{b}}{2}\right)\alpha\right],$$

$$[C]^\alpha = \left[\underline{C}_\alpha, \overline{C}_\alpha\right] = \left[\underline{c} + \left(\frac{\overline{c} - \underline{c}}{2}\right)\alpha, \overline{c} - \left(\frac{\overline{c} - \underline{c}}{2}\right)\alpha\right].$$

In this paper, (i,j) solution means that u is (i) differentiable, $u^{'}$ is (j) differentiable.
Case 1) If u and $u^{'}$ are (1) differentiable, since

$$s^2 L(u_\alpha(t)) \ominus s u_\alpha(0) \ominus u^{'}_\alpha(0) + L(u_\alpha(t)) = L\left([A]^\alpha\right),$$

and using the fuzzy arithmetic and Hukuhara difference, yields the equations

$$s^2 L(\underline{u}_\alpha(t)) - s\underline{u}_\alpha(0) - \underline{u}^{'}_\alpha(0) + L(\underline{u}_\alpha(t)) = L(\underline{A}_\alpha),$$

$$s^2 L(\overline{u}_\alpha(t)) - s\overline{u}_\alpha(0) - \overline{u}^{'}_\alpha(0) + L(\overline{u}_\alpha(t)) = L\left(\overline{A}_\alpha\right).$$

Using the initial values, we get

$$L(\underline{u}_\alpha(t)) = \frac{\underline{A}_\alpha}{s(s^2+1)} + \frac{s\underline{B}_\alpha}{s^2+1} + \frac{\underline{C}_\alpha}{s^2+1},$$

$$L(\overline{u}_\alpha(t)) = \frac{\overline{A}_\alpha}{s(s^2+1)} + \frac{s\overline{B}_\alpha}{s^2+1} + \frac{\overline{C}_\alpha}{s^2+1}.$$

From this, taking the inverse Laplace transform of the above equations, the lower solution and the upper solution are obtained as

$$\underline{u}_\alpha(t) = \underline{A}_\alpha(1 - \cos(t)) + \underline{B}_\alpha \cos(t) + \underline{C}_\alpha \sin(t),$$

$$\overline{u}_{\alpha}(t) = \overline{A}_{\alpha}(1-\cos(t)) + \overline{B}_{\alpha}\cos(t) + \overline{C}_{\alpha}\sin(t).$$

Case 2) If u is (1) differentiable and $u^{'}$ is (2) differentiable, since

$$-u_{\alpha}^{'}(0) \ominus \left(-s^{2}\right) L(u_{\alpha}(t)) - su_{\alpha}(0) + L(u_{\alpha}(t)) = L\left([A]^{\alpha}\right),$$

and using the fuzzy arithmetic and Hukuhara difference, the equations

$$-\overline{u}_{\alpha}^{'}(0) - \left(-s^{2}L(\overline{u}_{\alpha}(t))\right) - s\overline{u}_{\alpha}(0) + L(\underline{u}_{\alpha}(t)) = L(\underline{A}_{\alpha}), \tag{3.3}$$

$$-\underline{u}_{\alpha}^{'}(0) - \left(-s^{2}L(\underline{u}_{\alpha}(t))\right) - s\underline{u}_{\alpha}(0) + L(\overline{u}_{\alpha}(t)) = L\left(\overline{A}_{\alpha}\right) \tag{3.4}$$

are obtained. If $L(\overline{u}_{\alpha}(t))$ in the equation (3.4) is replaced by the equation (3.3) and making the necessary operations, we have

$$L(\underline{u}_{\alpha}(t)) = \frac{\underline{A}_{\alpha}}{s(1-s^{4})} + \frac{\overline{C}_{\alpha}}{1-s^{4}} + \frac{s\left(\overline{B}_{\alpha}-\overline{A}_{\alpha}\right)}{1-s^{4}} - \frac{s^{2}\underline{C}_{\alpha}}{1-s^{4}} - \frac{s^{3}\underline{B}_{\alpha}}{1-s^{4}}. \tag{3.5}$$

Taking inverse Laplace transform of the equation (3.5), the lower solution is obtained as

$$\begin{aligned}\underline{u}_{\alpha}(t) = \underline{A}_{\alpha}&\left(1-\frac{1}{4}e^{t}-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right) + \overline{C}_{\alpha}\left(-\frac{1}{4}e^{t}+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\ &+\left(\overline{B}_{\alpha}-\overline{A}_{\alpha}\right)\left(-\frac{1}{4}e^{t}-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right) - \underline{C}_{\alpha}\left(-\frac{1}{4}e^{t}+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\ &-\underline{B}_{\alpha}\left(-\frac{1}{4}e^{t}-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right).\end{aligned}$$

Similarly, the upper solution is obtained as

$$\begin{aligned}\overline{u}_{\alpha}(t) = \overline{A}_{\alpha}&\left(1-\frac{1}{4}e^{t}-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right) + \underline{C}_{\alpha}\left(-\frac{1}{4}e^{t}+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\ &+\left(\underline{B}_{\alpha}-\underline{A}_{\alpha}\right)\left(-\frac{1}{4}e^{t}-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right) - \overline{C}_{\alpha}\left(-\frac{1}{4}e^{t}+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\ &-\overline{B}_{\alpha}\left(-\frac{1}{4}e^{t}-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right).\end{aligned}$$

