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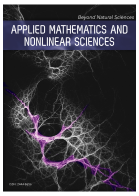

Intermittent transition to chaos in vibroimpact system

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Abstract

Chaotic behaviour of dynamical systems, their routes to chaos, and the intermittency in particular are interesting and investigated subjects in nonlinear dynamics. The studying of these phenomena in non-smooth dynamical systems is of the special scientists' interest. In this paper we study the type-III intermittency route to chaos in strongly nonlinear non-smooth discontinuous 2-DOF vibroimpact system. We apply relatively new mathematical tool – continuous wavelet transform CWT – for investigation this phenomenon. We show that CWT applying allows to detect and determine the chaotic motion and the intermittency with great confidence and reliability, gives the possibility to demonstrate intermittency route to chaos, to distinguish and analyze the laminar and turbulent phases.

Keywords: : vibroimpact system, chaotic behaviour, route to chaos, intermittency, continuous wavelet transform, surface of wavelet coefficients
AMS 2010 codes: Put here the AMS 2010 codes of the paper.

1 Introduction

At present chaotic dynamics is one of the most interesting and investigated subjects in nonlinear dynamics. Just deterministic chaos is not an exceptional mode of dynamical systems behaviour; on the contrary, such regimes are observed in many dynamical systems in mathematics, physics, chemistry, biology and medicine. Therefore, the studying of chaotic dynamics is one of the main ways of modern natural science development. Many monographs, papers and textbooks are devoted to chaos studying [1–4].

The routes to chaos in nonlinear dynamical systems are of the special scientists' interest. It is known three main routes to chaos in dynamical systems [1, 3]:

1) period-doubling route to chaos – the most celebrated scenario for chaotic vibrations, it is Feigenbaum scenario;

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2) quasiperiodic route to chaos;

3) intermittency route to chaos by Pomeau and Manneville.

So the transition from the periodic oscillatory regimes to chaotic ones via intermittency is one of the main routes to chaos in nonlinear dynamic systems. The studying of this phenomenon in non-smooth dynamical systems (in vibroimpact system in particular) is of the special scientists' interest. Intermittency was discovered and divided at three types by French scientists Yv.Pomeau and P.Manneville [5] in 1980 year. When intermittency occurs one observes long regions of periodic motion with bursts of chaos that is the zones of turbulent and laminar motion alternate in such regime under one value of control parameter. As one varies a control parameter the chaotic bursts become more frequent and longer. Intermittency classification is based on different types of local bifurcations after which the periodic motion loses the stability; so intermittency type is defined by multiplier Flouqet value [4, 5]. It is known that periodic motion loses the stability when at least one Floquet multiplier is larger then one. This may occur in three different ways: 1) a real Floquet multiplier crosses the unit circle at $(+1)$, intermittency that may occur in this case was called by Yv. Pomeau and P. Manneville as type-I intermittency; 2) two complex conjugate multipliers cross the unit circle simultaneously, intermittency that may occur in this case was called as type-II intermittency ; 3) a real Floquet multiplier crosses the unit circle at (-1), intermittency that may occur in this case was called as type-III intermittency.

In this paper we study the intermittent transition to chaos in strongly nonlinear non-smooth discontinuous system. It is 2-DOF two-body vibroimpact system (Fig. 1). We had studied its dynamical behaviour in our previous papers [6–9]. We had seen several zones of instability when the control parameter – external loading frequency – had been varying. We had found many interesting phenomena under this frequency changing, we had observed: discontinuous bifurcations, rare attractor, transient chaos, quasiperiodic route to chaos [9, 10].

Now for studying the intermittent transition to chaos in this system we apply relatively young mathematical tool – continuous wavelet transform CWT.

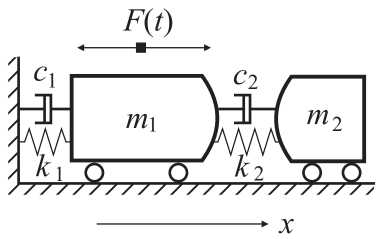

Fig. 1 Vibroimpact system model

The wavelet transform (WT) serves the purpose of analysis or synthesizing a wide variety of generic signals at different frequencies and with different resolution. WT arose in 80-th years of XX century. Now it is state-of-art technique for nonstationary signals analysis. There are quite a few articles, books, and textbooks written on them [11–14]. There is developed Software: Wavelet Toolbox in Matlab, Mathcad and so on [15].

Mathematical transformations are applied to signals to obtain a further information from signal that is not readily available in the raw signal. There is number of transformations that can be applied, among which the Fourier transforms (FT) are probably by far the most popular.

