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Oscillatory flow of a Casson fluid in an elastic tube with variable cross section

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Abstract

The aim of this paper was to study an oscillatory flow of a Casson fluid through an elastic tube of variable cross section. The radial displacement of tube wall is taken into consideration. The problem is modelled under the assumption that the variation of the cross section of the tube is slow in the axial direction. Cylindrical coordinate system is chosen to study the problem. The analytical expressions for axial velocity and mass flux as a function of pressure gradient are obtained. The change in pressure distribution for various pressure-radius relationships is analyzed by considering different geometries. The effects of elastic parameter, Womersley parameter and Casson parameter on excess pressure and pressure gradient along axial direction are discussed through graphs. The results reveal that the elastic parameter plays a key role in the variation of pressure along the tube. Womersley parameter has significant effect on pressure distribution. Another important observation is that the amplitude of pressure increases for growing values of Casson parameter for both tapered and constricted tubes. In addition, the pressure oscillates more for the case of locally constricted tube when compared to other geometries.

Keywords: Oscillatory flow, Casson fluid, Elasticity, Variable cross section, Womersley number.**AMS 2010 codes:** 76A05

1 Introduction

Recently, the study of flow with periodic variations has attracted much attention of researchers due to its various physiological and engineering applications. It helps to understand the characteristics of the blood flow through arteries. Oscillatory motion of a viscous liquid in a thin-walled elastic tube is investigated by Womersley (1955). Further, Womersley (1957) studied the elastic tube theory of pulse transmission and oscillatory flow in mammalian arteries. Rubinow and Keller (1972) analyzed the flow of a viscous fluid in an elastic tube with application to blood flow. Ramachandra Rao and Devanathan (1973) studied the pulsatile flow in tubes of varying cross section. Taylor and Gerrard (1977) presented a mathematical model to analyze the blood flow

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through arteries and expressed the different pressure radius relationships for elastic tube. Kaimal (1981) analyzed viscoelastic properties on oscillatory flow with consideration of pulsatile nature of tube wall. Furthermore, the significant effect of elasticity of the fluid and pulsation of the tube wall on flow characteristics was observed.

Ramachandra Rao (1983) investigated the oscillatory flow in an elastic tube of variable cross section. Furthermore, Ramachandra Rao (1983) studied the unsteady flow with attenuation in a fluid-filled elastic tube with a stenosis. An analytical solution by the method of linear approximation to describe the velocity fields for laminar periodic flow through porous walls is proposed by Chang et al. (1989). Sankar and Jayaraman (2001) discussed the nonlinear analysis of oscillatory flow in the annulus of an elastic tube. Misra and Ghosh (2003) considered the viscous fluid in a porous elastic vessel with variable cross section to explain the application of haemodynamic flows. In addition, they noticed that the velocity distribution in a small vessel depends significantly on geometry of the wall and its elastic nature.

Vajravelu et al. (2011) considered the case of inserting a catheter into an elastic tube to observe the changes in blood flow pattern by taking Herschel–Bulkley fluid. Unsteady flow of a Jeffrey fluid in an elastic tube with a stenosis was studied by Sreenadh et al. (2012). Omer and Staples (2012) analyzed the dynamics of pulsatile flows through elastic micro-tubes. Sochi (2014) proposed the expression for the volumetric flow as a function of pressure in elastic tube using two pressure area constitutive relationships. Recently, Vajravelu et al. (2016) analyzed the peristaltic transport of Casson fluid in an elastic tube.

Motivated by the abovementioned studies, oscillatory flow of a Casson fluid in an elastic tube with variable cross section is studied in the current paper. Slowly varying cross section of the tube wall and radial displacement are taken into consideration. Expressions for axial velocity and mass flux in terms of pressure gradient are derived. Different pressure radius relationships are considered to understand the pressure variation along the different tube geometries. The influence of physical parameters such as elastic parameter, Womersley number and Casson parameter on the variation of excess pressure is presented graphically.

