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Multidimensional BSDE with Poisson jumps of Osgood type

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Abstract

This paper is devoted to solve a multidimensional backward stochastic differential equation with jumps in finite time horizon. Under linear growth generator, we prove existence and uniqueness of solution.

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1 Introduction

It is well known that Backward stochastic differential equations (BSDEs in short) driven by random Poisson measure are natural extension of classical BSDEs. These equations, first discussed by Tang and Li [8] can be seen as a generalization of Pardoux and Peng's work [6], which constitute the key point of solving problem in financial mathematics and studying non linear partial differential equations (PDEs in short) by means of stochastic tools. Since then the interest in searching probabilistic formula of solution of other type of PDEs increases a lot. Some authors studying parabolic integral-partial differential equation (PIDE), interested in BSDEs with Poisson Process (BSDEP in short). Among them we mention the result of Barles *et al* [1] who establish a probabilistic interpretation of a solution of a PIDE. By means of a comparison theorem, they generalized the probabilistic representation of solution of quasilinear PDEs proved in [6] to PIDEs. But all these results are obtained either with a Lipschitz condition or a monotonicity one on the drift of the stochastic equation. Several authors investigate in weakening this restrictive assumption. Among others Mao [3] investigate successfully these equations with the Osgood condition. This one is introduced by specific function which allows the use of the well known Bihari's Lemma to get uniqueness. The limitation is that all these results are established in the one dimensional case.

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The study of multidimensional BSDEs with weak conditions on the generator was discussed recently by Fan *et al* [2]. Using a suitable sequence, they prove an existence and uniqueness result when the generator satisfies the Osgood condition. In this work we are interested in extending this result to multidimensional BSDEs driven by random Poisson measure (MBSDEPs in short) satisfying the Osgood condition. Inspired by the method introduced by Fan *et al* [2], we prove existence and uniqueness of solution of a MBSDEP. The paper is organized as follows. In section 2, we recall some important results on MBSDEs driven by Poisson random measure. In section 3, we establish our main result.

2 MBSDEP with Poisson Jumps

2.1 Definitions and preliminary results

Let Ω be a non-empty set, $\mathcal{F}$ a σ -algebra of sets of Ω and $\mathbf{P}$ a probability measure defined on $\mathcal{F}$. The triplet $(\Omega, \mathcal{F}, \mathbf{P})$ defines a probability space, which is assumed to be complete. We assume given two mutually independent processes :

- a d -dimensional Brownian motion $(B_t)_{t \geq 0}$,
- a random Poisson measure μ on $E \times \mathbf{R}_+$ with compensator $\nu(dt, de) = \lambda(de)dt$

where the space $E = \mathbf{R} - \{0\}$ is equipped with its Borel field $\mathcal{E}$ such that $\{\tilde{\mu}([0, t] \times A) = (\mu - \nu)[0, t] \times A\}$ is a martingale for any $A \in \mathcal{E}$ satisfying $\lambda(A) < \infty$. λ is a σ -finite measure on $\mathcal{E}$ and satisfies

$$\int_E (1 \wedge |e|^2) \lambda(de) < \infty.$$

We consider the filtration $(\mathcal{F}_t)_{t \geq 0}$ given by $\mathcal{F}_t = \mathcal{F}_t^B \vee \mathcal{F}_t^\mu$, where for any process $\{\eta_t\}_{t \geq 0}$, $\mathcal{F}_{s,t}^\eta = \sigma\{\eta_r - \eta_s, s \leq r \leq t\} \vee \mathcal{N}$, $\mathcal{F}_t^\eta = \mathcal{F}_{0,t}^\eta$. Here $\mathcal{N}$ denotes the class of $\mathbf{P}$ -null sets of $\mathcal{F}$.

For $Q \in \mathbf{N}^*$, $|\cdot|$ stands for the Euclidian norm in $\mathbf{R}^Q$.

