Applied Mathematics and Nonlinear Sciences 4(2) (2019) 315–330

Applied Mathematics and Nonlinear Sciences

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Dynamics of the Modified n-Degree Lorenz System

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Submission Info

Communicated by Juan Luis García Guirao
Received March 24th 2019
Accepted May 24th 2019
Available online August 22nd 2019

Abstract

The Lorenz model is one of the most studied dynamical systems. Chaotic dynamics of several modified models of the classical Lorenz system are studied. In this article, a new chaotic model is introduced and studied computationally. By finding the fixed points, the eigenvalues of the Jacobian, and the Lyapunov exponents. Transition from convergence behavior to the periodic behavior (limit cycle) are observed by varying the degree of the system. Also transiting from periodic behavior to the chaotic behavior are seen by changing the degree of the system.

Keywords: Modified Chaotic System, Chaos, Periodic, Lorenz System & Dynamical Systems.
AMS 2010 codes: 39A10; 39A11.

The non-linear dynamical system is well known for its various applications such as in population growth, economics and so on. In the early 1963, Lorenz introduced the first weather forecasting dynamical system [1]. He showed that small changes in the initial points of the weather system could alter the outcome with significantly surprising results. This phenomena is known as the "butterfly effect" as the system is dependent on its initial conditions. There are plenty of research has been carried out in order to understand the chaotic behavior of the chaotic dynamical systems [2–10]. Edward Lorenz has published reports of a "strange attractor" where he discovered this attractor as a result of using computers to find the approximate numerical solutions to a system of differential equations in the weather model. Centering this classical model, there are several modifications of the model are introduced and studied their chaotic behaviors [11–20]. Some of the previous work by Zhou et al., Qi et al. and Yan have presented the modified Lorenz system and has discussed the system in terms of stability and dynamical behavior [21, 22].

Over the decades chaos theory has matured as a science that has given us an intense insights of some natural phenomena of the nonlinear systems. The term chaos is coined formally by Li and Yorke in 1975, where they established a simple criterion for chaos in one-dimensional difference equations, the well-known "period three implies chaos" [23]. There are a lot of applications of chaos in circuit design and

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ISSN 2444-8656 doi:10.2478/AMNS.2019.2.00028

many other field including various biological systems. It is well accepted among modern mathematicians that a chaotic dynamical system is not only important but also useful from the very pure mathematics views [24].

In the present study, a variant model is defined which corresponds to a natural number n, we call it as the degree of the system and the dynamical system $D_{a,r,b}[n]$ is defined as follows:

$$\frac{dx}{dt} = a(by-x); \frac{dy}{dt} = rx - xz; \frac{dz}{dt} = (xy)^n - bz \tag{1}$$

where x, y and z are real variables and a, r and b are real parameters. Obviously the above system Eq.(1) is dissipative with an exponent contraction rate of

$$\frac{dV}{dt} = e^{-at-bt}$$

since the $divV = \frac{\delta}{\delta x}(\frac{dx}{dt}) + \frac{\delta}{\delta y}(\frac{dy}{dt}) + \frac{\delta}{\delta z}(\frac{dz}{dt}) = -a - b < 0$ where V is volume. It is worth mentioning that the modified model when $n = 2$ is studied in [25].

This new chaotic model is studied here by finding the fixed points, the eigenvalues of the Jacobian, and the Lyapunov exponents. The numerical simulations, the time series analysis, and the projections to the xy-plane, xz-plane, and yz-plane are conducted to highlight the chaotic behaviour. Also the role of the degree of system is taken into account to see the dominance behavior in characterizing the dynamical behavior of the system. This study fetches a new chaotic attractor, found by modification of the Lorenz system by a n-degree term. The detailed numerical and theoretical analysis reveals that the proposed system shows chaotic behaviour and the property of a two-scroll attractor like the Lorenz attractor.

It is has been seen in literatures that modified Lorenz system have stumbled upon or sought to discover the applications in reality. Tigan [26] shows a promising modified Lorenz system that has the potential application in secure communications. There are a lot of applications that chaos can be applied to other science and engineering problems [27–30].

1 Stability of the Fixed Points of the Model $D_{a,r,b}[n]$

Prior to the local stability analysis of the fixed points, let us see how does the 3D phase portrait looks like for different values of the degree (n) when the parameters a, r and b are fixed. The 3D phase portrait can be built by analogy with 2D as shown in the following Fig. 1. It is noted that the parameters are set to be $a = 2, r = 4, b = 1$ and the variable $(x, y, z) \in [-10, 10] \times [-10, 10] \times [-10, 10] \subset \mathbb{R}^3$ with $n = 1, 34, 67$ and 100. In the first, second and third rows of the Fig.1 the x-stacked planes, y-stacked planes and z-stacked planes respectively are shown.