2 Mathematical Formulation

Consider the oscillatory flow of a Casson fluid through elastic tube with variable cross section. The cylindrical coordinate system (r, θ, z) , where z is taken along the flow direction and r is perpendicular to it. The rheological equation for the Casson fluid is given as (Nakamura and Sawada (1988), Elabde et al., (2001))

$$\tau_{ij} = \begin{cases} (\mu_b + P_y/\sqrt{2\pi}) 2e_{ij}, & \pi > \pi_c \\ (\mu_b + P_y/\sqrt{2\pi_c}) 2e_{ij}, & \pi < \pi_c \end{cases} \quad (1)$$

where $\tau_{ij} = e_{ij}e_{ij}$ and $P_y = \frac{\mu_b\sqrt{2\pi}}{\beta}$

here, $\tau_{ij} = (i, j)^{th}$ component of stress tensor, $e_{ij} = (i, j)^{th}$ component of deformation rate, $\pi =$ Product of component of deformation rate with itself, $\pi_c =$ Critical value of this product depends on the non-Newtonian fluid, μ_b is the plastic dynamic viscosity of the non-Newtonian fluid, $\beta =$ Casson parameter and $P_y =$ Yield stress of the fluid.

The equations governing the motion are

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial \bar{r}} + \nu \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) \quad (2)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial \bar{z}} + \nu \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right) \quad (3)$$

here, the radius of the cross section of the tube is $r = a(z)$. The velocity components along radial and axial directions are u, w , respectively, ρ_0 is the fluid density, ν is the kinematic viscosity and p is the pressure.

The continuity equation is

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (4)$$

The radial displacement of tube wall ξ is given as (Ramachandra Rao (1983))

$$\frac{\partial^2 \bar{\xi}}{\partial \bar{t}^2} = \frac{1}{h\rho} \left(p - 2\nu\rho_0 \frac{\partial \bar{u}}{\partial \bar{r}} \right)_{r=a} - \frac{B}{\rho} \frac{\xi}{a^2} \quad (5)$$

where $B = \frac{E}{(1-\sigma^2)}$ here, h and ρ are the thickness and density of the material tube, respectively, E is the Youngs modulus and σ is Poisson's ratio.

The corresponding boundary conditions are

$$\bar{u} = \frac{\partial \bar{\xi}}{\partial \bar{t}}, \quad \bar{w} = 0; \quad \text{at} \quad \bar{r} = a_0 S(z) \quad (6)$$

where a_0 = Tube radius in the absence of elasticity.

The non-dimensional quantities are

$$\begin{aligned} u &= \frac{\bar{u}}{\varepsilon U_0}, & w &= \frac{\bar{w}}{U_0}, & t &= \omega \bar{t}, & \xi &= \frac{\bar{\xi}}{a_0}, \\ r &= \frac{\bar{r}}{a_0}, & z &= \frac{\bar{z}}{a_0}, & p &= \frac{\varepsilon a_0 \bar{p}}{\rho_0 \nu U_0}, \end{aligned} \quad (7)$$

where $\varepsilon = \frac{a_0}{L} (\ll 1)$.

here, U_0 is the characteristic velocity, ω is the frequency of oscillatory flow and L is characteristic length.

3 Solution of the Problem

Substituting the abovementioned non-dimensional quantities, Eqs. (2)–(6) takes the form

$$\varepsilon^2 \alpha^2 \frac{\partial u}{\partial t} + \varepsilon^3 R \left(u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \varepsilon^2 \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) + \varepsilon^4 \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial z^2} \quad (8)$$

$$\alpha^2 \frac{\partial w}{\partial t} + \varepsilon R \left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + \varepsilon^2 \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 w}{\partial z^2} \quad (9)$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (10)$$

$$\varepsilon^2 \frac{\rho h}{\rho_0 a_0} \frac{\partial^2 \xi}{\partial t^2} = \frac{1}{R S_t^2} \left(p - 2\varepsilon^2 \frac{\partial u}{\partial r} \right)_{r=S} - \frac{1}{\lambda_1^2} \frac{\xi}{S^2} \quad (11)$$

$$u = S_t \frac{\partial \xi}{\partial t} \quad \text{at} \quad r = S \quad (12)$$

$$w = 0 \quad \text{at} \quad r = S \quad (13)$$

where $\alpha = a_0 \left(\frac{\omega}{\nu} \right)^{\frac{1}{2}}$, $R = \frac{U_0 a_0}{\nu}$, $S_t = \frac{\omega a_0}{U_0}$, $\lambda_1^2 = \frac{\omega^2 L^2 \rho_0 a_0}{c^2 \rho h}$ and $c^2 = \frac{B}{\rho}$
here, α is the Womersley parameter, R is the Reynolds number and S_t is the Strouhal number.
Now, neglecting the higher-order terms of ε and the steady oscillatory flow, we have

$$(u, w, p, \xi) = e^{it} (\tilde{u}, \tilde{w}, \tilde{p}, \tilde{\xi}) \quad (14)$$