We consider the following sets (where $\mathbf{E}$ denotes the mathematical expectation with respect to the probability measure $\mathbf{P}$), and a non-random horizon time $0 < T < +\infty$:

- $\mathcal{S}^2(\mathbf{R}^Q)$ the space of $\mathcal{F}_t$ -adapted càdlàg processes

$$\Psi : [0, T] \times \Omega \longrightarrow \mathbf{R}^Q, \|\hat{E}\Psi\|_{\mathcal{S}^2(\mathbf{R}^Q)}^2 = \mathbf{E} \left(\sup_{0 \leq t \leq T} |\Psi_t|^2 \right) < \infty.$$

- $\mathcal{M}^2(\mathbf{R}^Q)$ the space of $\mathcal{F}_t$ -progressively measurable processes

$$\Psi : [0, T] \times \Omega \longrightarrow \mathbf{R}^{Q \times d}, \|\hat{E}\Psi\|_{\mathcal{M}^2(\mathbf{R}^Q)}^2 = \mathbf{E} \int_0^T |\Psi_t|^2 dt < \infty.$$

- $\mathcal{L}^2(\tilde{\mu}, \mathbf{R}^Q)$ the space of mappings $U : \Omega \times [0, T] \times E \longrightarrow \mathbf{R}^Q$ which are $\mathcal{P} \otimes \mathcal{E}$ -measurable such that

$$\|\hat{E}U\|_{\mathcal{L}^2(\mathbf{R}^Q)}^2 = \mathbf{E} \int_0^T \|U_t\|_{L^2(E, \mathcal{E}, \lambda, \mathbf{R})}^2 dt < \infty,$$

where $\mathcal{P}$ denotes the σ -algebra of $\mathcal{F}_t$ -predictable sets of $\Omega \times [0, T]$ and

$$\|U_t\|_{L^2(E, \mathcal{E}, \lambda, \mathbf{R})}^2 = \int_E |U_t(e)|^2 \lambda(de).$$

We may often write $|\cdot|$ instead of $\|\cdot\|_{L^2(E, \mathcal{E}, \lambda)}$ for a sake of simplicity.

Let $k \geq 1$ and define $\mathcal{A} = \mathbf{R}^k \times \mathbf{R}^{k \times d} \times L^2(E, \mathcal{E}, \lambda, \mathbf{R})$. Notice that the space $\mathcal{B}^2(\mathbf{R}^k) = \mathcal{S}^2(\mathbf{R}^k) \times \mathcal{M}^2(\mathbf{R}^{k \times d}) \times \mathcal{L}^2(\tilde{\mu}, \mathbf{R}^k)$ endowed with the norm

$$\|\hat{E}(Y, Z, U)\|_{\mathcal{B}^2(\mathbf{R}^k)}^2 = \|\hat{E}Y\|_{\mathcal{S}^2(\mathbf{R}^k)}^2 + \|\hat{E}Z\|_{\mathcal{M}^2(\mathbf{R}^{k \times d})}^2 + \|\hat{E}U\|_{\mathcal{L}^2(\mathbf{R}^k)}^2$$

is a Banach space.

Finally let $\mathbf{S}$ be the set of all non-decreasing and concave function $\varphi(\cdot) : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ satisfying $\varphi(0) = 0$, $\varphi(s) > 0$ for $s > 0$ and $\int_{0+} \varphi^{-1}(u) du = +\infty$.

Given $f : \Omega \times [0, T] \times \mathcal{A} \rightarrow \mathbf{R}^k$ a jointly measurable function and $\xi \in L^2(\mathcal{F}_T, \mathbf{R}^k)$ the set of all $\mathbf{R}^k$ -valued, square integrable and $\mathcal{F}_T$ -measurable random vectors, we are interested in the MBSDEP with parameters (ξ, f, T) :

$$Y_t = \xi + \int_t^T f(s, \Theta_s) ds - \int_t^T Z_s dB_s - \int_t^T \int_E U_s(e) \tilde{\mu}(ds, de), \quad 0 \leq t \leq T, \quad (2.1)$$

where Θ_s stands for the triple (Y_s, Z_s, U_s) .