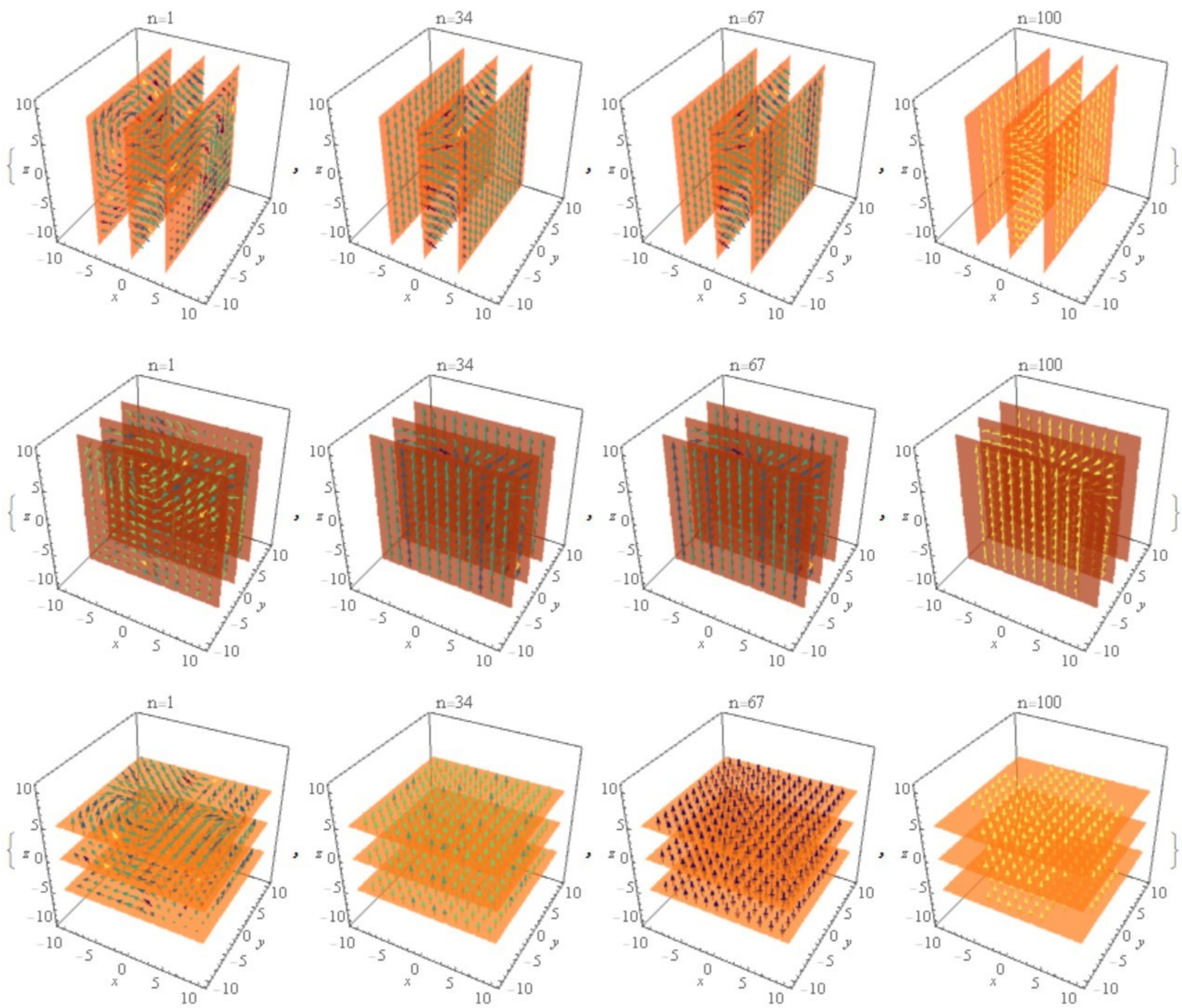

Fig. 1 First, second and third rows from top, x-stacked planes, y-stacked planes and z-stacked planes respectively are shown.

Now we shall describe the local stability of the fixed points of the model Eq.(1). The fixed points can be obtained by setting the derivatives Eq.(1) equal to zero.

The system Eq.(1) has three fixed points $(0,0,0)$, $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ and $(\sqrt{b}\sqrt{(br)^{1/n}}, \frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$.

Here the Jacobian evaluated at the above fixed point say (x^*, y^*, z^*) has the form

$$\begin{pmatrix} -a & ab & 0 \\ r - z^* & 0 & -x^* \\ ny^*(x^*y^*)^{n-1} & nx^*(x^*y^*)^{n-1} & -b \end{pmatrix}$$

In order to understand the local behaviour of the fixed points one needs to determine the signs of the real parts of the eigenvalues of the Jacobian evaluated at the fixed point. The following *Theorem 1.1* is useful for checking the signs of the real parts of the eigenvalues of a 3×3 matrix.

Theorem 1.1. *Routh-Hurwitz Criterion*

The Routh-Hurwitz test [9] applied to a general third degree polynomial

$$a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

states that the number of sign changes in the sequence $\{a_3, a_2, H, a_0\}$ *where* $H = a_2a_1 - a_3a_0$ *is equal to the number of roots of the polynomial having positive real part, and if all entries in the sequence are nonzero and of the same sign, then all roots have negative real part.*

1.1 Stability of the Fixed Point (0,0,0) of the System $D_{a,r,b}[n]$

The Jacobian about the equilibrium point $(0,0,0)$ of the system Eq.(1) is

$$\begin{pmatrix} -a & ab & 0 \\ r & 0 & 0 \\ 0 & 0 & -b \end{pmatrix}$$

and it has three eigenvalues $-b, \frac{1}{2}\left(-\sqrt{a}\sqrt{a+4br} - a\right)$ and $\frac{1}{2}\left(\sqrt{a}\sqrt{a+4br} - a\right)$.

Theorem 1.2. *The equilibrium point* $(0,0,0)$ *of the system Eq.(1) is locally asymptotically stable if*

$$a > 0, b > 0, -\frac{a}{4b} \leq r < 0$$

Proof. It can be easily seen by doing a simple algebraic calculation that the eigenvalues are negative provided

$$a > 0, b > 0, -\frac{a}{4b} \leq r < 0$$

satisfied. Then the equilibrium point $(0,0,0)$ is locally asymptotically stable. □

We have fetched a set of parameters $(a, r, b) \in \mathbb{R}^3$ such that the origin is locally asymptotically stable. The 3D plot of the parameter subspace is given below in the Fig.2.

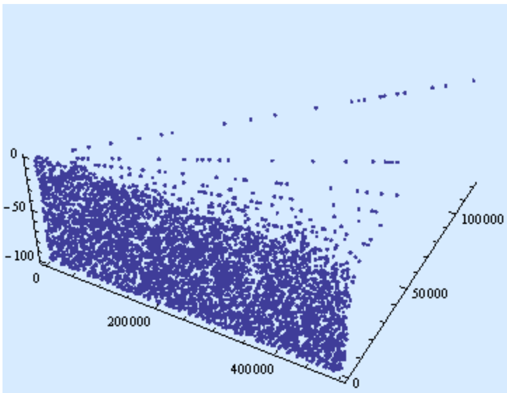

Fig. 2 Parameter Subspace of $\mathbb{R}^3$ such that the origin is locally asymptotically stable.

It is importantly noted that there is no parameter (a, b, r) in the positive octant of $\mathbb{R}^3$. If both a and b are positive then necessarily the parameter r has to be negative in order to get local asymptotically stability of the origin.

Here we illustrate some example of stability and instability of the equilibrium point $(0,0,0)$. For $a = 267, r = -\frac{267}{116}$ and $b = 29$, the equilibrium point becomes $(0,0,0)$ which is locally asymptotically stable as shown in Fig.3.