The Eqs.(8)–(13) reduces to

$$\frac{\partial \tilde{p}}{\partial r} = 0 \quad (15)$$

$$\left(1 + \frac{1}{\beta}\right) \left(\frac{\partial^2 \tilde{w}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{w}}{\partial r}\right) + \lambda^2 \tilde{w} = \frac{\partial \tilde{p}}{\partial z} \quad (16)$$

$$\frac{\partial \tilde{u}}{\partial r} + \frac{\tilde{u}}{r} + \frac{\partial \tilde{w}}{\partial z} = 0 \quad (17)$$

$$\xi = \frac{\lambda_1^2}{RS_t^2} S^2 p_e \quad (18)$$

$$\tilde{u} = iS_t \xi \quad \text{at } r = S \quad (19)$$

$$\tilde{w} = 0, \quad \text{at } r = S \quad (20)$$

here, $\lambda^2 = -i\alpha^2$ and p_e is the excess pressure on the tube wall.

Solving Eq. (16) using the Eq. (20), we have

$$\tilde{w} = \frac{1}{\lambda^2} \frac{dp_e}{dz} \left(1 - \frac{J_0\left(\lambda \sqrt{\frac{\beta}{1+\beta}} r\right)}{J_0\left(\lambda \sqrt{\frac{\beta}{1+\beta}} S\right)} \right) \quad (21)$$

The flux across any cross section of the tube is

$$\begin{aligned} Q &= \int_0^{S(z)} 2\pi r w \, dr \\ &= -\frac{\pi S^2}{\lambda^2} e^{it} \frac{dp_e}{dz} \left[\frac{J_2\left(\lambda \sqrt{\frac{\beta}{1+\beta}} S\right)}{\lambda \sqrt{\frac{\beta}{1+\beta}} S J_0\left(\lambda \sqrt{\frac{\beta}{1+\beta}} S\right)} \right] \end{aligned} \quad (22)$$

The equations of continuity for longitudinal motion in non-dimensional form effected by area changes are given as (Lighthill (1978))

$$\frac{\omega L}{U_0} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial z} = 0 \quad (23)$$

here, A is the cross sectional area at any axial point of the tube. The elasticity of the tube wall is taken in to the consideration by expressing a relationship between the excess pressure $p - p_0$ and known area A . The pressure radius relation for thick and thin walled elastic tube are presented by (Taylor and Gerrard (1977)).

$$\frac{\partial p}{\partial t} = \frac{\partial p}{\partial A} \cdot \frac{\partial A}{\partial t} \quad (24)$$

$$c^2(z) = \frac{A}{\rho_0} \frac{\partial p}{\partial A} = \frac{S}{2\rho_0} \frac{\partial p}{\partial S} \quad (25)$$

here, $c(z)$ is the local value of the wave speed. From Eqs. (24) and (25), we have

$$\frac{\partial A}{\partial t} = \frac{A}{c^2} \frac{\nu U_0 L}{a_0^2} i e^{it} p_e \quad (26)$$

using Eqs. (22) and (26) in Eq. (23), we get

$$\frac{d^2 p_e}{dz^2} + \frac{2}{S} \frac{dS}{dz} a_1 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right) \frac{dp_e}{dz} - \frac{w^2 L^2}{c^2} a_2 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right) p_e = 0 \quad (27)$$

where

$$a_1 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right) = \frac{J_1^2 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right)}{J_0 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right) J_2 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right)}, \quad a_2 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right) = \frac{J_0 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right)}{J_2 \left(\lambda \sqrt{\frac{\beta}{1+\beta}} S \right)}$$

Equation (27) is similar to the shell equation for a thin walled elastic tube derived by Ramachandra Rao (1983)

$$\frac{d^2 p_e}{dz^2} + \frac{2}{S} \frac{dS}{dz} a_1 (\lambda S) \frac{dp_e}{dz} - 2S \lambda_1^2 a_2 (\lambda S) p_e = 0 \quad (28)$$

where $\lambda_1^2 = \frac{\omega^2 L^2 \rho_0 a_0}{c_0^2 \rho h}$ and $c_0^2 = B/\rho$

The above relation can also be written as

$$\lambda_a^2 = 2\lambda_1^2 = \frac{\omega^2 L^2}{c_1^2} \quad (29)$$

here, $c_1^2 = \frac{Bh}{2\rho_0 a_0}$ is the classical Mones-Korteweg speed.