The FT gives the frequency information of the signal, which means that it tells us how much of each frequency exists in the signal, but it does not tell us when in time these frequency components exist. This information is not required when the signal is stationary. When the signal is not stationary it is suitably to use the WT, more exactly when the time localization of the spectral components are needed, a transform giving the time-frequency representation of the signal is needed. The Wavelet transform is a transform of this type. It provides the time-frequency representation. (There are other transforms which give this information too, such as short time Fourier transform, Wigner distributions, etc.). Wavelet transform is capable of providing the time and frequency information simultaneously, hence giving a time-frequency representation of the signal. The WT was developed as an alternative to the short time Fourier Transform (STFT).

Like the FT the continuous wavelet transform (CWT) uses inner products to measure the similarity between a signal and an analyzing function. In the FT the analyzing functions are the complex exponents $e^{j\omega t}$. The resulting transform is a function of a single variable ω. In the STFT the analyzing functions are windowed complex exponentials $w(t)e^{j\omega t}$, and the result is the function of two variables. The STFT coefficients $F(\omega,\tau)$ represent the match between the signal and a sinusoid with angular frequency ω in an interval of a specified length centered at τ.

In CWT the analyzing function is a wavelet ψ. The CWT compares the signal to shifted and compressed or stretched versions of a wavelet. Stretching or compressing a function is collectively referred to as dilatation or scaling and corresponds to the physical notion of scale. By comparing the signal to the wavelet at various scales and positions we obtain a function of two variables. There are many different admissible wavelets that can be used in the CWT. While it may seen confusing that there are so many choices for the analyzing wavelet it is actually a strength of wavelet analysis. Depending on what signal features we are trying to detect, we are free to select a wavelet that facilitates our detection of that feature.

Intermittency route to chaos has some complexity for analysis. At first it occurs much less then period doubling route (which occurs the most often and is studied in the best way). At second "the catching" of intermittency in system motion is not such simple task. The continuous wavelet transform CWT is useful exactly for this task solving.

The chaotic motion and the intermittency in different mechanical and physical systems were studied in [16–22] with WT applying.

The goals of this paper are the following:

1. To study the intermittency route to chaos in 2-DOF two-body vibroimpact system.

2. To apply the continuous wavelet transform CWT for this studying and to show its use for intermittency "catching" and chaoticity anlysis.

2 The background for studying the intermittent transition to chaos in vibroimpact system

For these goals achievement we consider the model of 2-DOF two-body vibroimpact system (Fig. 1) which we have studied it our previous works [6, 7, 9] and have obtained the amplitude-frequency response [7] in wide range of control parameter by parameter continuation method (Fig. 2). The regions of unable motion are drawn by grey colour. Here we'll give only short model description.

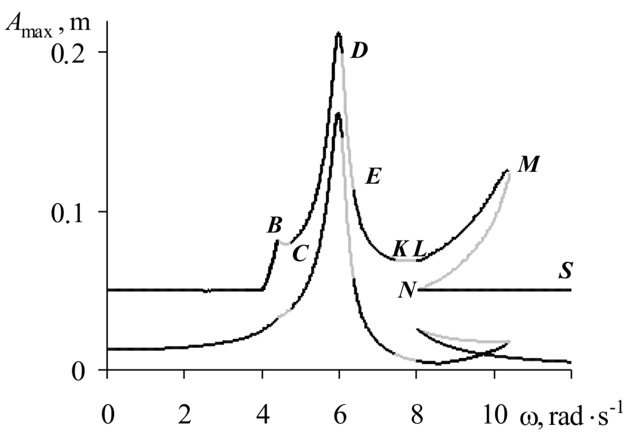

Fig. 2 Amplitude-frequency response in wide range of excitation frequency

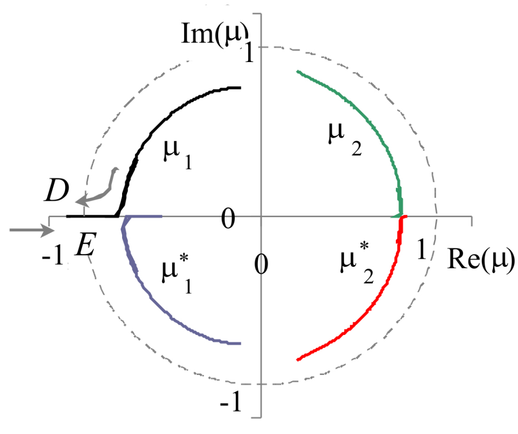

Fig. 3 Floquet multipliers behaviour at *DE* region

This model is formed by the main body m_1 and attached one m_2 , which can play the role of percussive or non-percussive dynamic damper. Bodies are connected by linear elastic springs with stiffness k_1 and k_2 and dampers with damping coefficients c_1 and c_2. (The damping force is taken as proportional to first degree of velocity with coefficients c_1 and c_2.)

