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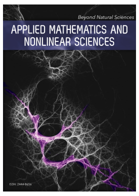

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Visibility intervals between two artificial satellites under the action of Earth oblateness

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Abstract

This paper presents an analytical method to determine the rise-set times of satellite-satellite visibility periods in different orbits. The Visibility function in terms of the orbital elements of the two satellites versus the time were derived explicitly up to e^4. The line-of-sight corrected for Earth Oblateness up to J_2, were considered as a perturbation to the orbital elements. The visibility intervals of the satellites were calculated for some numerical examples in order to test the results of the analytical work.

Keywords: Visibility function – line of site – Earth Oblateness – Rise and Set times.

1 Introduction

The long term and highly accurate orbit estimation, especially rise-and-set time computation, plays a key part in the pre-request information for mission analysis and on-board resources management in more general communication, Earth observation and scientific spacecraft, there has been a big trend to use low cost, fast access and multi-functional small satellites to provide and exchange information for a wide range of military and civil applications such as communication in a very remote area for changing conditions, low cost store-and-forward communication, disaster warning for global shipping service and some Earth Observation missions. Therefore, this requires accurate estimation of when the satellites will start to be visible (rise) to a given location on the Earth or to other satellite and the time when the satellite disappears from the horizon (set) over a time-scale of months in some cases.

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The rise/set problem may be defined as the process of determining the times at which a satellite rises and sets with respect to a ground location. The easiest solution uses a numerical method to determine visibility periods for the site and satellite by evaluating UK position vectors of each. It advances vectors by a small time increment, Δt, and checks visibility at each step. A drawback to this method is computation time, especially when modeling many perturbations and processing several satellites. Escobal [1], [2] proposed a faster method to solve the rise/set problem by developing a closed-form solution for unrestricted visibility periods about an oblate Earth. He assumes infinite range, azimuth, and elevation visibility for the site. Escobal transforms the geometry for the satellite and tracking station into a single transcendental equation for time as a function of eccentric anomaly. He then uses numerical methods to find the rise and set anomalies, if they exist. Lawton [3] has developed another method to solve for satellite-satellite and satellite-ground station visibility periods for vehicles in circular or near circular orbits by approximating the visibility function, by a Fourier series.

More recently, Alfano, Negron, and Moore [4] derived an analytical method to obtain rise/set times of a satellite for a ground station and includes restrictions for range, azimuth, and elevation. The algorithm uses pairs of fourth-order polynomials to construct functions that represent the restricted parameters (range, azimuth, and elevation) versus time for an oblate Earth. It can produce these functions from either uniform or arbitrarily spaced data points. The viewing times are obtained by extracting the real roots of localized quantic.

Palmar [5], introduced a new method to predict the passes of satellite to a specific target on the ground which is useful for solving the satellite visibility problem. he firstly described a coarse search phase of this method including two-body motion, secular perturbation and atmospheric drag, then he described the second phase (refinement), which uses a further developed controlling equation $F(\alpha)=0$ based on the epicycle equations

In this work, a fast method for satellite-satellite visibility periods for the rise-and-set time prediction for two satellites in terms of classical orbital elements of the two satellites versus time were established. The secular variations of the orbital elements due to Earth Oblateness were taken into account in order to consider the changes in the nodal period of satellite and the changes in the long term prediction of maximum elevation angle. In the following description we will introduce the formulae for satellite rise-and-set times of the two satellites. The derived visibility function provides high accuracy over a long period, and provides direct computation of rise-set times.

2 Visibility Analysis

The location of a satellite is determined by the Kepler's laws, in addition, orbit perturbations due to Earth Oblateness are considered. A set of six orbital parameters is used to fully describe the position of a satellite in a point in space at any given time: semi-major axis a, eccentricity e, inclinations of the orbit plane i, right ascension of the nodeΩ, the argument of perigee ω, and true anomaly f. The above parameters are shown in the Figure1.

The links between two satellites are determined by the visibility analysis presented as follows:

Referring to Fig.1, the position vectors of satellites 1, and 2 with respect to the ECI coordinate system are $\vec{r}_1$ and $\vec{r}_2$. The position vector from satellite 1 to satellite 2 will be denoted by $\vec{\rho}=\vec{r}_2-\vec{r}_1$.