For thin walled elastic tubes, different pressure radius relationship are given as (Taylor and Gerrard (1977))

$$(i) \quad p = \rho_0 c_1^2 \left(1 - \frac{1}{S}\right) \quad (30)$$

$$(ii) \quad p = \frac{8}{3} \rho_0 c_1^2 \left(1 - \frac{1}{S^2}\right) \quad (31)$$

$$(iii) \quad p = 2\rho_0 c_1^2 \frac{1}{S^2} (S-1) \quad (32)$$

$$(iv) \quad p = \frac{8}{3} \rho_0 c_1^2 \frac{1}{S^2} (S-1) \quad (33)$$

$$(v) \quad p = \frac{2\rho_0}{3} c_1^2 \left(1 - \frac{1}{S^4}\right) \quad (34)$$

From the above pressure radius relationships, $c^2(z)$ at any axial station is represented by $c_1^2/F(S)$, where $F(S)$ is a function of s depends on the selected pressure radius relationship.

Now, Eq. (28) can be written in the general form as

$$\frac{d^2 p_e}{dz^2} + \frac{2}{S} \frac{dS}{dz} a_1 \frac{dp_e}{dz} - \lambda_a^2 F(S) a_2 p_e = 0 \quad (35)$$

The expression of excess pressure for large Womersley parameter is

$$\frac{d^2 p_e}{dz^2} + \frac{2}{S} \frac{dS}{dz} \left(1 - i \left/ \lambda \sqrt{\frac{\beta}{1+\beta}} S \right. \right) \frac{dp_e}{dz} + \lambda_a^2 F(S) \left(1 - 2i \left/ \lambda \sqrt{\frac{\beta}{1+\beta}} S \right. \right) p_e = 0 \quad (36)$$

The excess pressure given by Eq. (36) is complex, by taking $p_e = p_r + ip_i$ and comparing real and imaginary parts, above expression reduced to two coupled second order ordinary differential equations of p_r and p_i . Now, the obtained two equations are reduced to four first order equations and solved using Mathematica by defining the initial conditions at some point of the z . With these values of p_r and p_i , we calculate

$$|p_e| = (p_r^2 + p_i^2)^{1/2}, \quad \left| \frac{dp_e}{dz} \right| = \left(\left(\frac{dp_r}{dz} \right)^2 + \left(\frac{dp_i}{dz} \right)^2 \right)^{1/2} \quad (37)$$

The modulus of excess pressure and pressure gradient for some geometries such as (i) straight tube defined by $S = 1, 0 < z < 10$ (ii) tapered tube defined by $S(z) = \exp(-0.025z), 0 < z < 10$ (iii) locally constricted tube defined by $S(z) = 1 - 0.5 \exp(-(z-6)^2), 0 < z < 10$ with prescribed initial conditions

$$p_r = 0.1, \quad p_i = 0, \quad \frac{dp_e}{dz} = 0, \quad \frac{dp_i}{dz} = -0.01 \quad \text{at} \quad z = 0 \quad (38)$$

Fig. 1 Pressure along Straight Tube for Different Values of λ_a with $\alpha = 20, \beta = 0.5$

4 Results and Discussion

In this paper, the oscillatory motion of a Casson fluid through elastic tube with varying cross section is studied. The effects of various pertinent parameters on pressure variation are analyzed through graphs. Figs. 1–3 represent the pressure distribution along the axial direction of the tube for pressure radius relation defined by Eq. (30). Fig. 1 illustrates the variation in pressure distribution along straight tube for various values of λ_a . It is found that the pressure oscillates more as λ_a takes higher values. The influence of elastic parameter on pressure distribution for tapered tube is shown in Fig. 2. It shows that the amplitude of pressure increases for increasing values of λ_a . Fig. 3 shows the distribution of excess pressure along the locally constricted tube for various values of elastic parameter. We observe that pressure oscillates more for increasing values of λ_a with the maximum point of constriction at $z = 6$.