The differential equations of its movement are:

$$\begin{aligned} \ddot{x}_1 &= -2\xi_1\omega_1\dot{x}_1 - \omega_1^2 x_1 - 2\xi_2\omega_2\chi(\dot{x}_1 - \dot{x}_2) - \\ &\quad - \omega_2^2\chi(x_1 - x_2 + D) + \frac{1}{m_1}[F(t) - F_{con}(x_1 - x_2)], \\ \ddot{x}_2 &= -2\xi_2\omega_2(\dot{x}_2 - \dot{x}_1) - \omega_2^2(x_2 - x_1 - D) + \frac{1}{m_2}F_{con}(x_1 - x_2), \end{aligned} \tag{1}$$

where $\omega_1 = \sqrt{\dfrac{k_1}{m_1}}$, $\omega_2 = \sqrt{\dfrac{k_2}{m_2}}$; $\xi_1 = \dfrac{c_1}{2m_1\omega_1}$, $\xi_2 = \dfrac{c_2}{2m_2\omega_2}$; $\chi = \dfrac{m_2}{m_1}$.

External loading is periodic one: $F(t) = P\cos(\omega t + \varphi_0)$, $T = \dfrac{2\pi}{\omega}$ is its period.

Impact is simulated by contact interaction force F_{con} according to contact quasistatic Hertz's law:

$$\begin{aligned} F_{con}(z) &= K[H(z)z(t)]^{3/2}, \\ K &= \frac{4}{3}\frac{q}{(\delta_1+\delta_2)\sqrt{A+B}}, \delta_1 = \frac{1-\nu_1{}^2}{E_1\pi}, \delta_2 = \frac{1-\nu_2{}^2}{E_2\pi}, \end{aligned} \tag{2}$$

where $z(t)$ is the relative closing in of bodies, $z(t) = x_2 - x_1$; A , B , and q are constants characterizing the local geometry of the contact zone; ν_i and E_i are respectively Poisson's ratios and Young's modulus for both bodies, $H(z)$ is the discontinuous step Heaviside function. The numerical parameters of this system are following:

m_1=1000 kg,	ω_1=6.283 rad·s^{-1},	ξ_1=0.036,	E_1=2.1 ·10^{11} N·m^2,	ν_1=0.3,
m_2= 100 kg,	ω_2=5.646 rad·s^{-1},	ξ_2=0.036,	E_2=2.1·10^{11} N·m^2,	ν_2=0.3,
P=500 N,	A=B=0.5 m^{-1},	q=0.318,	φ_0=0.	

3 The numerical results

We had considered the region KL of unstable motion at amplitude-frequency response (Fig. 2) before [9,10]. There we had observed quasiperiodic route to chaos and transient chaos. Now we examine the region DE of unstable motion. At points D and E the stable (1,1)-regime loses the stability, real Floquet multiplier crosses the unit circle at (-1) (Fig. 3). (1,1)-regime – is periodic regime with T period and one impact per cycle.

There are two plots which show the whole motion picture very visibly and obviously (Fig. 4,5).

At these Figures we see the change of system dynamic states when the control parameter is varied. Let us have an attentive look at dependence of the largest Lyapunov exponent on control parameter that is external loading frequency (Fig. 4). Lyapunov exponents characterize the kind of dynamical system motion because they measure the divergence rate of nearby phase trajectories. In order to have a criterion for chaos one need only calculate the largest exponent which tells whether nearby trajectories diverge ($\lambda > 0$) or converge ($\lambda < 0$) on the average. Its sign is chaos criterion. For regular motions $\lambda \leq 0$, but for chaotic motion $\lambda > 0$ that is positive Lyapunov exponent imply chaotic dynamics. We had written about the largest Lyapunov exponent estimation in non-smooth vibroimpact system in [9,23]. We see that the positive largest Lyapunov exponent corresponds to the motion in external frequency range 6.07 rad·s^{-1}< ω <6.29 rad·s^{-1}. Here the motion is chaotic one. Further we'll show other motion characteristics in order to confirm its chaoticity.

At Fig. 5 the bifurcation diagram is depicted. The bifurcation diagram is a widely used technique for investigation different states of a dynamical system as parameter is varied. At Fig. 5 the value of control parameter (a forcing frequency) is plotted on the horizontal axis and the values of phase coordinate $x_2(t)$ at Poincaré points are plotted on the vertical axis. There is only one value of one point coordinate in Poincaré map for (1,1)-regime, we see one point along vertical line at bifurcation diagram for ω <6.07 rad·s^{-1} and ω >6.38 rad·s^{-1}. There are 2 separate points along vertical line for (2,2) and (2,3)-periodic regimes. They are the regimes with 2T period and 2 or 3 impacts per cycle. There are unbroken vertical lines for chaotic regime.