Let h = OP = R_e +Δh, be the perpendicular from the dynamical center of the earth to the range vector , cutting the earth surface at Q and the range vector $\vec{\rho}$, where R_e is the mean radius of the Earth, and Δh is the thickness of the atmosphere above the surface of the Earth to P. From the geometry of Fig.1, we have the following relations:

$$Area\,of\;\Delta OS_1S_2=\frac{1}{2}\left|\vec{r}_1\wedge\vec{r}_2\right|=\frac{1}{2}\left|\vec{r}_1\right|\left|\vec{r}_2\right|\sin\psi \tag{1}$$

On the other hand, the area can be calculated from the relation:

$$Area\,of\;\Delta OS_2S_2=\frac{1}{2}\left|\vec{r}_2-\vec{r}_1\right|\cdot h \tag{2}$$

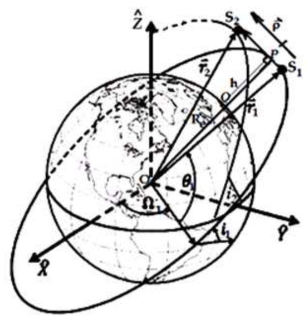

Fig. 1 Geometry of Satellites Visibility

From Eqs. 1 and 2, we conclude that:

$$h = R_e + \Delta h = \frac{|\vec{r}_1|\,|\vec{r}_2|\,\sin\psi}{|\vec{r}_2 - \vec{r}_1|} \tag{3}$$

Where ψ is the angle between $\vec{r}_1$ and $\vec{r}_2$ of the two satellites. The condition for direct visibility between the two satellites can be determined from Eq. 3 by putting

$$\Delta h = \frac{|\vec{r}_1|\,|\vec{r}_2|\,\sin\psi}{|\vec{r}_2 - \vec{r}_1|} - R_e$$

Where

$$\psi = \cos^{-1}\left(\frac{\vec{r}_1.\vec{r}_2}{|\vec{r}_1|\cdot|\vec{r}_2|}\right)$$

After the determination of the locations of satellites at any time in the space, they can only achieve visibility when they are both above the same plane which is tangent to the earth surface. The extreme situation is that both of them are in the tangent plane.

When $\Delta h > 0$, the two satellites can achieve visibility. Otherwise, there is no visibility.

$$\Delta h > 0: \ \Delta h = \frac{|\vec{r}_1|\cdot|\vec{r}_2|\cdot\sin\psi}{|\vec{r}_2 - \vec{r}_1|} - R_e = \sqrt{\frac{{r_1}^2{r_2}^2 - (\vec{r}_1\cdot\vec{r}_2)^2}{({r_1}^2 + {r_2}^2) - 2(\vec{r}_1\cdot\vec{r}_2)}} - R_e$$

i.e. When

$$\frac{{r_1}^2{r_2}^2 - (\vec{r}_1\cdot\vec{r}_2)^2}{({r_1}^2 + {r_2}^2) - 2(\vec{r}_1\cdot\vec{r}_2)} \geq R_e^2$$

After rearrangement the terms, we obtain:

$${r_1}^2{r_2}^2 - (\vec{r}_1\cdot\vec{r}_2)^2 \geq R_e^2\left[({r_1}^2 + {r_2}^2) - 2(\vec{r}_1\cdot\vec{r}_2)\right]$$

Therefore, the visibility function, V, that describes whether these two satellites can achieve visibility is gained, as follows:

$$V = R_e^2\left[({r_1}^2 + {r_2}^2) - 2(\vec{r}_1\cdot\vec{r}_2)\right] - {r_1}^2{r_2}^2 + (\vec{r}_1\cdot\vec{r}_2)^2 \tag{4}$$

Where

$$V = \begin{cases} +ve\ ,\ Non-visibility\,case \\ 0\ ,\ rise\ or\ set \\ -ve\ ,\ direct-line\,of-sight \end{cases}$$

If the position relation between two satellites satisfies the visibility conditions, two satellites can communicate with each other over interstellar links.

3 Construction of The Visibility Function

The position vector of each satellite in the geocentric coordinate system,$\vec{r} = (x, y, z)$, can be calculated by the following formula [6]:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = r \begin{pmatrix} \cos\Omega\cos(\omega+f) - \sin\Omega\sin(\omega+f)\cos i \\ \sin\Omega\cos(\omega+f) + \cos\Omega\sin(\omega+f)\cos i \\ \sin(\omega+f)\sin i \end{pmatrix}$$

Where r denote the distance from the earth center O to the satellite, given by:

$$r = \frac{a(1-e^2)}{1+e\cos f}$$

Forming scalar product $(\vec{r}_1 \cdot \vec{r}_2)$, keeping terms up to $O(e^4)$ only, we obtain

$$\vec{r}_1 \cdot \vec{r}_2 = x_1 x_2 + y_1 y_2 + z_1 z_2$$

For the sake of simplification of calculations, we put the coordinates of the satellite as:

$$\begin{aligned} x_1 &= r_1 \left[\sigma_1{}^2 \cos(f_1+\omega_1+\Omega_1) + \gamma_1{}^2 \cos(f_1+\omega_1-\Omega_1)\right] \\ y_1 &= r_1 \left[\sigma_1{}^2 \sin(f_1+\omega_1+\Omega_1) - \gamma_1{}^2 \sin(f_1+\omega_1-\Omega_1)\right] \\ z_1 &= 2\, r_1\, \sigma_1\, \gamma_1 \cos(f_1+\omega_1) \end{aligned} \tag{5}$$