Case 3) If u is (2) differentiable and $u^{'}$ is (1) differentiable, since

$$-su_{\alpha}(0) \ominus \left(-s^{2}\right) L(u_{\alpha}(t)) \ominus u_{\alpha}^{'}(0) + L(u_{\alpha}(t)) = L\left([A]^{\alpha}\right),$$

we have the equations

$$-s\overline{u}_{\alpha}(0) - \left(-s^{2}L(\overline{u}_{\alpha}(t))\right) - \overline{u}_{\alpha}^{'}(0) + L(\underline{u}_{\alpha}(t)) = L(\underline{A}_{\alpha}), \tag{3.6}$$

$$-s\underline{u}_{\alpha}(0) - \left(-s^{2}L(\underline{u}_{\alpha}(t))\right) - \underline{u}_{\alpha}^{'}(0) + L(\overline{u}_{\alpha}(t)) = L\left(\overline{A}_{\alpha}\right), \tag{3.7}$$

If $L(\overline{u}_{\alpha}(t))$ in the equation (3.7) is replaced by the equation (3.6) , we get

$$L(\underline{u}_{\alpha}(t)) = \frac{\underline{A}_{\alpha}}{s(1-s^{4})} + \frac{\underline{C}_{\alpha}}{1-s^{4}} + \frac{s\left(\overline{B}_{\alpha}-\overline{A}_{\alpha}\right)}{1-s^{4}} - \frac{s^{2}\overline{C}_{\alpha}}{1-s^{4}} - \frac{s^{3}\underline{B}_{\alpha}}{1-s^{4}}.$$

From this, the lower solution is obtained as

$$\begin{aligned}\underline{u}_\alpha(t) &= \underline{A}_\alpha\left(1-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right)+\underline{C}_\alpha\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\&+\left(\overline{B}_\alpha-\overline{A}_\alpha\right)\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right)-\overline{C}_\alpha\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\&-\underline{B}_\alpha\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right).\end{aligned}$$

Similarly, the upper solution is obtained as

$$\begin{aligned}\overline{u}_\alpha(t) &= \overline{A}_\alpha\left(1-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right)+\overline{C}_\alpha\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\&+\left(\underline{B}_\alpha-\underline{A}_\alpha\right)\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right)-\underline{C}_\alpha\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\&-\overline{B}_\alpha\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right).\end{aligned}$$

Case 4) If u is (2) differentiable and u' is (2) differentiable, since

$$s^2L(u_\alpha(t))\ominus su_\alpha(0)-u'_\alpha(0)+L(u_\alpha(t))=L\left([A]^\alpha\right),$$

we have the equations

$$s^2L(\underline{u}_\alpha(t))-s\underline{u}_\alpha(0)-\underline{u}'_\alpha(0)+L(\underline{u}_\alpha(t))=L(\underline{A}_\alpha),$$

$$s^2L(\overline{u}_\alpha(t))-s\overline{u}_\alpha(0)-\overline{u}'_\alpha(0)+L(\overline{u}_\alpha(t))=L\left(\overline{A}_\alpha\right),$$

Using the initial values, the lower and upper solutions are obtained as

$$\begin{aligned}L(\underline{u}_\alpha(t))&=\frac{\underline{A}_\alpha}{s(s^2+1)}+\frac{s\underline{B}_\alpha}{s^2+1}+\frac{\overline{C}_\alpha}{s^2+1},\\L(\overline{u}_\alpha(t))&=\frac{\overline{A}_\alpha}{s(s^2+1)}+\frac{s\overline{B}_\alpha}{s^2+1}+\frac{\underline{C}_\alpha}{s^2+1}.\end{aligned}$$

From this, solutions are obtained as

$$\underline{u}_\alpha(t)=\underline{A}_\alpha(1-\cos(t))+\underline{B}_\alpha\cos(t)+\overline{C}_\alpha\sin(t),$$

$$\overline{u}_\alpha(t)=\overline{A}_\alpha(1-\cos(t))+\overline{B}_\alpha\cos(t)+\underline{C}_\alpha\sin(t).$$

Example 1. *Consider the problem*

$$u''(t)+u(t)=[0]^\alpha,\;\; y(0)=[1]^\alpha,\;\; y'(0)=[2]^\alpha \tag{3.8}$$

by fuzzy Laplace transform, where $[0]^\alpha=[-1+\alpha,1-\alpha],[1]^\alpha=[\alpha,2-\alpha],\,[2]^\alpha=[1+\alpha,3-\alpha].$

(1,1) solution is

$$\underline{u}_\alpha(t) = (-1+\alpha)(1-\cos(t)) + \alpha\cos(t) + (1+\alpha)\sin(t),$$
$$\overline{u}_\alpha(t) = (1-\alpha)(1-\cos(t)) + (2-\alpha)\cos(t) + (3-\alpha)\sin(t),$$
$$[u(t)]^\alpha = [\underline{u}_\alpha(t), \overline{u}_\alpha(t)],$$