For instance let us precise the notion of solution to eq.(2.1).

Definition 2.1. A triplet of processes $(Y_t, Z_t, U_t)_{0 \leq t \leq T}$ is called a solution to eq.(2.1), if $(Y_t, Z_t, U_t) \in \mathcal{B}^2(\mathbf{R}^k)$ and it satisfies eq.(2.1).

Now, let us introduce the following Proposition 2.2, which will play an important role in the proof of Theorem 3.4. We consider the following assumption on the generator f :

$$\begin{aligned} (\mathbf{A}) : \quad & d\mathbf{P} \times dt\text{-a.e.}, \quad \forall (y, z, u) \in \mathbf{R}^k \times \mathbf{R}^{k \times d} \times \mathbf{R}^k, \\ & \langle y, f(\omega, t, y, z, u) \rangle \leq \gamma(t) \psi(|y|^2) + \alpha(t) |y| (|z| + |u|) + |y| \varphi_t \end{aligned}$$

where $\gamma, \alpha : [0, T] \rightarrow \mathbf{R}_+$ satisfying $0 < \int_0^T [\alpha^2(s) + \gamma(s)] ds < \infty$, $\varphi \in \mathcal{M}^2(\mathbf{R})$ is non-negative and ψ is non-decreasing concave function from $\mathbf{R}_+$ to itself with $\psi(0) = 0$.

Proposition 2.2. Assume that f satisfies $(\mathbf{A})$ and let $(Y_t, Z_t, U_t)_{0 \leq t \leq T}$ be a solution to the MBSDEP (2.1). There exists a constant $c > 0$ depending only on α such that, for any $0 \leq t \leq T$,

$$\begin{aligned} & \mathbf{E} \left(\sup_{t \leq s \leq T} |Y_s|^2 \right) + \mathbf{E} \left(\int_t^T |Z_s|^2 ds \right) + \mathbf{E} \left(\int_t^T \int_E |U_s(e)|^2 \lambda(de) ds \right) \\ & \leq C_{2.2}(\alpha) \left[\mathbf{E} |\xi|^2 + \int_t^T \gamma(s) \psi(\mathbf{E} |Y_s|^2) ds + \mathbf{E} \int_t^T \varphi_s^2 ds \right]. \end{aligned} \quad (2.2)$$

Proof. Itô's formula applied to $|Y_t|^2$ reads to

$$\begin{aligned} |Y_t|^2 + \int_t^T |Z_s|^2 ds + \int_t^T \int_E |U_s(e)|^2 \lambda(de) ds + \sum_{t < s \leq T} (\Delta Y_s)^2 &= |\xi|^2 + 2 \int_t^T \langle Y_s, f(s, \Theta_s) \rangle ds \\ &- 2 \int_t^T \langle Y_s, Z_s dB_s \rangle - 2 \int_t^T \int_E \langle Y_{s-}, U_s(e) \tilde{\mu}(ds, de) \rangle, \quad 0 \leq t \leq T. \end{aligned} \quad (2.3)$$

By the assumption $(\mathbf{A})$ and the inequality $2ab \leq \theta a^2 + b^2/\theta$ for any $\theta > 0$, we have

$$\begin{aligned} 2 \langle Y_s, f(s, \Theta_s) \rangle &\leq 2\gamma(s) \psi(|Y_s|^2) + 2\alpha(s) |Y_s| |Z_s| + 2\alpha(s) |Y_s| |U_s| + 2|Y_s| \varphi_s \\ &\leq 2\gamma(s) \psi(|Y_s|^2) + (1 + 2\alpha^2(s)) \sup_{t \leq s \leq T} |Y_s|^2 + \varphi_s^2 + \frac{1}{2} (|Z_s|^2 + |U_s|^2). \end{aligned} \quad (2.4)$$