Where $\sigma_1 = \cos(i_1/2)$ *and* $\gamma_1 = \sin(i_1/2)$, with similar expressions for the other satellite. In order to obtain the visibility function as an explicit function of time, we transform the true anomaly f, to the mean anomaly M, using the following transformation formulas Brouwer [6] up to $O(e^4)$

$$\begin{aligned} r_1 = a_1 \Big[&\left(1+\tfrac{1}{2}e_1^2\right) + \left(-e_1+\tfrac{3}{8}e_1^3\right)\cos M_1 + \left(-\tfrac{1}{2}e_1^2+\tfrac{1}{3}e_1^4\right)\cos 2M_1 \\ &-\tfrac{3}{8}e_1^3\cos 3M_1 - \tfrac{1}{3}e_1^4\cos 4M_1\Big] \end{aligned} \tag{6}$$

$$\begin{aligned} r_1\cos f_1 = a_1 \Big[&-\tfrac{3}{2}e + \left(1-\tfrac{3}{8}e_1^2+\tfrac{5}{192}e_1^4\right)\cos M_1 - \left(\tfrac{1}{2}e_1+\tfrac{1}{3}e_1^3\right)\cos 2M_1 \\ &+\left(-\tfrac{1}{2}e_1^2+\tfrac{1}{3}e_1^4\right)\cos 2M_1 + \left(\tfrac{3}{8}e_1^2-\tfrac{45}{128}e_1^3\right)\cos 3M_1 + \tfrac{1}{3}e_1^3\cos 4M_1\Big] \end{aligned} \tag{7}$$

$$\begin{aligned} r_1\sin f_1 = a_1 \Big[&\left(1-\tfrac{5}{8}e_1^2-\tfrac{11}{192}e_1^4\right)\sin M_1 + \left(\tfrac{1}{2}e_1-\tfrac{5}{12}e_1^3\right)\sin 2M_1 \\ &+\left(\tfrac{3}{8}e_1^2-\tfrac{51}{128}e_1^4\right)\sin 3M_1 + \tfrac{1}{3}e_1^3\sin 4M_1\Big] \end{aligned} \tag{8}$$

With similar expressions for the other satellite.
Substituting Eqs. (6–8) into Eq. 5, and keeping terms up to $O(e^4)$, we obtain:

$$
\begin{aligned}
x = \tfrac{a}{24} \{ & \sigma^2 \left[\left(3e^2+e^4\right)\cos(M-\omega-\Omega) + e^3\cos(2M-\omega-\Omega) + \tfrac{9}{16}e^4\cos(3M-\omega-\Omega) \right.\\
& +12\left(2-e^2\right)\cos(M+\omega+\Omega) + \left(12e-9e^3\right)\cos(2M+\omega+\Omega)\\
& \left. +9\left(e^2-e^4\right)\cos(3M+\omega+\Omega) + 8e^4\cos(4M+\omega+\Omega) - 36e\cos(\omega+\Omega) \right]\\
& +\gamma^2 \left[\left(24-12e^2-\tfrac{3}{8}e^4\right)\cos(M+\omega-\Omega) + \left(12e-9e^3\right)\cos(2M+\omega-\Omega) \right.\\
& +9\left(e^2-e^4\right)\cos(3M+\omega-\Omega) + 8e^4\cos(4M+\omega-\Omega) + \left(3e^2+e^4\right)\cos(M-\omega+\Omega)\\
& \left. +e^3\cos(2M-\omega+\Omega) - \tfrac{9}{16}e^4\cos(3M-\omega+\Omega) - 36e\cos(\omega-\Omega) \right] \}\\
y = \tfrac{a}{24} \{ & \sigma^2 \left[\left(-3e^2-e^4\right)\sin(M-\omega-\Omega) - e^3\sin(2M-\omega-\Omega) - \tfrac{9}{16}e^4\sin(3M-\omega-\Omega) \right.\\
& + \left(24-12e^2-\tfrac{3}{8}e^4\right)\sin(M+\omega+\Omega) + \left(12e-9e^3\right)\sin(2M+\omega+\Omega)\\
& \left. +9\left(e^2-e^4\right)\sin(3M+\omega+\Omega) + 8e^4\sin(4M+\omega+\Omega) - 36e\sin(\omega+\Omega) \right]\\
& +\gamma^2 \left[\left(-24+12e^2+\tfrac{3}{8}e^4\right)\sin(M+\omega-\Omega) + \left(-12e+9e^3\right)\sin(2M+\omega-\Omega) \right.\\
& +9\left(-e^2+e^4\right)\sin(3M+\omega-\Omega) - 8e^4\sin(4M+\omega-\Omega) + \left(3e^2+e^4\right)\sin(M-\omega+\Omega)\\
& \left. +e^3\sin(2M-\omega+\Omega) + \tfrac{9}{16}e^4\sin(3M-\omega+\Omega) - 36e\sin(\omega-\Omega) \right] \}\\
z = \tfrac{a\sigma\gamma}{12} & \left[-36e\sin\omega - \left(3e^2+e^4\right)\sin(M-\omega) + \left(24-12e^2-\tfrac{3}{8}e^4\right)\sin(M+\omega) \right.\\
& -e^3\sin(2M-\omega) + \left(12e-9e^3\right)\sin(2M+\omega) + 9(e^2-e^4)\sin(3M+\omega)\\
& - \tfrac{9}{16}e^4\sin(3M-\omega) + 8e^4\sin(4M+\omega)
\end{aligned}
$$