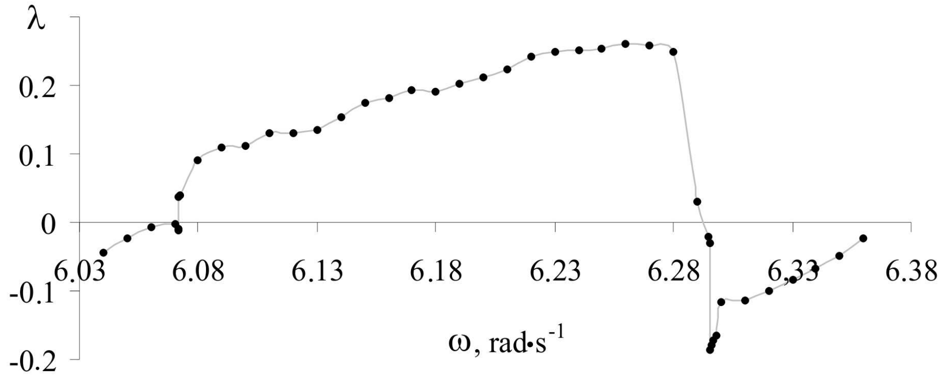

Fig. 4 The largest Lyapunov exponent dependence on control parameter

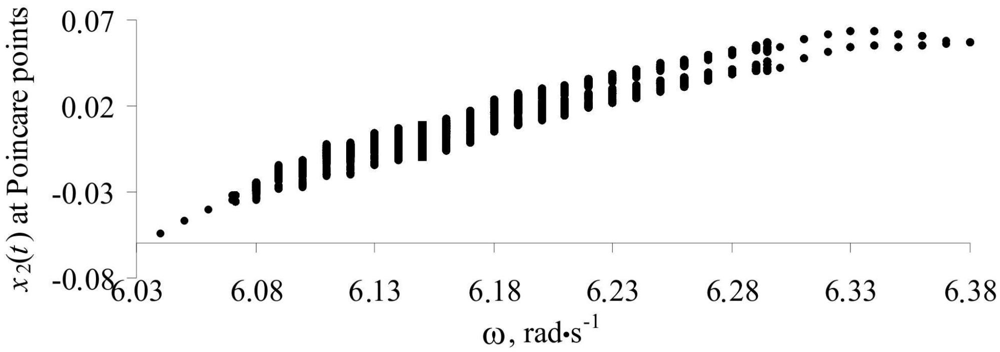

Fig. 5 Bifurcation diagram

Let us now discuss more in details the transition to chaos. At the left border under ω =6.07 rad·s^{-1} (1,1)-regime loses stability and becomes (2,2)-regime when the control parameter ω is increasing. Then intermittency appears simultaneously with period doubling. The amplitude of subharmonic is growing and the amplitude of main frequency is decreasing [4]. When the subharmonic amplitude becomes the big one signal loses regularity, and turbulent bursts arise.

At first among wide laminar phases the narrow and rare turbulent bursts occur. They are the bursts of vibrations with low frequencies and small amplitudes. These bursts have the pale non pronounced colour at the plots of the wavelet surface protections. It is type-III intermittency. The projections of wavelet surface in wide and narrow time ranges demonstrate this phenomenon very well (Fig. 6).

We use wavelet Morlet for continuous wavelet transform CWT. Here and further all plots are fulfilled for attached body. Its mass is much less the main body mass. So its oscillatory amplitudes are more big and their changes are seen better, so the plots are more obvious ones.

Then when ω is increasing the turbulence grows quickly, the turbulent bursts become more frequent. Sometimes in laminar phases subharmonic disappears and only main frequency remains.

We'll show the intermittency under ω =6.13 rad·s^{-1} where it is pronounced (Fig. 7). Oscillogram at this Figure is typical for type-III intermittency [4]. Let us have an attentive look at projection of wavelet surface. It is well seen the regions where chaotic motion in turbulent phases and it's high and low frequencies are interrupted and only one high frequency remains that is laminar regions occur interrupted by chaos.

At Fig. 8 we show the small time region that is picked out by red oval. At this Fig. we see very obviously the sharp change of chaotic motion into almost periodic one.