Hence it follows from eq.(2.3) and (2.4) that for any $0 \leq t \leq T$,

$$\frac{1}{2} \mathbf{E} \left[\int_t^T |Z_s|^2 ds + \int_t^T \int_E |U_s(e)|^2 \lambda(de) ds \right] \leq X_t^T \quad (2.5)$$

where for $0 \leq t \leq T$,

$$X_t^T = \mathbf{E}|\xi|^2 + \int_t^T (1 + 2\alpha^2(s)) \cdot \mathbf{E} \left(\sup_{s \leq r \leq T} |Y_r|^2 \right) ds + \mathbf{E} \left[\int_t^T (2\gamma(s)\psi(|Y_s|^2) + \varphi_s^2) ds \right].$$

Applying Burkholder-Davis-Gundy inequality, we derive that the process $\left\{ M_t = \int_0^t \langle Y_s, Z_s dB_s \rangle \right\}_{0 \leq t \leq T}$ is in fact a uniformly integrable martingale and there exists $\delta > 0$ such that for $0 \leq t \leq T$, we have

$$\begin{aligned} 2\mathbf{E} \left[\sup_{t \leq r \leq T} \left| \int_r^T \langle Y_s, Z_s dB_s \rangle \right| \right] &\leq \delta \mathbf{E} \left[\left(\sup_{t \leq s \leq T} |Y_s|^2 \right)^{\frac{1}{2}} \cdot \left(\int_t^T |Z_s|^2 ds \right)^{\frac{1}{2}} \right] \\ &\leq \frac{1}{4} \mathbf{E} \left[\sup_{t \leq s \leq T} |Y_s|^2 \right] + \delta \mathbf{E} \left[\int_t^T |Z_s|^2 ds \right]. \end{aligned} \quad (2.6)$$

Similarly for the discontinuous martingale, there exists $\delta' > 0$ such that for $0 \leq t \leq T$, we have

$$\begin{aligned} 2\mathbf{E} \left[\sup_{t \leq r \leq T} \left| \int_r^T \int_E \langle Y_{s-}, U_s(e) \tilde{\mu}(ds, de) \rangle \right| \right] \\ \leq \delta' \mathbf{E} \left[\left(\sup_{t \leq s \leq T} |Y_s|^2 \right)^{\frac{1}{2}} \cdot \left(\int_t^T \int_E |U_s(e)|^2 \lambda(de) ds \right)^{\frac{1}{2}} \right] \\ \leq \frac{1}{4} \mathbf{E} \left(\sup_{t \leq s \leq T} |Y_s|^2 \right) + \delta' \mathbf{E} \left(\int_t^T \int_E |U_s(e)|^2 \lambda(de) ds \right). \end{aligned} \quad (2.7)$$

By virtue of (2.5), (2.6) and (2.7), we deduce from eq.(2.3) (where $\bar{\delta} = \max(\delta, \delta')$)

$$\begin{aligned} \mathbf{E} \left(\sup_{t \leq s \leq T} |Y_s|^2 \right) + \mathbf{E} \left(\int_t^T |Z_s|^2 ds \right) + \mathbf{E} \left(\int_t^T \int_E |U_s(e)|^2 \lambda(de) ds \right) \\ \leq 2X_t^T + \bar{\delta} \left[\mathbf{E} \left(\int_t^T |Z_s|^2 ds \right) + \mathbf{E} \left(\int_t^T \int_E |U_s(e)|^2 \lambda(de) ds \right) \right] \\ \leq 2(\bar{\delta} + 1)X_t^T. \end{aligned} \quad (2.8)$$

Hence combining the above inequality with (2.5), we deduce that there exists a constant $c > 0$ depending only on $\bar{\delta}$ such that

$$g(t) \leq c \left[\mathbf{E}|\xi|^2 + \int_t^T (1 + 2\alpha^2(s))g(s)ds + \mathbf{E} \left(\int_t^T [\gamma(s)\psi(|Y_s|^2) + \varphi_s^2] ds \right) \right]$$

where $g(t)$ stands for the left hand side of (2.8).