Using Eq. 6, we can expand the term $r_1^2\, r_2^2$ in Eq. 4 up to the 4^{th} degree in the eccentricities as:

$$
\begin{aligned}
r_1^2\, r_2^2 = \tfrac{a_1^2 a_2^2}{4} & \left[\left(4+4e_1^2+4e_2^2+4e_1^2e_2^2+e_1^4+e_2^4\right) - \left(8e_1+4e_1^3+12e_1e_2^2\right)\cos(M_1) \right.\\
& - \left(8e_2+4e_2^3+12e_1^2e_2\right)\cos(M_2) + \left(2e_1^2-e_1^2e_2^2\right)\cos(2M_1) + \left(2e_2^2-e_1^2e_2^2\right)\cos(2M_2)\\
& -e_1^3e_2\cos(3M_1-M_2) - \tfrac{1}{2}e_1^2e_2^2\cos(2M_1-2M_2) - \tfrac{1}{2}e_1^2e_2^2\cos(2M_1+2M_2)\\
& + \left(8e_1e_2+3e_1^3e_2+3e_1e_2^3\right)\cos(M_1+M_2) - e_1e_2^3\cos(M_1-3M_2) - e_1^3e_2\cos(3M_1+M_2)\\
& \left. + \left(8e_1e_2+3e_1^3e_2+3e_1e_2^3\right)\cos(M_1-M_2) - e_1e_2^3\cos(M_1+3M_2) \right]
\end{aligned}
\tag{9}
$$

Scalar product in Eq. 4 can be formed by using Eq. 8, we obtain:

$$
\vec{r}_1 \cdot \vec{r}_2 = \frac{a_1\, a_2}{32} \sum_{i=0}^{11} T_i \tag{10}
$$

$$T_0 = 19e_1^3e_2 + 19e_1e_2^3$$

$$\begin{aligned}
T_1 = &-\left(-72\,e_1e_2 + 8\,e_1e_2s_1^2 + 8\,e_1e_2s_2^2\right)\cos\left(\omega_1 - \omega_2\right)\\
&+\left(-48e_2 - 8\,e_2e_1^2 + 2e_2^3 + 16s_1^2 + 16s_2^2\right)\cos(M_1 + \omega_1 - \omega_2)\\
&-\left(24e_1e_2 + 27e_1^3e_2 + 13e_1e_2^3 + 8e_1e_2s_1^2 + 8e_1e_2s_2^2\right)\cos(2M_1 + \omega_1 - \omega_2)\\
&+10e_1^2e_2\cos\left(3M_1 + \omega_1 - \omega_2\right) + 13e_1^3e_2\cos(4M_1 + \omega_1 - \omega_2)\\
&-18e_2^3\cos(M_1 - 4M_2 + \omega_1 - \omega_2) + 22e_1e_2^2\cos\left(2M_1 - 3M_2 + \omega_1 - \omega_2\right)
\end{aligned}$$

$$\begin{aligned}
T_2 = &+13e_1e_2^3\cos\left(2M_1 - 4M_2 + \omega_1 - \omega_2\right) + 18e_1^2e_2^2\cos\left(3M_1 - 3M_2 + \omega_1 - \omega_2\right)\\
&+\left(20e_2^2 + 22e_1^2e_2^2 + 18e_2^4\right)\cos\left(M_1 - 3M_2 + \omega_1 - \omega_2\right)\\
&+\left(2e_2^2 - e_1^2e_2^2\right)\cos(2M_2) + 18e_1e_2^2\cos(M_2 + \omega_1 - \omega_2)\\
&+\left(16e_2 + 14e_2^3 - 8e_1^2e_2^2 + 16e_2s_1^2 + 16e_2s_2^2\right)\cos(M_1 - 2M_2 + \omega_1 - \omega_2)\\
&+\left(8e_1e_2 + 5e_1^3e_2 + 5e_1e_2^3 - 8e_1e_2s_1^2 - 8e_1e_2s_2^2\right)\cos(2M_1 - 2M_2 + \omega_1 - \omega_2)
\end{aligned}$$