The surface of wavelet coefficients is shown at Fig. 9. We see very clearly how chaotic motion with many different high and low frequencies (which are not constant in time) is changing by the periodic motion with only

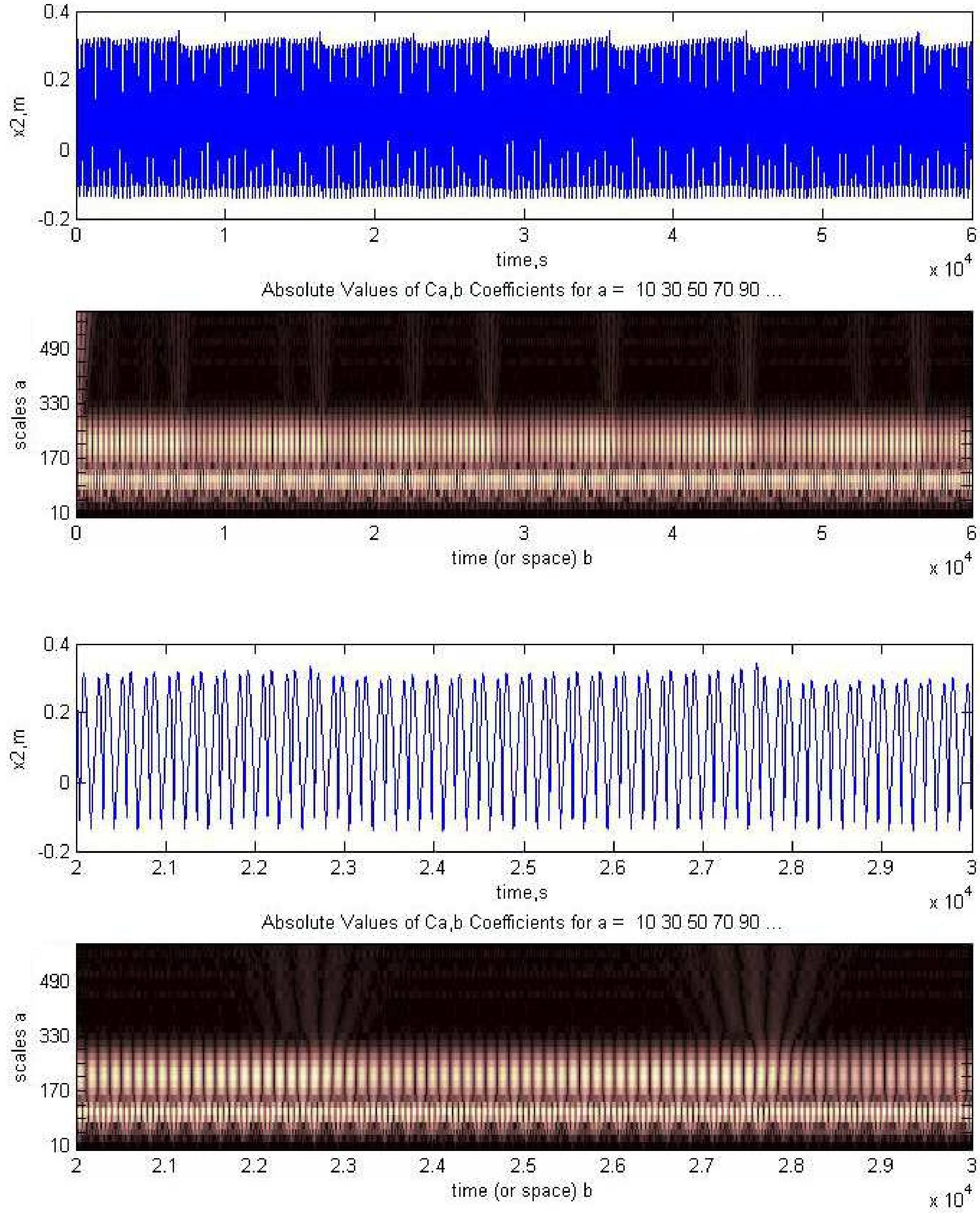

Fig. 6 Time series and wavelet surface projection for intermittency under ω =6.076 rad·s^{-1} (Color online)

one high frequency.

At Fig. 10 the phase trajectories and Poincaré maps are shown for regions of chaotic (turbulent phase) and periodic (laminar phase) motions under intermittency (ω =6.13 rad·s^{-1}). These plots underline the regimes changing. They confirm and give the confidence in presence of almost periodic motion at this region.

Thus we see that surfaces of wavelet coefficients and their projections obtained by continuous wavelet transform CWT give the possibility to find and "catch" the intermittency with great confidence and reliability.