The variation in Pressure gradient for different values of elastic parameter for the pressure radius relation defined by Eq. 3.30 is presented in Figs. 4–6. The influence of elastic parameter on pressure gradient for straight tube is depicted in Fig. 4. It is noticed that the amplitude of the pressure gradient increases along the axial direction with increasing values of λ_a . The variation in pressure gradient along axial direction for tapered tube is shown in Fig. 5. It is clear that as growing values of λ_a , the pressure gradient oscillates more. The distribution of $\left| \frac{dp_e}{dz} \right|$ for different values of λ_a for locally constricted tube is depicted in Fig. 6. It is observed that the amplitude of pressure gradient increases as elastic parameter increases.

Figs. 7 and 8 shows the variation in excess pressure for the different pressure radius relations defined in Eqs.(31)–(34). Fig. 7 denotes the pressure distribution along tapered tube for different pressure radius relations for fixed value of λ_a . It is noticed that there is no much change in amplitude of the pressure for all the mentioned relations. The similar behavior is observed for the case of constricted tube which is given in Fig. 8.

The distribution of pressure along the tapered and constricted tubes for various values of Womersley parameter α for the pressure radius relationship defined by Eq. (30) is presented in Figs. 9 and 10, respectively. From Fig. 9, it is clear that for fixed values of elastic parameter, the amplitude of pressure increases as α increases. The increasing values of Womersley number decreases the amplitude of the pressure for the constricted tube which is shown in Fig. 10.

The influence of Casson parameter on pressure distribution for tapered and constricted tube for pressure radius relation defined by Eq. (30) is represented in Figs.11 and 12 respectively. It is noticed that for fixed values of α and λ_a , the amplitude of pressure enhances with increasing Casson parameter for tapered tube whereas the opposite behavior is found for constricted tube.

Fig. 2 Pressure along Tapered Tube for Various Values of λ_a with $\alpha = 20, \beta = 0.5$

Fig. 3 Pressure along Locally Constricted Tube for Various Values of λ_a with $\alpha = 20, \beta = 0.5$

Fig. 4 Pressure Gradient along Straight Tube for Various Values of λ_a with $\alpha = 20, \beta = 0.5$

Fig. 5 Pressure Gradient along Tapered Tube for Various Values of λ_a with $\alpha = 20, \beta = 0.5$

Fig. 6 Pressure Gradient along Locally Constricted Tube for Various Values of λ_a with $\alpha = 20, \beta = 0.5$

Fig. 7 Pressure along Tapered Tube for Various Pressure-Radius Relations with $\lambda_a = 0.75, \alpha = 15, \beta = 0.5$

Fig. 8 Pressure along Constricted Tube for Various Pressure-Radius Relations with $\lambda_a = 0.75, \alpha = 15, \beta = 0.5$

Fig. 9 Pressure along Tapered Tube for Various Values of α with $\lambda_a = 0.5, \beta = 0.5$

Fig. 10 Pressure along Constricted Tube for Various Values of α with $\lambda_a = 0.5, \beta = 0.5$

Fig. 11 Pressure along Tapered Tube for Various Values of β with $\lambda_a = 0.5$, $\alpha = 15$

Fig. 12 Pressure along Constricted Tube for Various Values of β with $\lambda_a = 0.5$, $\alpha = 15$

5 Conclusions

In the present chapter, the oscillatory flow of a Casson fluid in an elastic tube with variable cross section is investigated. The analytic expressions for axial velocity and mass flux are derived. The effects of different pertinent parameters on pressure distribution along the tube for different geometries are analyzed graphically. When elastic parameter increases, the amplitude of pressure increasing and pressure oscillates more in all cases such as straight tube, tapered tube and locally constricted tube. For various geometries the pressure gradient increases as elastic parameter increases. There is no much change in the amplitude of pressure for all defined pressure-radius relations. The variation in distribution of pressure for tapered tube increases as Womersley parameter takes higher values. Increasing Casson parameter enhances the amplitude of pressure for tapered tube where the opposite behavior is noticed for constricted tube.

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