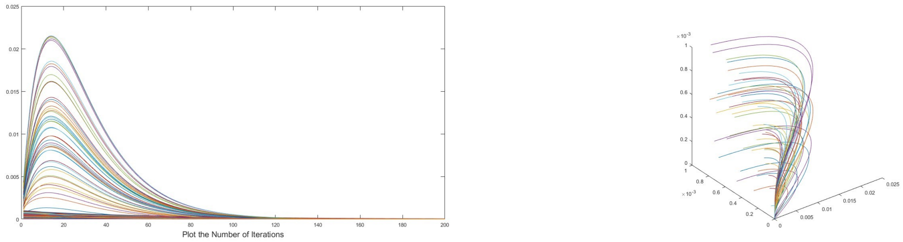

Fig. 3 Left: Locally asymptotically stable trajectories and in Right: the 3D plot of the trajectories of the equilibrium point (0,0,0) where the initial values are taken from the neighbourhood of the origin.

For the parameters $a = 56, r = 15$ and $b = 16$ the equilibrium point $(0,0,0)$ is unstable and forming strange attractor. The trajectories are shown in Fig.4.

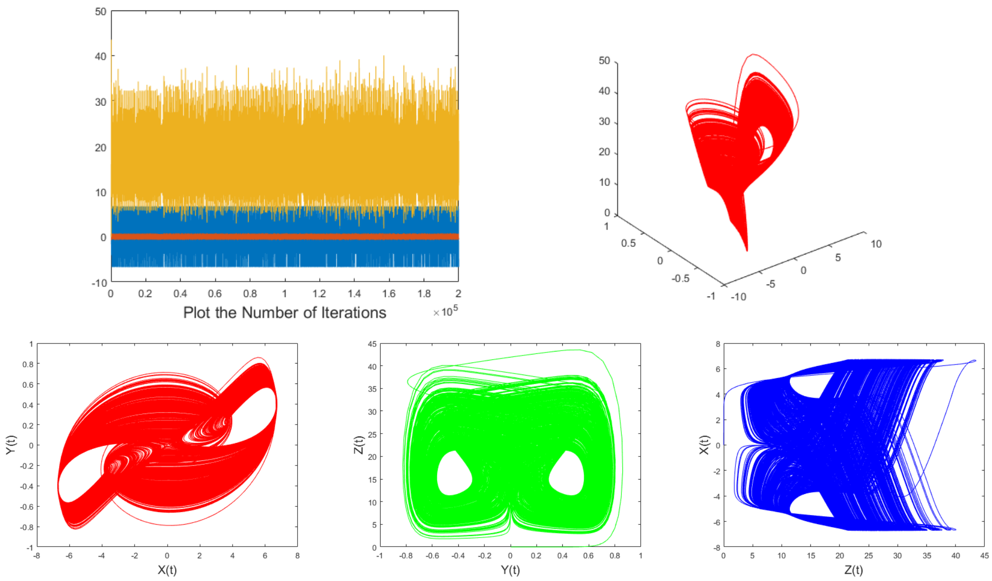

Fig. 4 Unstable trajectories which are away from the origin in the system $D_{56,15,16}$[6].

For the parameters $a = 26, r = 93$ and $b = 36$ the equilibrium point $(0,0,0)$ is unstable and forming strange attractor. The trajectories are shown in Fig.5.

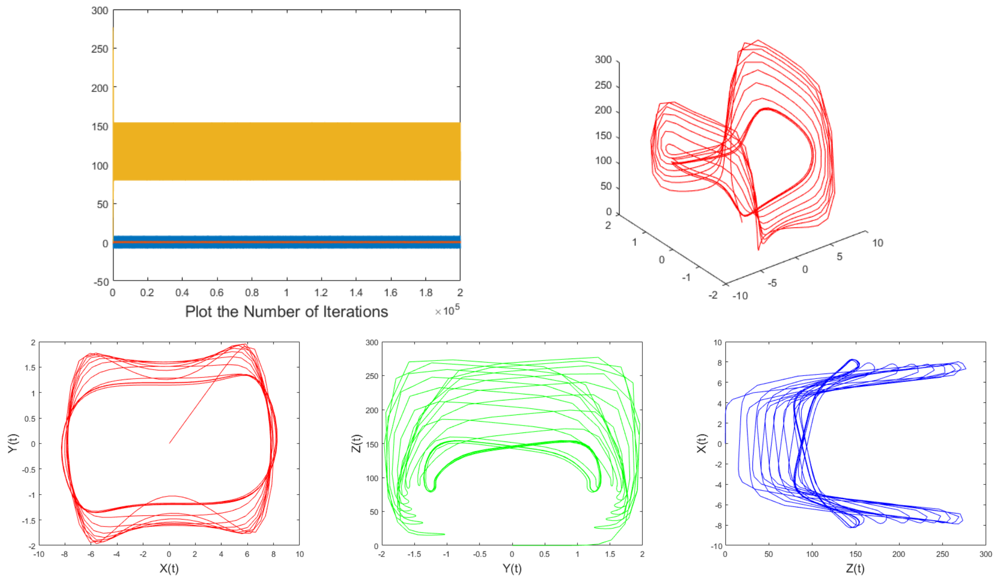

Fig. 5 Unstable trajectories (limit cycle) which are away from the origin in the system $D_{26,93,36}[5]$.

For the system $D_{9,7,54}[16]$, the eigenvalues of the Jacobian are $-63, -54$ and 54 where real part of the eigenvalues are not all negative and hence the equilibrium point $(0,0,0)$ is unstable. The trajectories are shown in Fig.6 with many initial values taken from the neighbourhood of the equilibrium point $(0,0,0)$.

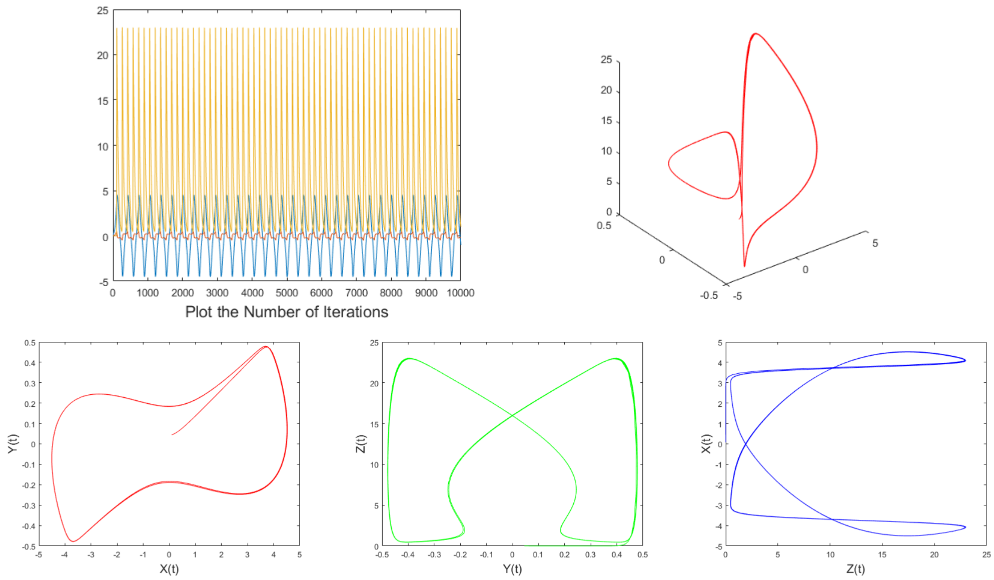

Fig. 6 Unstable trajectories (limit cycle) which are away from the origin in the system $D_{9,7,54}[16]$.