We succeeded in finding the intermittency in non-smooth strongly nonlinear vibroimpact system. The CWT

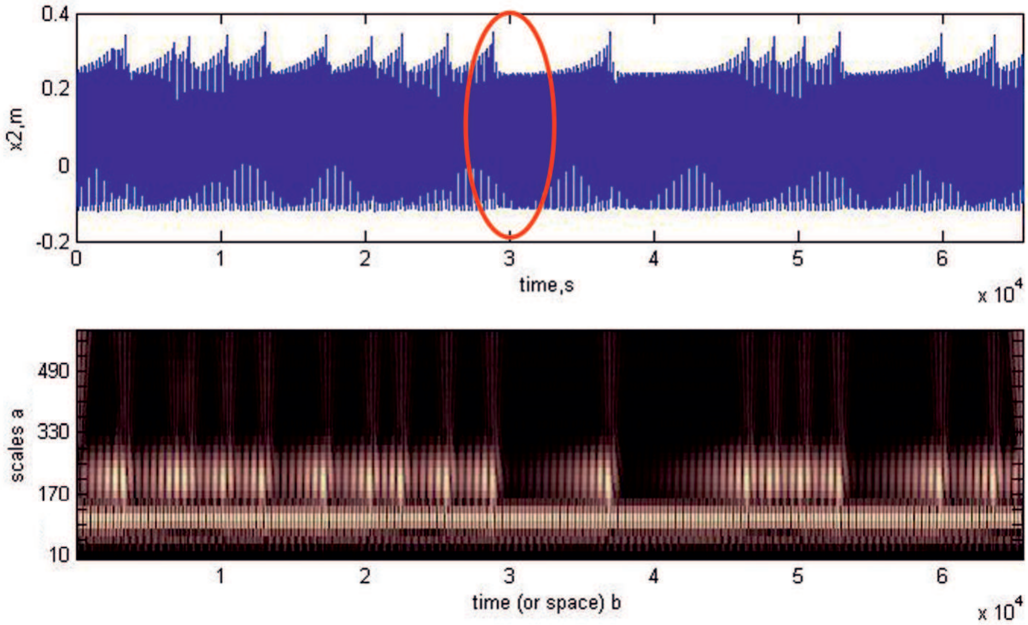

Fig. 7 Time series and wavelet surface projection for intermittency under ω =6.13 rad·s^{-1} (Color online)

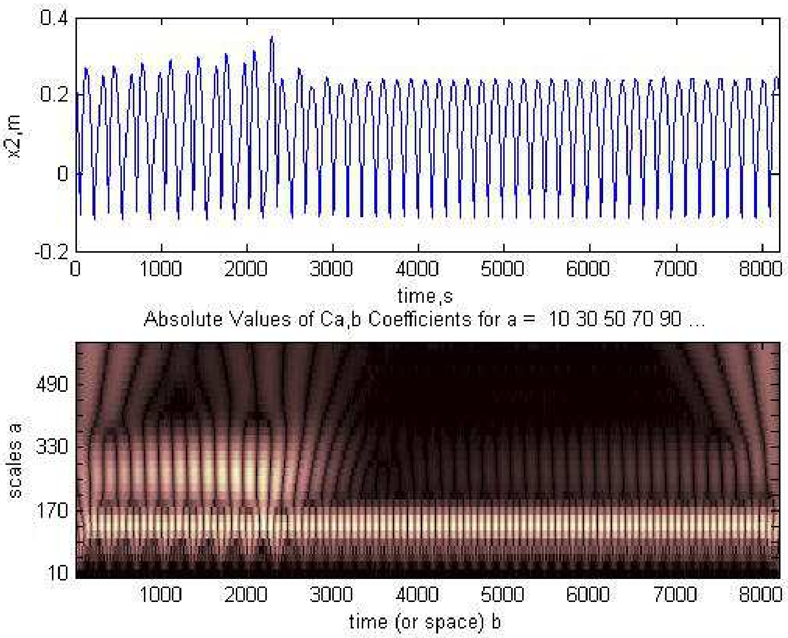

Fig. 8 Time series and wavelet surface projection for intermittency under ω=6.13 rad·s^{-1} (region inside red oval at Fig. 7)