$$\begin{aligned}
T_3 = &-e_1^3e_2\cos\left(3M_1 + M_2 + \omega_1 - \omega_2\right) - 22\,e_2e_1^2\cos\left(3M_1 - 2M_2 + \omega_1 - \omega_2\right)\\
&+13\,e_2e_1^3\cos\left(4M_1 - 2M_2 + \omega_1 - \omega_2\right)] - 18e_1^3\cos(4M_1 - M_2 + \omega_1 - \omega_2)\\
&+\left(12e_2^2 - 2e_2^4 + 2e_1^2e_2^2\right)\cos(M_1 + M_2 + \omega_1 - \omega_2)\\
&+\left(32 - 16e_1^2 - 16e_2^2 + 16e_1^2e_2^2 - 32s_1^2 - 32s_2^2\right)\cos\left(M_1 - M_2 + \omega_1 - \omega_2\right)\\
&+\left(16e_1 + 14e_1^3 + 24e_1e_2^2 + 16e_1s_1^2 + 16e_1s_2^2\right)\cos(2M_1 - M_2 + \omega_1 - \omega_2)\\
&+\left(20e_1^2 + 18e_1^4 + 6e_1^2e_2^2\right)\cos(3M_1 - M_2 + \omega_1 - \omega_2)
\end{aligned}$$

$$\begin{aligned}
T_4 = &-\left(16e_1 + 18e_1^3 + 24e_1e_2^2 - 16e_1s_1^2 - 16e_1s_2^2\right)\cos(M_2 - \omega_1 + \omega_2)\\
&+\left(12e_2^2 + 2e_1^2e_2^2 - 2e_2^4\right)\cos(M_1 + M_2 + \omega_1 - \omega_2) + 14e_1e_2^2\cos(2M_1 + M_2 + \omega_1 - \omega_2)
\end{aligned}$$

$$\begin{aligned}
T_5 = &-18e_1e_2^2\cos(M_2 + \omega_1 - \omega_2) - 5e_1e_2^3\cos(2M_2 + \omega_1 - \omega_2) - 18e_1^2e_2\cos(M_1 - \omega_1 + \omega_2)\\
&+2e_1^2e_2^2\cos(M_1 - M_2 - \omega_1 + \omega_2) + \left(12e_1^2 + 18e_1^2e_2^2 - 2e_1^4\right)\cos(M_1 + M_2 - \omega_1 + \omega_2)
\end{aligned}$$

$$\begin{aligned}
T_6 = &+\left(40e_1e_2 + 2e_1^3e_2 + 5e_1e_2^3 - 8e_1e_2s_1^2 - 8e_1e_2s_2^2\right)\cos(2M_2 - \omega_1 + \omega_2)\\
&-5e_1^3e_2\cos(2M_1 + 2M_2 - \omega_1 + \omega_2) + 2e_1^3\cos(2M_1 + M_2 - \omega_1 + \omega_2)\\
&-18e_1^2e_2\cos(M_1 + 2M_2 - \omega_1 + \omega_2) - 16\,e_1s_1^2\cos(M_2 + \omega_1 + \omega_2 - 2\Omega_1)
\end{aligned}$$

$$T_7 = +8e_1e_2s_1^2\cos(\omega_1+\omega_2-2\Omega_1)+13e_1e_2^3\cos(4M_2-\omega_1+\omega_2)$$
$$+10e_1^2e_2^2\cos(M_1+3M_2-\omega_1+\omega_2)-16e_1s_1^2\cos(2M_1+M_2+\omega_1+\omega_2-2\Omega_1)$$
$$-16e_2s_1^2\cos(M_1+\omega_1+\omega_2-2\Omega_1)+8e_1e_2s_1^2\cos(2M_1+\omega_1+\omega_2-2\Omega_1)$$
$$+(32s_1^2+16e_2^2s_1^2+16e_1^2s_1^2)\cos(M_1+M_2+\omega_1+\omega_2-2\Omega_1)-22e_1e_2^2\cos(3M_2-\omega_1+\omega_2)$$