Applying Fubini's theorem and Jensen's inequality, we deduce by Gronwall's lemma (see Lemma 3.1 below)

$$g(t) \leq \left[\mathbf{E}|\xi|^2 + \int_t^T \gamma(s)\psi(\mathbf{E}|Y_s|^2)ds + \mathbf{E} \int_t^T \varphi_s^2 ds \right] \times c \exp \left(c \int_t^T (1 + 2\alpha^2(s))ds \right).$$

Putting $C_{2.2}(\alpha) = c \exp \left(c \int_0^T (1 + 2\alpha^2(s))ds \right)$, the result follows. $\square$

We are in position to investigate our main result.

3 Existence and uniqueness of solution

Let us introduce the following assumptions on the generator f . We say that f satisfies assumptions **(H1)** if the following hold (where we define for $0 \leq s \leq T$, $f(s, 0) = f(s, 0, 0, 0)$ to ease the reading):

- **(H1.1)**: f satisfies the weak Lipschitz condition in y , i.e., there exists $\rho \in \mathbf{S}$ such that $d\mathbf{P} \times dt$ -a.e., $\forall (y, y') \in (\mathbf{R}^k)^2$, $z \in \mathbf{R}^{k \times d}$, $u \in \mathbf{R}^k$,

$$|f(\omega, t, y, z, u) - f(\omega, t, y', z, u)| \leq \gamma(t)\rho(|y - y'|).$$

- **(H1.2)**: f is Lipschitz continuous in (z, u) uniformly with respect to (ω, t, y) , i.e., there exists a function $\beta : [0, T] \rightarrow \mathbf{R}_+$ such that $d\mathbf{P} \times dt$ -a.e., $y \in \mathbf{R}^k$, $(z, z') \in (\mathbf{R}^{k \times d})^2$ and $(u, u') \in (\mathbf{R}^k)^2$

$$|f(\omega, t, y, z, u) - f(\omega, t, y, z', u')| \leq \beta(t)(|z - z'| + |u - u'|).$$

- **(H1.3)**: The integrability condition holds a.s.

$$\mathbf{E} \left[\left(\int_0^T |f(s, 0)| ds \right)^2 \right] + \int_0^T [\beta^2(s) + \gamma(s)] ds < +\infty.$$

We recall the following results, which will be useful in the proof of uniqueness.

Lemma 3.1 (Gronwall). Assume given $T \geq 0$, $K \geq 0$ and $\Phi, \Psi : [0, T] \rightarrow \mathbf{R}^+$ such that $\int_0^T \Psi(s) ds < \infty$. If

$$\forall 0 \leq t \leq T, \quad \Phi(t) \leq K + \int_0^t \Psi(s)\Phi(s) ds < \infty,$$

then we have

$$\forall 0 \leq t \leq T, \quad \Phi(t) \leq K \exp \left(\int_0^t \Psi(s) ds \right).$$

Lemma 3.2 (Bihari's inequality). Let $T > 0$, u, v continuous non-negative functions on $[0, T]$ and a continuous function $H \in \mathbf{S}$. If

$$u(t) \leq \int_0^t v(s)H(u(s))ds, \quad 0 \leq t \leq T,$$

then $u(t) = 0$ for all $0 \leq t \leq T$.

As in [2], Theorem 1, we consider the sequence $(f_n)_{n \geq 1}$ defined by

$$f^n = (f_1^n, f_2^n, \dots, f_k^n)$$

where

$$\forall i = 1, \dots, k, \quad f_i^n(\omega, t, y, z, u) = \inf_{v \in \mathbf{R}^k} \{f_i(\omega, t, v, z, u) + (n + A)\gamma(t)|v - y|\}$$

We have the following result whose proof is omitted since it is an adaptation of step 1 of Theorem 1 in [2].