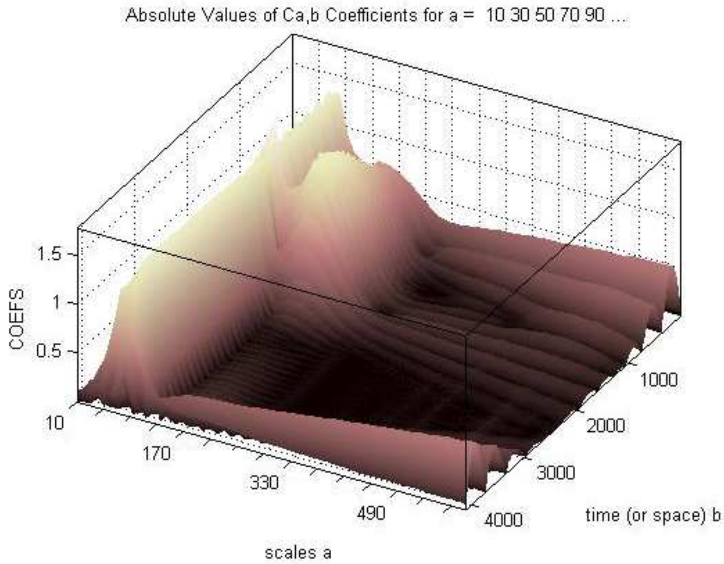

Fig. 9 Surface of wavelet coefficients for intermittency under ω =6.13 rad·s^{-1} (Color online)

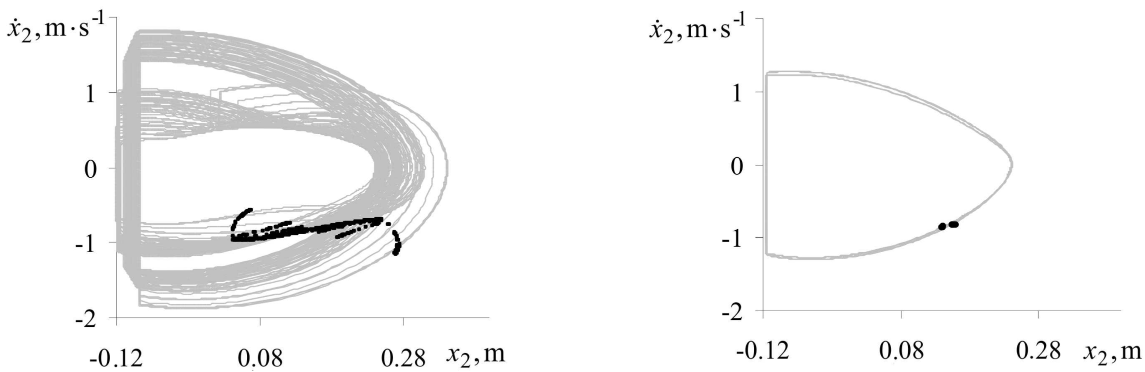

Fig. 10 Phase trajectories and Poincare maps for the regions of chaotic and periodic motions under intermittency (ω =6.13 rad·s^{-1})

was very useful for this studying.

Then the whole chaos is beginning. .For chaotic motion under ω =6.2 rad·s^{-1} we show time series and wavelet surface projection at Fig. 11.

We see many low frequencies which guarantee continuous Fourier spectrum. They are not constant in time. The more high frequency (subharmonic) is not constant in time too. It is typical for non regular motion.

For confirming the chaoticity of this motion we show its phase trajectories and Poincare map at Fig. 12.

At Fig. 13 the surface of wavelet coefficients is depicted. It is seen well not a regular set of frequencies which are not constant in time, they change in time. We see also many not regular low frequencies which are not constant in time too.

Now let us have a look at the right border of bifurcation diagram at Fig. 5. Under ω =6.37 rad·s^{-1} (1,1)-regime loses stability and becomes at first (2,2)-regime and then (2,3)-regime (under ω =6.3 rad·s^{-1}) when the control parameter ω is decreasing. Under further decreasing of external frequency ω the vibroimpact system immediately finds itself in chaotic motion. We don't observe intermittent regime. Already under ω =6.295 rad·s^{-1} (even 6.2951 rad·s^{-1}) we see the whole chaos with continuous Fourier spectrum, with lot of low frequencies which are not constant in time. The view of wavelet surface projection is analogous to Fig. 11.