1.2 Stability of the Fixed Point $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ of the System $D_{a,r,b}[n]$

The Jacobian about the equilibrium point $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ of the system Eq.(1) is

$$\begin{pmatrix} -a & ab & 0 \\ 0 & 0 & \sqrt{b}\sqrt{(br)^{1/n}} \\ -\frac{1.^{n-1}n\left((br)^{1/n}\right)^{n-\frac{1}{2}}}{\sqrt{b}} & -1.^{n-1}\sqrt{b}n\left((br)^{1/n}\right)^{n-\frac{1}{2}} & -b \end{pmatrix}$$

Theorem 1.3. *The equilibrium point* $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ *of the system Eq.(1) is locally asymptotically stable if* $n > [\frac{a^2+ab}{3abr+b^2r}]$ *where* a, b *and* $r \in \mathbb{N}$. *Here* [] *is the greatest integer function.*

Proof. The equilibrium point $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ of the system Eq.(1) is locally asymptotically stable if real part of all the eigenvalues are negative.
Here the characteristic polynomial of the above Jacobian is

$$\lambda\left(-ab - a\lambda - b\lambda - \lambda^2\right) + bn(-2.a - \lambda)\left((br)^{1/n}\right)^n$$

Assuming all the parameters are positive, the coefficients are

$$a_3 = -1 < 0$$

$$a_2 = -a - b < 0$$

$$a_1 = bn\left((br)^{1/n}\right)^n - ab$$

$$a_0 = -2abn\left((br)^{1/n}\right)^n$$

and hence

$$H = bn(-3a - b)\left((br)^{1/n}\right)^n + ab(a+b)$$

By the *Theorem 1.1*, if sign of H as well as a_0 are both negative then the equilibrium point $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ would be locally asymptotically stable since already a_3 and a_2 are negative.
It is easy to reduce by simple algebra, the coefficient a_0 and H will be negative only if $n > [\frac{a^2+ab}{3abr+b^2r}]$ where a, b and $r \in \mathbb{N}$.
Hence the result follows.

□

For the system $D_{15,17,28}$[6], the eigenvalues of the Jacobian are $-6.50559 + 282.764i, -6.50559 - 282.764i, -29.9888$ where real part of the eigenvalues are all negative and hence the equilibrium point $(-8.84531, -0.315904, 17)$ is locally asymptotically stable. The trajectories are shown in Fig.7 with many initial values taken from the neighbourhood of the equilibrium point $(-8.84531, -0.315904, 17)$.

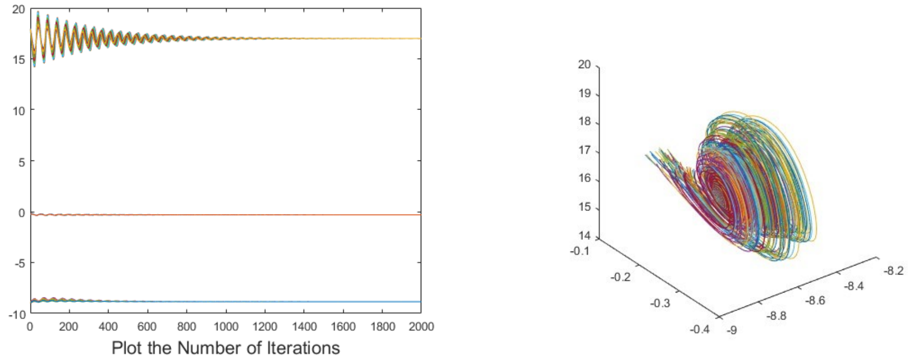

Fig. 7 Left: Locally asymptotically stable trajectories and in Right: the 3D plot of the trajectories of the equilibrium point $(-8.84531,-0.315904,17)$ where the initial values are taken from the neighbourhood of the origin.

For the system $D_{52,41,9}[15]$, the eigenvalues are $17.7339+231.063i, 17.7339-231.063i, -96.4679$ where real part of the eigenvalues are not all negative and hence the equilibrium point $(3.65333, 0.405925, 41)$ is unstable. The trajectories are shown in Fig.8 with many initial values taken from the neighbourhood of the equilibrium point $(3.65333, 0.405925, 41)$.

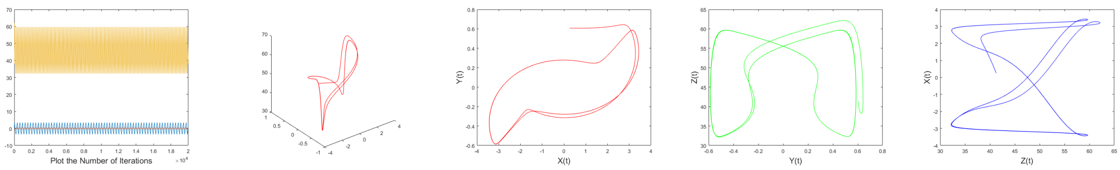

Fig. 8 Unstable trajectories which are away from the equilibrium point $(3.65333, 0.405925, 41)$ in the system $D_{52,41,9}[15]$.