Lemma 3.3. The sequence of $\mathcal{F}_t$ -progressively measurable function f^n satisfies:

$$(i) \forall (y, z, u) \in \mathcal{A}, \quad |f^n(\omega, t, y, z, u) - f(\omega, t, y, z, u)| \leq k\gamma(t)\rho\left(\frac{2A}{n}\right)$$

$$(ii) \forall (y, y') \in (\mathbf{R}^k)^2, (z, u) \in \mathcal{A},$$

$$|f^n(\omega, t, y, z, u) - f^n(\omega, t, y', z, u)| \leq k\gamma(t)\rho(|y - y'|).$$

$$(iii) \forall y \in \mathbf{R}^k, (z, u) \in \mathcal{A}, \forall (z', u') \in \mathcal{A},$$

$$|f^n(\omega, t, y, z, u) - f^n(\omega, t, y, z', u')| \leq k\beta(t)[|z - z'| + |u - u'|].$$

$$(iii) \text{ The integrability condition holds } \mathbf{E} \left[\left(\int_0^T |f^n(t, 0)| dt \right)^2 \right] < +\infty.$$

The following Theorem 3.4 is the main result in this section.

Theorem 3.4. Given f satisfying assumptions **(H1)** and $\xi \in L^2(\mathcal{F}_T, \mathbf{R}^k)$, the MBSDEP (2.1) has a unique solution.

Proof. (i) Uniqueness. Let $(Y_t^i, Z_t^i, U_t^i)_{0 \leq t \leq T}$ ($i = 1, 2$) be two solutions of eq.(2.1) and define for $\delta \in \{Y, Z, U\}$, $\widehat{\delta} = \delta^1 - \delta^2$. Then the triple $(\widehat{Y}_t, \widehat{Z}_t, \widehat{U}_t)_{0 \leq t \leq T}$ is a solution to the following MBSDEP with parameters $(0, T, \widehat{f})$:

$$\widehat{Y}_t = \int_t^T \widehat{f}(s, \widehat{Y}_s, \widehat{Z}_s, \widehat{U}_s) ds - \int_t^T \widehat{Z}_s dB_s - \int_t^T \int_E \widehat{U}_s(e) \widetilde{\mu}(ds, de), \quad 0 \leq t \leq T, \quad (3.1)$$

where

$$\forall (y, z, u) \in \mathbf{R}^k \times \mathbf{R}^{k \times d} \times \mathbf{R}^k, \widehat{f}(s, y, z, u) = f(s, y + Y_s^2, z + Z_s^2, u + U_s^2) - f(s, Y_s^2, Z_s^2, U_s^2).$$

It follows from assumptions **(H1.1)** and **(H1.2)** that $d\mathbf{P} \times dt$ -a.e.,

$$\forall (y, z, u) \in \mathbf{R}^k \times \mathbf{R}^{k \times d} \times \mathbf{R}^k, \langle y, \widehat{f}(s, y, z, u) \rangle \leq \gamma(s)|y|\rho(|y|) + \beta(s)|y|(|z| + |u|).$$

By Remark 2 in [2], the function $H(u) = \sqrt{u}\rho(\sqrt{u})$ belongs in **S**. Then the generator $\widehat{g}$ of eq.(3.1) satisfies the assumption **(A)** with

$$\psi(u) = H(u), \quad \alpha(t) = \beta(t) \quad \text{and} \quad \varphi_t \equiv 0.$$

Thus applying Proposition 2.2, we have for $0 \leq t \leq T$,

$$\begin{aligned} \mathbf{E} \left[\sup_{t \leq s \leq T} |\widehat{Y}_s|^2 \right] + \mathbf{E} \left[\int_t^T |\widehat{Z}_s|^2 ds \right] + \mathbf{E} \left[\int_t^T \int_E |\widehat{U}_s(e)|^2 \lambda(de) ds \right] \\ \leq C_{2.2}(\beta) \int_t^T \gamma(s) H \left(\mathbf{E} \left[\sup_{s \leq r \leq T} |\widehat{Y}_r|^2 \right] \right) ds. \end{aligned}$$

Bihari's inequality (see Lemma 3.2) yields that, for any $0 \leq t \leq T$,

$$\mathbf{E} \left[\sup_{t \leq s \leq T} |\widehat{Y}_s|^2 \right] + \mathbf{E} \left[\int_t^T |\widehat{Z}_s|^2 ds \right] + \mathbf{E} \left[\int_t^T \int_E |\widehat{U}_s(e)|^2 \lambda(de) ds \right] = 0.$$

Uniqueness follows.