$$T_8 = -16e_1s_2^2\cos(2M_1+M_2+\omega_1+\omega_2-2\Omega_2)+8e_1e_2s_2^2\cos(2M_2+\omega_1+\omega_2-2\Omega_2)$$
$$-16e_2s_2^2\cos(M_1+2M_2+\omega_1+\omega_2-2\Omega_2)+8e_1e_2s_2^2\cos(2M_1+2M_2+\omega_1+\omega_2-2\Omega_2)$$
$$-16e_1e_2s_1s_2\cos(\omega_1+\omega_2-\Omega_1-\Omega_2)+32e_2s_1s_2\cos(M_1+\omega_1+\omega_2-\Omega_1-\Omega_2)$$
$$-16e_1e_2s_1s_2\cos(2M_1+\omega_1+\omega_2-\Omega_1-\Omega_2)+32e_1s_1s_2\cos(M_2+\omega_1+\omega_2-\Omega_1-\Omega_2)$$

$$T_9 = +8e_1e_2s_1^2\cos(2M_1+\omega_1+\omega_2-2\Omega_1)-16e_2s_1^2\cos(M_1+2M_2+\omega_1+\omega_2-2\Omega_1)$$
$$+8e_1e_2s_1^2\cos(2M_1+2M_2+\omega_1+\omega_2-2\Omega_1)+8e_1e_2s_2^2\cos(\omega_1+\omega_2-2\Omega_2)$$
$$-16e_2s_2^2\cos(M_1+\omega_1+\omega_2-2\Omega_2)+8e_1e_2s_2^2\cos(2M_1+\omega_1+\omega_2-2\Omega_2)$$
$$-16e_1s_2^2\cos(M_2+\omega_1+\omega_2-2\Omega_2)+(32s_2^2+16e_1^2s_2^2+16e_2^2s_2^2)\cos(M_1+M_2+\omega_1+\omega_2-2\Omega_2)$$

$$T_{10} = -\left(64s_1s_2+32e_1^2s_1s_2+32e_2^2s_1s_2\right)\cos(M_1+M_2+\omega_1+\omega_2-\Omega_1-\Omega_2)$$
$$+32e_1s_1s_2\cos(2M_1+M_2+\omega_1+\omega_2-\Omega_1-\Omega_2)-16e_1e_2s_1s_2\cos(2M_2+\omega_1+\omega_2-\Omega_1-\Omega_2)$$
$$+32e_2s_1s_2\cos(M_1+2M_2+\omega_1+\omega_2-\Omega_1-\Omega_2)-16e_1e_2s_1s_2\cos(2M_1+2M_2+\omega_1+\omega_2-\Omega_1-\Omega_2)$$

$$T_{11} = -32e_1s_1s_2\cos(M_2-\omega_1+\omega_2+\Omega_1-\Omega_2)+16e_1e_2s_1s_2\cos(\omega_1-\omega_2-\Omega_1+\Omega_2)$$
$$-32e_2s_1s_2\cos(M_1+\omega_1-\omega_2-\Omega_1+\Omega_2)+16e_1e_2s_1s_2\cos(2M_1+\omega_1-\omega_2-\Omega_1+\Omega_2)$$
$$-32e_2s_1s_2\cos(M_1-2M_2+\omega_1-\omega_2-\Omega_1+\Omega_2)+16e_1e_2s_1s_2\cos(2M_1-2M_2+\omega_1-\omega_2-\Omega_1+\Omega_2)$$
$$-32e_1s_1s_2\cos(2M_1-M_2+\omega_1-\omega_2-\Omega_1+\Omega_2)$$
$$+\left(64s_1s_2+32e_1^2s_1s_2+32e_2^2s_1s_2\right)\cos(M_1-M_2+\omega_1-\omega_2-\Omega_1+\Omega_2)]$$

Substituting Eqs. 9 and 10 into Eq. 4, we obtain the complete expression of the visibility function in terms of the orbital elements in the form:

$$V=\sum_{i=0}^{15}V_i$$

$$V_1 = A_0 + A_1\cos(M_1) + A_2\cos(2M_1) + A_3\cos(3M_1) + A_4\cos(4M_1) + A_5\cos(M_2)$$
$$+A_6\cos(2M_2) + A_7\cos(3M_2) + A_8\cos(4M_2) + A_9\cos(M_1 - M_2) + A_{10}\cos(M_1 - 2M_2)$$
$$+A_{11}\cos(M_1 - 3M_2) + A_{12}\cos(2M_1 - M_2) + A_{13}\cos(3M_1 - M_2) + A_{14}\cos(2M_1 - 2M_2)$$
$$+A_{15}\cos(M_1 + M_2) + A_{16}\cos(2M_1 + M_2) + A_{17}\cos(3M_1 + M_2) + A_{18}\cos(M_1 + M_2)$$
$$+A_{19}\cos(2M_1 + M_2) + A_{20}\cos(M_1 + 3M_2) + A_{21}\cos(2\omega_1 - 2\Omega_1) + A_{22}\cos(2\omega_2 - 2\Omega_2)$$