1.3 Stability of the Fixed Point $(\sqrt{b}\sqrt{(br)^{1/n}}, \frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ of the System $D_{a,r,b}[n]$

The Jacobian about the equilibrium point $(\sqrt{b}\sqrt{(br)^{1/n}}, \frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ of the system Eq.(1) is

$$\begin{pmatrix} -a & ab & 0 \\ 0 & 0 & -\sqrt{b}\sqrt{(br)^{1/n}} \\ \frac{n\left((br)^{1/n}\right)^{n-\frac{1}{2}}}{\sqrt{b}} & \sqrt{b}n\left((br)^{1/n}\right)^{n-\frac{1}{2}} & -b \end{pmatrix}$$

Theorem 1.4. *The equilibrium point* $(\sqrt{b}\sqrt{(br)^{1/n}}, \frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ *of the system Eq.(1) is locally asymptotically stable if* $n > [\frac{a^2+ab}{abr-b^2r}]$ *where* a, b *and* $r \in \mathbb{N}$. *Here* $[]$ *is the greatest integer function.*

Proof. The equilibrium point $(\sqrt{b}\sqrt{(br)^{1/n}}, \frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ of the system Eq.(1) is locally asymptotically stable if real part of all the eigenvalues are negative.
Here the characteristic polynomial of the above Jacobian is

$$-\lambda(a+\lambda)(b+\lambda) - bn(2a+\lambda)\left((br)^{1/n}\right)^n$$

Assuming all the parameters are positive, the coefficients are

$$a_3 = -1 < 0$$

$$a_2 = -a - b < 0$$

$$a_1 = -b\left(a + n\left((br)^{1/n}\right)^n\right)$$

$$a_0 = -2abn\left((br)^{1/n}\right)^n$$

and hence

$$H = bn(b-a)\left((br)^{1/n}\right)^n + ab(a+b)$$

By the *Theorem 1.1*, if sign of H as well as a_0 are both negative then the equilibrium point $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ would be locally asymptotically stable since already a_3 and a_2 are negative.
As before it is easy to reduce by simple algebra, the coefficient a_0 and H will be negative only if $n > [\frac{a^2+ab}{abr-b^2r}]$ where a, b and $r \in \mathbb{N}$.
Hence the result follows.

□

It is noted that both the *Theorem 1.3* and *Theorem 1.4* hold good for certain ranges of parameters a, r and b involved in the system. Here computationally we have fetched a set of parameters $(a, r, b) \in \mathbf{R}^3$ which is plotted in the following Fig.9.

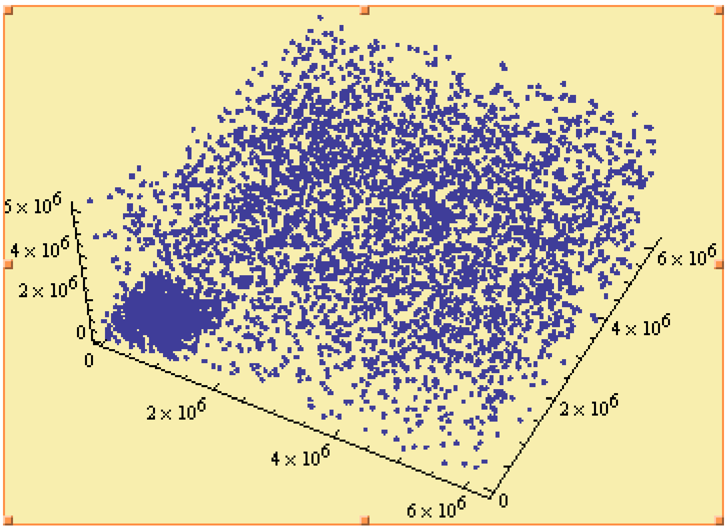

Fig. 9 Parameter Plot

For the system $D_{15,17,28}$[6], the eigenvalues are $-6.50559 + 282.764i, -6.50559 - 282.764i, -29.9888$ where real part of the eigenvalues are all negative and hence the equilibrium point $(8.84531, 0.315904, 17)$ is locally asymptotically stable. The trajectories are shown in Fig.10 with many initial values taken from the neighbourhood of the equilibrium point $(8.84531, 0.315904, 17)$.

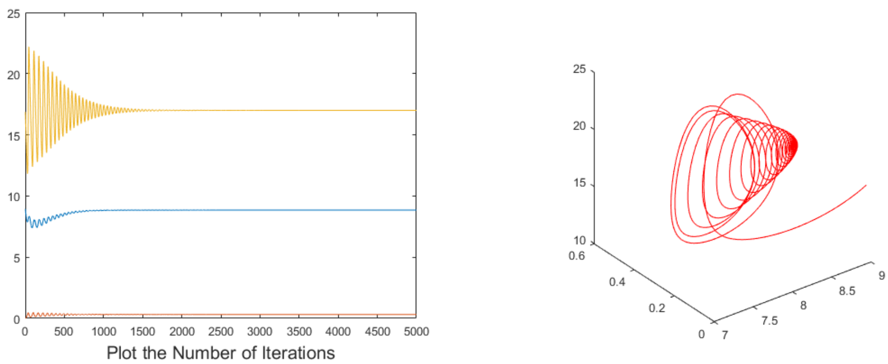

Fig. 10 Left: Locally asymptotically stable trajectories and in Right: the 3D plot of the trajectories of the equilibrium point $(8.84531, 0.315904, 17)$ where the initial values are taken from the neighbourhood of the origin.

For the system $D_{52,41,9}[13]$, the eigenvalues are $17.3142+215.992i, 17.3142-215.992i, -95.6285$ where real part of the eigenvalues are not all negative and hence the equilibrium point $(3.76576, 0.418418, 41)$ is unstable. The trajectories are shown in Fig.11 with many initial values taken from the neighbourhood of the equilibrium point $(3.76576, 0.418418, 41)$.

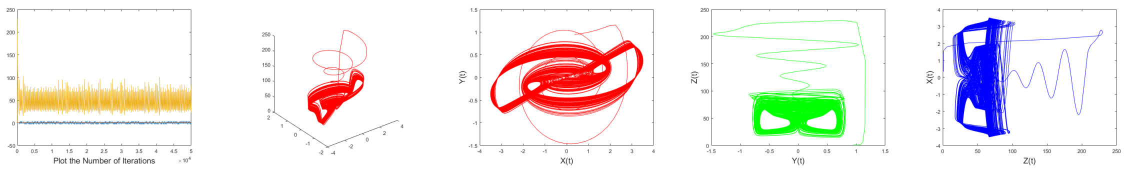

Fig. 11 Unstable trajectories which are away from the equilibrium point $(3.76576, 0.418418, 41)$ in the system $D_{52,41,9}[13]$.