(ii) Existence. We consider the sequence $(f^n)_{n \geq 1}$ defined in Lemma 3.3 and the following MBSDEP with parameters (ξ, f^n, T) :

$$Y_t^n = \xi + \int_t^T f^n(s, \Theta_s^n) ds - \int_t^T Z_s^n dB_s - \int_t^T \int_E U_s^n(e) \widetilde{\mu}(ds, de). \quad (3.2)$$

Since f^n satisfies Lipschitz condition, it follows from Proposition 2.4 in [9] (putting $g \equiv 0$), that the sequence $\Theta^n = (Y^n, Z^n, U^n)$ is well defined. In addition for any $n \geq 1$ and $m \geq 1$, define for $\delta \in \{Y, Z, U\}$, $\widehat{\delta}^{n,m} = \delta^n - \delta^m$. Then the triplet $(\widehat{Y}^{n,m}, \widehat{Z}^{n,m}, \widehat{U}^{n,m})$ solves the following MBSDEP

$$\widehat{Y}_t^{n,m} = \int_t^T f^{n,m}(s, \widehat{\Theta}_s^{n,m}) ds - \int_t^T \widehat{Z}_s^{n,m} dB_s - \int_t^T \int_E \widehat{U}_s^{n,m}(e) \widetilde{\mu}(ds, de) \quad (3.3)$$

where for any (y, z, u) ,

$$f^{n,m}(s, y, z, u) = f^n(s, y + Y_s^m, z + Z_s^m, u + U_s^m) - f^m(s, Y_s^m, Z_s^m, U_s^m) \quad (3.4)$$

satisfies

$$\langle y, f^{n,m}(t, y, z, u) \rangle \leq k\gamma(t)|y| \left[\rho(|y|) + \rho \left(\frac{2A}{n} \right) + \rho \left(\frac{2A}{m} \right) \right] + k\beta(t)|y|(|z| + |u|). \quad (3.5)$$

Putting $H(u) = k\sqrt{u} \rho(\sqrt{u})$. Then the generator $f^{n,m}$ satisfies the assumption (A) with

$$\psi(u) = H(u), \quad \alpha(t) = k\beta(t) \quad \text{and} \quad \varphi_t = k\gamma(t) \left(\rho\left(\frac{2A}{n}\right) + \rho\left(\frac{2A}{m}\right) \right).$$

By Proposition 2.2, there exists a constant $c > 0$ depending on k, ρ, n, m and β such that for $0 \leq t \leq T$,

$$\begin{aligned} \mathbf{E} \left[\sup_{t \leq s \leq T} |\widehat{Y}_s^{n,m}|^2 \right] + \mathbf{E} \left[\int_t^T |\widehat{Z}_s^{n,m}|^2 ds \right] + \mathbf{E} \left[\int_t^T \int_E |\widehat{U}_s^{n,m}(e)|^2 \lambda(de) ds \right] \\ \leq c \int_t^T \gamma(s) H \left(\mathbf{E} \left[\sup_{s \leq r \leq T} |\widehat{Y}_r^{n,m}|^2 \right] \right) ds + c \times \int_0^T \gamma^2(s) ds. \end{aligned}$$

Then using the same arguments as in [2], we deduce that the sequence $(\Theta^n) = (Y^n, Z^n, U^n)$ is a Cauchy sequence in the space $\mathcal{B}^2(\mathbf{R}^k)$. Letting $n \rightarrow \infty$ in eq.(3.2) in uniform convergence in probability, implies that the triple (Y, Z, U) is solution to eq.(2.1). This completes the proof. $\square$

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