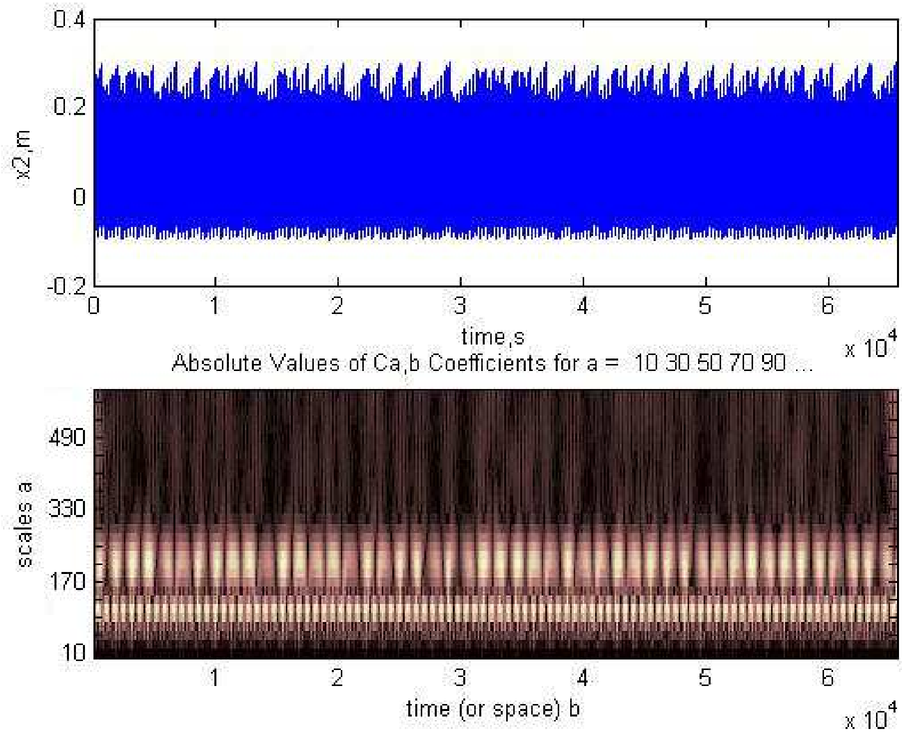

Fig. 11 Time series and wavelet surface projection for chaotic regime under ω =6.2 rad·s^{-1} (Color online)

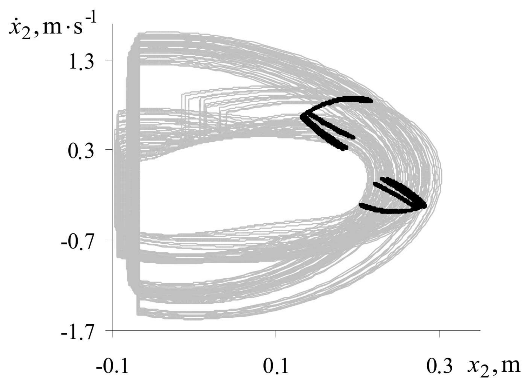

Fig. 12 Phase trajectories and Poincare map for chaotic motion under ω =6.2 rad·s^{-1}

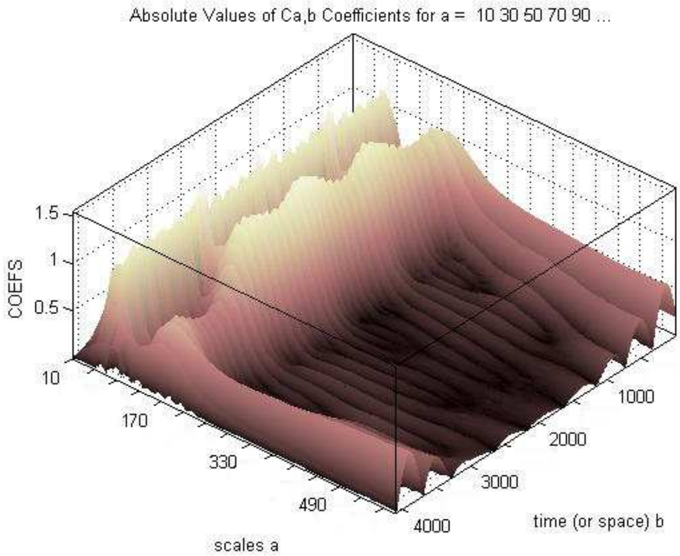

Fig. 13 Surface of wavelet coefficients for chaotic motion under ω =6.2 rad·s^{-1} (Color online)

4 Conclusions

1. Strongly nonlinear non-smooth discontinuous vibroimpact system demonstrates type-III intermittency route to chaos when control parameter (frequency of external loading) is increasing. Intermittency is observing after period doubling. Intermittency occurs only under increasing the external frequency after the loss of stability by main oscillatory regime – at the left border of bifurcation diagram. At its right border when the external frequency is decreasing intermittency isn't observed – the vibroimpact system immediately after period doubling finds itself in chaotic motion.

2. The continuous wavelet transform CWT is very useful for studying of chaotic motion and intermittency. Its theory and existing Software allow to detect and determine these phenomena with great confidence and reliability. Wavelet transform applying gives the possibility to demonstrate intermittency route to chaos and to distinguish and analyze the laminar and turbulent phases. The plots of wavelet coefficients surfaces and their projections give very obvious presentation of these regimes, especially the color plots online.

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