1.4 Trajectories Away from the Fixed points of the System $D_{a,r,b}[n]$

Here we set parameters $a = 52, r = 41$ and $b = 9$ and also chosen three different degree $n = 13, 21$ and 14. For any initial values taken from the neighbourhood of the origin, the system $D_{52,41,9}[13]$ possesses to a limit cycle. Similarly, For any initial values taken from the neighbourhood of the other two equilibrium points $(\sqrt{b}\sqrt{(br)^{1/n}}, \frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$ and $(-\sqrt{b}\sqrt{(br)^{1/n}}, -\frac{\sqrt{(br)^{1/n}}}{\sqrt{b}}, r)$, the system $D_{52,41,9}[21]$ and $D_{52,41,9}[14]$ possess limit cycles respectively. This has been depicted graphically in the following Fig.11. The trajectory plots are given too in the Fig.12.

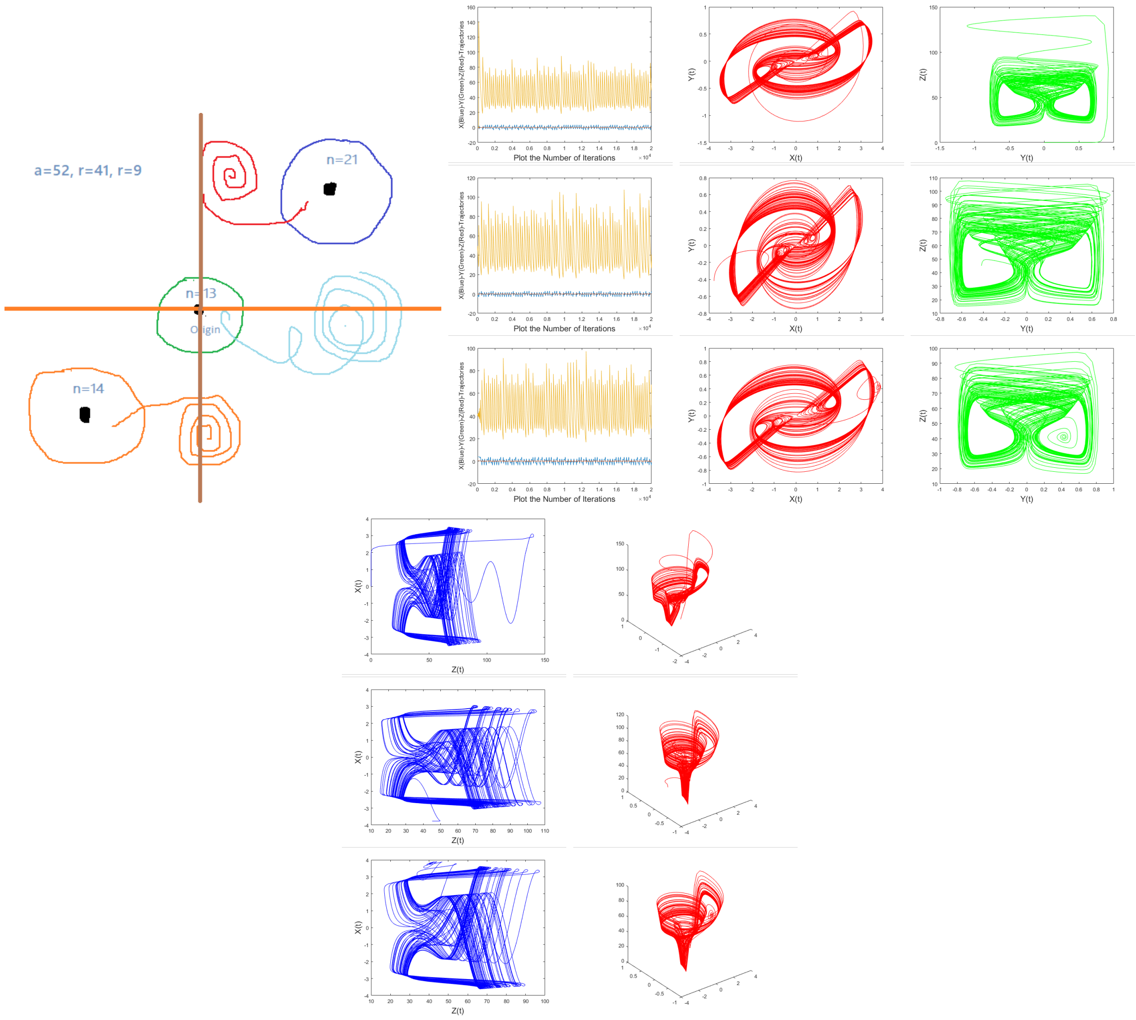

Fig. 12 Unstable trajectories which are away from the all the equilibrium points of the system $D_{52,41,9}[n]$.

All the parameters $a = 52$, $r = 41$ and $r = 9$ keeps away the trajectories from all the three equilibrium points of the system and forming limit cycles, whereas the degree of the system is controlling the repelling behavior of those equilibrium points as observed.

2 Dominance of Parameters Involved in the system $D_{a,r,b}[n]$

In this section, we would like to experience what happens by varying the parameters involved in the system. This would computationally able to understand the dominating governing parameters among all the parameters involved.

2.1 Transiting from Convergence Behavior to Periodic Behavior of the System $D_{a,r,b}[n]$

Here we shall discover while changing the degree n of the system $D_{a,r,b}[n]$ takes the system behavior from convergence of the trajectories to the periodic trajectories (limit cycles).

Here we have equilibrium three parameters $a = 51, r = 3$ and $b = 4$ and five different set initial values are taken randomly from the neighbourhood of the origin. Different colors in the trajectory plots represents different trajectories corresponding to different set of initial values.

In the system $D_{51,3,4}[n]$, we vary the degree of the system n from 1 to 15 one by one and see their corresponding trajectories which are plotted in the Fig.12.