$$V_2 = A_{23}\cos(M_1 + 2\omega_1 - 2\Omega_1) + A_{24}\cos(2M_1 + 2\omega_1 - 2\Omega_1) + A_{25}\cos(4M_1 + 2\omega_1 - 2\Omega_1)$$
$$+A_{26}\cos(2M_1 - 2M_2 + 2\omega_1 - 2\Omega_1) + A_{27}\cos(M_1 - M_2 + 2\omega_1 - 2\Omega_1)$$
$$+A_{28}\cos(M_1 + M_2 + 2\omega_1 - 2\Omega_1) + A_{29}\cos(2M_1 + M_2 + 2\omega_1 - 2\Omega_1)$$
$$+A_{30}\cos(2\omega_1 - 2\omega_2) + A_{31}\cos(M_1 + 2\omega_1 - 2\omega_2) + A_{32}\cos(2M_1 + 2\omega_1 - 2\omega_2)$$
$$+A_{33}\cos(3M_1 + 2\omega_1 - 2\omega_2) + A_{34}\cos(4M_1 + 2\omega_1 - 2\omega_2)$$

$$V_3 = A_{35}\cos(2M_1 - 6M + 2\omega_1 - 2\omega_2) + A_{36}\cos(M_1 - 5M_2 + 2\omega_1 - 2\omega_2)$$
$$+A_{37}\cos(3M_1 - 5M_2 + 2\omega_1 - 2\omega_2) + A_{38}\cos(M_1 - 4M_2 + \omega_1 - \omega_2)$$
$$+A_{39}\cos(3M_1 - 4M + 2\omega_1 - 2\omega_2) + A_{40}\cos(4M_1 - 4M_2 + \omega_1 - \omega_2)$$
$$+A_{41}\cos(2M_1 - 4M_2 + 2\omega_1 - 2\omega_2) + A_{42}\cos(M_1 - 3M_2 + 2\omega_1 - 2\omega_2)$$
$$+A_{43}\cos(2M_1 - 3M + 2\omega_1 - 2\omega_2) + A_{44}\cos(3M_1 - 3M_2 + 2\omega_1 - 2\omega_2)$$
$$+A_{45}\cos(4M_1 - 3M_2 + 2\omega_1 - 2\omega) + A_{46}\cos(5M_1 - 3M_2 + 2\omega_1 - 2\omega_2)$$
$$+A_{47}\cos(2M_1 - 2M_2 + 2\omega_1 - 2\omega_2) + A_{48}\cos(3M_1 - 2M_2 + 2\omega_1 - 2\omega)$$

$$V_4 = A_{49}\cos(5M_1 - 3M_2 + 2\omega_1 - 2\omega_2) + A_{50}\cos(4M_1 - 2M_2 + 2\omega_1 - 2\omega_2)$$
$$+A_{51}\cos(6M_1 - 2M_2 + 2\omega_1 - 2\omega_2) + +A_{52}\cos(M_1 - M_2 + 2\omega_1 - 2\omega_2)$$

$$V_5 = A_{53}\cos(2M_1 - M_2 + 2\omega_1 - 2\omega_2) + A_{54}\cos(3M_1 - M_2 + 2\omega_1 - 2\omega)$$
$$+A_{55}\cos(4M_1 - M_2 + 2\omega_1 - 2\omega_2) + A_{56}\cos(5M_1 - M_2 + 2\omega_1 - 2\omega_2)$$
$$+A_{57}\cos(M_1 + M_2 + 2\omega_1 - 2\omega) + A_{58}\cos(2M_1 + M_2 + 2\omega_1 - 2\omega_2)$$

$$V_6 = A_{59}\cos(3M_1+M_2+2\omega_1-2\omega_2)+A_{60}\cos(2M_1+2M_2+2\omega_1-2\omega)$$
$$+A_{61}\cos(2\omega_1-2\Omega_2)+A_{62}\cos(2M_1-2M_2+2\omega_1-2\omega_2)$$
$$+A_{63}\cos(M_1-M_2+\omega_1-\omega_2)+A_{64}\cos(M_1+M_2+\omega_1+\omega_2)$$
$$+A_{65}\cos(M_2-2\omega_1+2\omega_2)+A_{66}\cos(M_1+M_2-2\omega_1+2\omega_2)$$
$$+A_{67}\cos(M_1+3M_2-2\omega_1+2\omega_2)+A_{68}\cos(M_1+2M_2-2\omega_1+2\omega_2)$$
$$+A_{69}\cos(2M_1+2M_2-2\omega_1+2\omega_2)+A_{70}\cos(3M_2-2\omega_1+2\omega_2)$$
$$+A_{71}\cos(4M_2-2\omega_1+2\omega_2)+A_{72}\cos(M_2-2\Omega_1+2\omega_2)$$
$$+A_{73}\cos(M_1+M_2-2\Omega_1+2\omega_2)+A_{74}\cos(2M_2-2\omega_1+2\omega_2)$$