(1,2) solution is

$$\begin{aligned}\underline{u}_\alpha(t) = {} & (-1+\alpha)\left(1-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right)+(3-\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\ & +\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right)-(1+\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\ & -\alpha\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right),\end{aligned}$$

$$\begin{aligned}\overline{u}_\alpha(t) = {} & (1-\alpha)\left(1-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right)+(1+\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\ & +\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right)-(3-\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\ & -(2-\alpha)\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right),\end{aligned}$$

$$[u(t)]^\alpha = [\underline{u}_\alpha(t), \overline{u}_\alpha(t)],$$

(2,1) solution is

$$\begin{aligned}\underline{u}_\alpha(t) = {} & (-1+\alpha)\left(1-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right)+(1+\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\ & +\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right)-(3-\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\ & -\alpha\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right),\end{aligned}$$

$$\begin{aligned}\overline{u}_\alpha(t) = {} & (1-\alpha)\left(1-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right)+(3-\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}+\frac{1}{2}\sin(t)\right)\\ & +\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}+\frac{1}{2}\cos(t)\right)-(1+\alpha)\left(-\frac{1}{4}e^t+\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)\right)\\ & -(2-\alpha)\left(-\frac{1}{4}e^t-\frac{1}{4}e^{-t}-\frac{1}{2}\cos(t)\right),\end{aligned}$$

$$[u(t)]^\alpha = [\underline{u}_\alpha(t), \overline{u}_\alpha(t)],$$

and (2,2) solution is

$$\underline{u}_\alpha(t) = (-1+\alpha)(1-\cos(t)) + \alpha\cos(t) + (3-\alpha)\sin(t),$$
$$\overline{u}_\alpha(t) = (1-\alpha)(1-\cos(t)) + (2-\alpha)\cos(t) + (1+\alpha)\sin(t),$$

$$[u(t)]^{\alpha} = [\underline{u}_{\alpha}(t), \overline{u}_{\alpha}(t)].$$

If

$$\frac{\partial \underline{u}_{\alpha}(t)}{\partial \alpha} > 0,\ \frac{\partial \overline{u}_{\alpha}(t)}{\partial \alpha} < 0,\ \underline{u}_{\alpha}(t) \leq \overline{u}_{\alpha}(t),$$

the (i,j) solution (i=1,2) is a valid $\alpha-$*level set. According to this, since* $\sin(t) \geq -1$, *(1,1) solution is a valid* $\alpha-$*level set, since* $e^{t} - e^{-t} > 0$, *(1,2) solution is a valid* $\alpha-$*level set, (2,1) solution is not a valid* $\alpha-$*level set as* $e^{t} - e^{-t} - 2 > 0$, *that is (2,1) solution is not a valid* $\alpha-$*level set for* $t > 0.881374$, *since* $\sin(t) \leq 1$, *(2,2) solution is a valid* $\alpha-$*level set.*

Also, since for (1,1) solution,

$$\underline{u}_{1}(t) = \cos(t) + 2\sin(t) = \overline{u}_{1}(t),$$

$$\underline{u}_{1}(t) - \underline{u}_{\alpha}(t) = (1-\alpha)(1+\sin(t)) = \overline{u}_{\alpha}(t) - \overline{u}_{1}(t),$$

for (1,2) solution,

$$\underline{u}_{1}(t) = \cos(t) + 2\sin(t) = \overline{u}_{1}(t),$$

$$\underline{u}_{1}(t) - \underline{u}_{\alpha}(t) = (1-\alpha)\left(1 + \frac{1}{2}e^{t} - \frac{1}{2}e^{-t}\right) = \overline{u}_{\alpha}(t) - \overline{u}_{1}(t),$$

for (2,1) solution,

$$\underline{u}_{1}(t) = \cos(t) + 2\sin(t) = \overline{u}_{1}(t),$$

$$\underline{u}_{1}(t) - \underline{u}_{\alpha}(t) = (1-\alpha)\left(1 - \frac{1}{2}e^{t} + \frac{1}{2}e^{-t}\right) = \overline{u}_{\alpha}(t) - \overline{u}_{1}(t),$$

for (2,2) solution,

$$\underline{u}_{1}(t) = \cos(t) + 2\sin(t) = \overline{u}_{1}(t),$$

$$\underline{u}_{1}(t) - \underline{u}_{\alpha}(t) = (1-\alpha)(1-\sin(t)) = \overline{u}_{\alpha}(t) - \overline{u}_{1}(t),$$

all of the solutions are symmetric triangular fuzzy numbers.

4 Conclusions

In this paper, solutions of a fuzzy problem with symmetric triangular fuzzy number inital values are investigated by fuzzy Laplace transform. Generalized differantiability, fuzzy arithmetic are used. Example is solved. It is shown whether the solutions are valid $\alpha-$level sets or not. If inital values are symmetric triangular fuzzy numbers, then the solutions are symmetric triangular fuzzy numbers for any time.

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