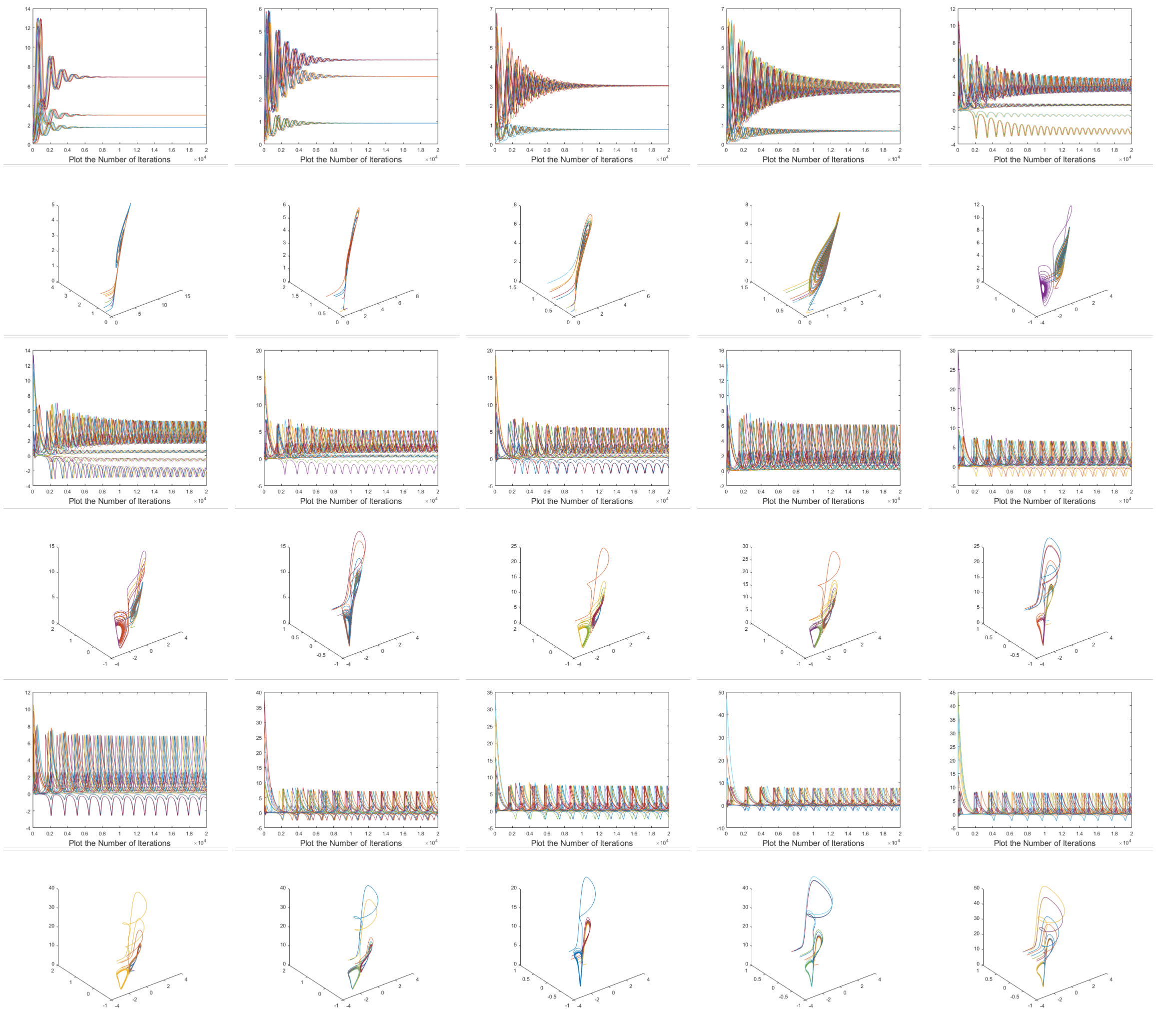

Fig. 13 Trajectories (Up) and three dimensional plots (Down) respectively for the system $D_{51,3,4}[n]$ where the degree of the system n varying from 1 to 15.

Sl. No.	System $D_{51,3,4}[n]$	Character of the System
1	$D_{51,3,4}[1]$	converges to the equilibrium point $(6.9282, 1.7321, 3)$
2	$D_{51,3,4}[2]$	converges to the equilibrium point $(3.7225, 0.9306, 3)$
3	$D_{51,3,4}[3]$	converges to the equilibrium point $(3.0262, 0.7565, 3)$
4	$D_{51,3,4}[4]$	converges to the equilibrium point $(2.7285, 0.6821, 3)$
5	$D_{51,3,4}[5]$	Periodic with high period.
6	$D_{51,3,4}[6]$	Periodic with high period.
7	$D_{51,3,4}[7]$	Periodic with high period.
8	$D_{51,3,4}[8]$	Periodic with high period.
9	$D_{51,3,4}[9]$	Periodic with high period.
10	$D_{51,3,4}[10]$	Periodic with high period.
11	$D_{51,3,4}[11]$	Periodic with high period.
12	$D_{51,3,4}[12]$	Periodic with high period.
13	$D_{51,3,4}[13]$	Periodic with high period.
14	$D_{51,3,4}[14]$	Periodic with high period.
15	$D_{51,3,4}[15]$	Periodic with high period.

Table 1 The system $D_{51,3,4}[n]$ behavior according as the degree of the system n varying from 1 to 15.

2.2 Transiting from Periodic Behavior to Chaotic Behavior of the System $D_{a,r,b}[n]$

Here in the system $D_{76,36,31}[n]$, we vary the degree of the system n from 1 to 15 one by one and see their corresponding trajectories which are plotted in the Fig.13. The chaotic behavior is detected through the positive Lyapunov exponents of the three dimensional trajectory as per the algorithm described in the article [31].

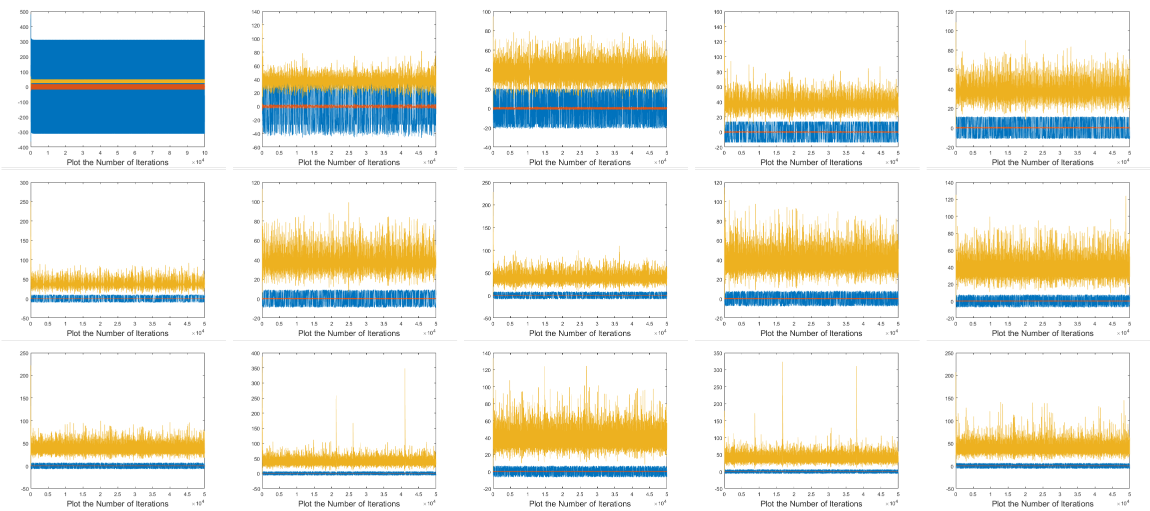

Fig. 14 Trajectories are plotted for the system $D_{76,36,31}[n]$ where the degree of the system n varying from 1 to 15.

