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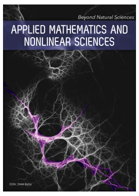

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New Exact Solutions for Generalized (3+1) Shallow Water-Like (SWL) Equation

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Abstract

In this study, we use the improved Bernoulli sub-equation function method for exact solutions to the generalized (3+1) shallow water-like (SWL) equation. Some new solutions are successfully constructed. We carried out all the computations and the graphics plot in this paper by Wolfram Mathematica.

Keywords: Generalized (3+1) shallow water-like (SWL) equation ,Improved Bernoulli sub-equation function method, Exact solution
AMS 2010 codes: 35Q35, 37N10

1 Introduction

In various fields of physical sciences, nonlinear evolution equations (NLEEs) and their exact solutions are important for non-linear phenomena. In this paper, generalized (3 + 1) shallow water-like (SWL) equation [1,2] which is one of these equations will be discussed and new solutions will be examined.

$$u_{xxxy} + 3u_{xx}u_y + 3u_x u_{xy} - u_{yt} - u_{xz} = 0. \tag{1}$$

There are some studies on this equation. Rational solutions and lump solutions are obtained for equation(1) by Zhang et al. [1] and Grammian and Pfaffian solutions are obtained by Tang et al. [2]. Also, this equation solved by Tian and Gao [3] via the tanh method,by Zayed [4] via the (G′/G) expansion method. Lump-type solutions and their interaction solutions are generated by Sadat [5]. In this context, various papers were presented to the literature [6–23]. The organization of this paper is as follows: firstly, we give the methodology of the improved Bernoulli sub-equation function method. Then we apply this method to the SWL equation for finding new exact solutions. At last, we give a conclusion.

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2 Material ve Method

In this part, we use the improved Bernoulli sub-equation function method [24–28] for solutions eq. (1).
Step 1. Let's consider the following partial differential equation;

$$P(u,u_x,u_y,u_z,u_t,u_{xx},u_{xy},...)=0. \tag{2}$$

and take the wave transformation

$$u(x,y,z,t)=U(\xi),\xi=x+ky+mz-wt, \tag{3}$$

where k,m and w are nonzero constants.Substituting Eq. (2) into Eq. (3), we obtain the following nonlinear ordinary differential equation;

$$N=(U,U',U'',U''',...)=0. \tag{4}$$

Step 2. Considering trial equation of solution in Eq. (4), it can be written as following;

$$U(\xi)=\frac{\sum_{i=0}^{n}a_iF^i(\xi)}{\sum_{j=0}^{m}b_jF^j(\xi)} \tag{5}$$

According to the Bernoulli theory, we can consider the general form of Bernoulli differential equation for as following;

$$F'=\alpha F+\beta F^M,\alpha,\beta\neq 0,M\in R-0,1 \tag{6}$$

where F is Bernoulli differential polynomial. Substituting Eq. (5-b6) into Eq. (4), it converts an equations of polynomial $\sigma(F)$ as following;

$$\sigma(F)=\rho_sF^s+...+\rho_1F+\rho_0=0 \tag{7}$$

According to the balance principle, we can determine the relationship between n,m and M.
Step 3. The coefficients of $\sigma(F)$ all be zero will yield us an algebraic system of equations;

$$\rho_i=0,i=0,...,s \tag{8}$$

Solving this system of equation, we reach the values of$a_0,...,a_n$ and $b_0,...,b_m$. Step 4. When we solve nonlinear Bernoulli differential equation Eq. (6), we obtain the following two situations according to α and β ;

$$F(\xi)=[\frac{-\beta}{\alpha}+\frac{E}{e^{\alpha(M-1)\xi}}]^{\frac{1}{1-M}},\alpha\neq\beta \tag{9}$$

$$F(\xi)=[\frac{(E-1)+(E+1)tanh(\frac{\alpha(1-M)\xi}{2}}{1-tanh(\frac{\alpha(1-M)\xi}{2}}]^{\frac{1}{1-M}},\alpha=\beta,E\in R. \tag{10}$$

3 Findings

In this section, application of the improved Bernoulli sub-equation function method to SWL equation is presented. Using the wave transformation on Eq. (1)

$$u(x,y,z,t)=U(\xi),\xi=x+ky+mz-wt, \tag{11}$$

we get the following nonlinear ordinary differential equation:

$$kU^{(4)}+6kU'U''+(kw-m)U''=0. \tag{12}$$

Integrating the equation in (12), we get

$$kU''' + 3k[U']^2 + (kw - m)U' = 0. \tag{13}$$

Finally,if we write V instead of U', the equation (13) becomes a second order nonlinear ordinary differential equation:

$$kV'' + 3kV^2 + (kw - m)V = 0. \tag{14}$$

Balancing Eq. (14) by considering the highest derivative V'' and the highest power V^2, we obtain

$$n + 2 = 2M + m.$$

Choosing $M = 2, m = 1$, gives $n = 3$. Thus, the trial solution to Eq. (1) takes the following form:

$$U(\xi) = \frac{a_0 + a_1 F(\xi) + a_2 F^2(\xi) + a_3 F^3(\xi)}{b_0 + b_1 F(\xi)}. \tag{15}$$

where $F' = \alpha F + \beta F^2, \alpha, \beta \neq 0$. Substituting Eq. (15), its second derivative and power along with $F' = \alpha F + \beta F^2, \alpha, \beta \neq 0$, into Eq. (14), yields a polynomial in F. Solving the system of the algebraic equations, yields the values of the parameter involved. Substituting the obtained values of the parameters into Eq. (15), yields the solutions to Eq. (1). We can find following coefficients:

Case 1

$$a_0 = -\frac{(1+k)wb_0}{3k}, a_1 = \frac{2w^{3/2}b_0}{\sqrt{\frac{k}{1+k}}} - \frac{(1+k)wb_1}{3k}, a_2 = -2w^2 b_0 + \frac{2w^{3/2}b_1}{\sqrt{\frac{k}{1+k}}}, a_3 = -2w^2 b_1, \sigma = -\frac{\sqrt{w}}{\sqrt{\frac{k}{1+k}}}; \tag{16}$$

Case 2

$$a_0 = 0, a_1 = 0, a_2 = \frac{2iw^{3/2}b_1}{\sqrt{\frac{k}{1+k}}}, a_3 = -2w^2 b_1, b_0 = 0, \sigma = -\frac{i\sqrt{w}}{\sqrt{\frac{k}{1+k}}}; \tag{17}$$

Case 3

$$a_0 = 0, a_1 = -\frac{mb_1}{3k}, a_2 = -\frac{4m^{3/2}b_1}{k^{3/2}}, a_3 = -\frac{8m^2 b_1}{k^2}, b_0 = 0, w = \frac{2m}{k}, \sigma = \frac{\sqrt{m}}{\sqrt{k}}; \tag{18}$$

Case 4

$$a_0 = 0, a_1 = \frac{(-kw)b_1}{3k}, a_2 = -\frac{2iw\sqrt{m-kw}b_1}{\sqrt{k}}, a_3 = -2w^2 b_1, b_0 = 0, \sigma = \frac{i\sqrt{m-kw}}{\sqrt{k}}; \tag{19}$$

Choosing the suitable values of parameters, we performed the numerical simulations of the obtained solutions for (16,17) case by plotting their 2D and 3D.

4 Result and Discussion

In this article, new solutions are obtained for the SWL equation using the IBSEFM method. We have seen that the results we obtained are new solutions when we compare them with previous ones. The results may be useful to explain the physical effects of various nonlinear models in non-linear sciences. IBSEFM is a powerful and efficient mathematical tool that can be used to process various nonlinear mathematical models.

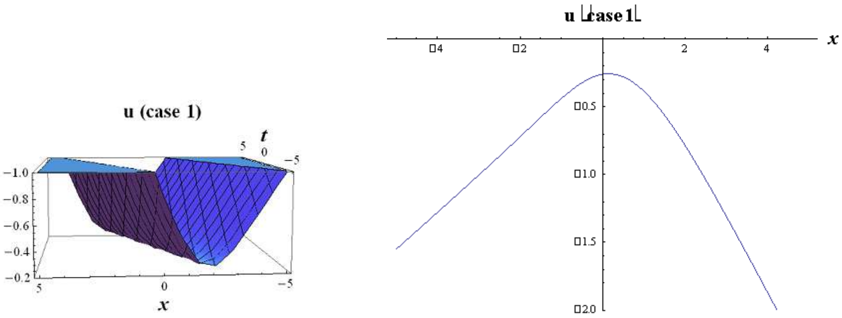

Fig. 1 The 3D and 2D surfaces of the solution for (16) for suitable values

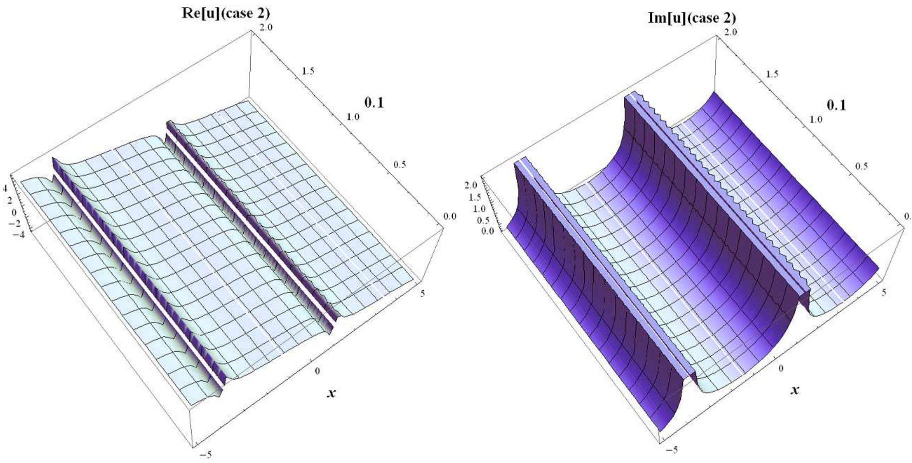

Fig. 2 The 3D and 2D surfaces of the solution for (17) for suitable values

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