It is seen by computation that the system is possessing to a periodic trajectory only if $n = 1$ otherwise for all the other values of n ranging from 2 to 15 the system approaches either to the chaotic trajectories or it is unbounded. Now we consider a set of parameters a=3, r=12 and b=24 and varying n from 1 to 10. The trajectories and their corresponding 3D plots for each of the system with different degree ranging from 1 to 10 are given in the following Fig. 14.

Sl. No.	System $D_{51,3,4}[n]$	Character of the System
1	$D_{76,36,31}[1]$	Periodic with high period.
2	$D_{76,36,31}[2]$	Chaotic with Positive Lyapunov exponent 0.6324
3	$D_{76,36,31}[3]$	Chaotic with Positive Lyapunov exponent 0.7634
4	$D_{76,36,31}[4]$	Chaotic with Positive Lyapunov exponent 0.4589
5	$D_{76,36,31}[5]$	Chaotic with Positive Lyapunov exponent 0.3412
6	$D_{76,36,31}[6]$	Chaotic with Positive Lyapunov exponent 0.2875
7	$D_{76,36,31}[7]$	Chaotic with Positive Lyapunov exponent 0.2337
8	$D_{76,36,31}[8]$	Chaotic with Positive Lyapunov exponent 0.5643
9	$D_{76,36,31}[9]$	Chaotic with Positive Lyapunov exponent 0.2981
10	$D_{76,36,31}[10]$	Chaotic with Positive Lyapunov exponent 0.6873
11	$D_{76,36,31}[11]$	Chaotic with Positive Lyapunov exponent 0.5438
12	$D_{76,36,31}[12]$	Chaotic with Positive Lyapunov exponent 0.9865
13	$D_{76,36,31}[13]$	Chaotic with Positive Lyapunov exponent 0.7541
14	$D_{76,36,31}[14]$	Chaotic with Positive Lyapunov exponent 0.6987
15	$D_{76,36,31}[15]$	Chaotic with Positive Lyapunov exponent 0.3086

Table 2 Transition of the dynamical behavior of the system $D_{51,3,4}[n]$ according as the degree of the system n varying from 1 to 15.

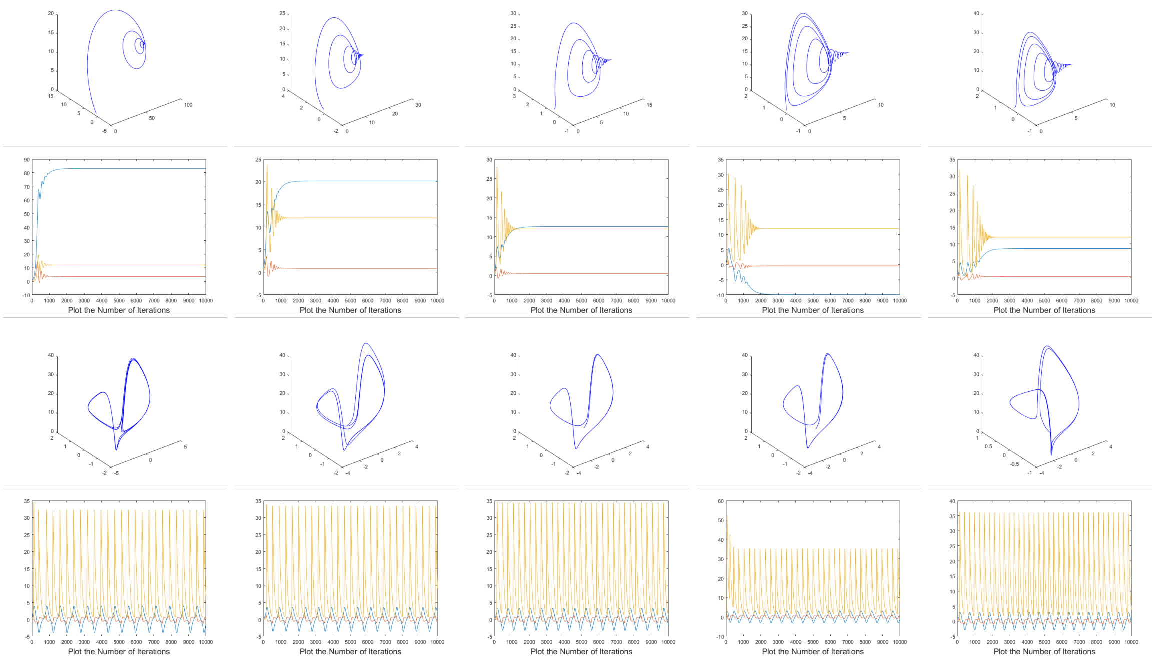

Fig. 15 Trajectories and 3D plots are plotted for the system $D_{3,12,24}[n]$ where the degree of the system n varying from 1 to 10.

We have observed that for $n = 1,2,3,4$ and 5 the system $D_{3,12,24}[n]$ converges to the equilibrium point $(83.1384, 3.4641, 12)$, $(20.1815, 0.8409, 12)$, $(12.5894, 0.5246, 12)$, $(9.9433, 0.4143, 12)$ and $(8.6307, 0.3596, 12)$ respectively. For any other values of n from 6 onwards (up to 10 are shown in Fig.14) the system possesses high periodic trajectories (limit cycles). Hence it is understood that the degree (n) of the system plays a governing role (one of the controlling parameters of the system) in transiting the behavior of the dynamics.

3 Concluding Remarks & Future Endeavours

In this article, we have found a new modified Lorenz system with some computational results and analysis. Also, we have studied the dynamic behaviour of the system and basic dynamic analysis. Throughout the study the vibrant role of the degree is adumbrated through by illustrating the transition from convergence behavior to limit cycles and from limit cycles to chaotic behavior. Different set of parameters are fetched which produce chaotic trajectories/attractors. Other classical chaotic models like Rosler system, Chen system also could be modified by introducing the degree and can be studied in the similar manners.

Acknowledgement

The first author thanks to the *Pingla Thana Mahavidyalaya, Maligram, Paschim Medinipur, West Bengal* for generous support in executing the present work.

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