$$V_7 = A_{75}\cos(2M_2-2\Omega_1+2\omega_2)+A_{76}\cos(M_1+2M_2-2\Omega_1+2\omega_2)$$
$$+A_{77}\cos(2M_1+2M_2-2\Omega_1+2\omega_2)+A_{78}\cos(M_1+3M_2-2\Omega_1+2\omega_2)$$
$$+A_{79}\cos(4M_2-2\Omega_1+2\omega_2)+A_{80}\cos(2M_1+2M_2+2\omega_1+2\omega_2-2\Omega_2)$$
$$+A_{81}\cos(2M_1+2M_2+2\omega_1+2\omega_2-\Omega_1-3\Omega_2)+A_{82}\cos(2M_2+\Omega_1+2\omega_2-3\Omega_2)$$
$$+A_{83}\cos(M_1+2\omega_1-2\Omega_2)+A_{84}\cos(2M_1+2\omega_1-2\Omega_2)$$
$$+A_{85}\cos(4M_1+2\omega_1-2\Omega_2)+A_{86}\cos(2M_1-2M_2+2\omega_1-2\Omega_2)$$
$$+A_{87}\cos(M_1-M_2+2\omega_1-2\Omega_2)+A_{88}\cos(M_1+M_2+2\omega_1-2\Omega_2)$$
$$+A_{89}\cos(2M_1+M_2+2\omega_1-2\Omega_2)+A_{90}\cos(M_2+2\omega_2-2\Omega_2)$$

$$V_8 = +A_{91}\cos(M_1+M_2+2\omega_2-2\Omega_2)+A_{92}\cos(2M_2+2\omega_2-2\Omega_2)$$
$$+A_{93}\cos(2M_2+2\omega_2-2\Omega_2)+A_{94}\cos(2M_1+2M_2+2\omega_2-2\Omega_2)$$
$$+A_{95}\cos(M_1+3M_2+2\omega_2-2\Omega_2)+A_{96}\cos(4M_2+2\omega_2-2\Omega_2)$$

$$V_9 = A_{97}\cos(2M_1+2M_2+2\omega_1+2\omega_2-2\Omega_1-2\Omega_2)+A_{98}\cos(2\Omega_1-2\Omega_2-2\omega_2)$$
$$+A_{99}\cos(M_1+2\omega_1-\Omega_1-\Omega_2)+A_{100}\cos(2M_1+2\omega_1-\Omega_1-\Omega_2)$$
$$+A_{101}\cos(4M_1+2\omega_1-\Omega_1-\Omega_2)+A_{102}\cos(2M_1-2M_2+2\omega_1-\Omega_1-\Omega_2)$$
$$+A_{103}\cos(M_1-M_2+2\omega_1-\Omega_1-\Omega_2)+A_{104}\cos(M_1+M_2+2\omega_1-\Omega_1-\Omega_2)$$
$$+A_{105}\cos(2M_1+M_2+2\omega_1-\Omega_1-\Omega_2)+A_{106}\cos(2M_1+2M_2+2\omega_1-\Omega_1-\Omega_2)$$
$$+A_{107}\cos(\Omega_1-\Omega_2)+A_{108}\cos(2M_1+\Omega_1-\Omega_2)+A_{109}\cos(M_2+\Omega_1-\Omega_2)$$

$$V_{10} = A_{110}\cos(2M_2+\Omega_1-\Omega_2)+A_{111}\cos(\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{112}\cos(M_1+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{113}\cos(2M_1+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{114}\cos(3M_1+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{115}\cos(M_1-4M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{116}\cos(2M_1-3M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{117}\cos(3M_1-3M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{118}\cos(M_1-2M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{119}\cos(M_1-2M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{120}\cos(3M_1-2M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{121}\cos(2M_1-2M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{122}\cos(M_1-M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{123}\cos(2M_1-M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{124}\cos(3M_1-M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$

$$V_{11} = A_{125}\cos(4M_1-M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{126}\cos(M_1+M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{127}\cos(2M_1+M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{128}\cos(3M_1+M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{129}\cos(2M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{130}\cos(M_1+2M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$

$$V_{12} = A_{131}\cos(2M_1+2M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)+A_{132}\cos(M_1+3M_2+\omega_1-\omega_2+\Omega_1-\Omega_2)$$
$$+A_{133}\cos(\omega_1+\omega_2+\Omega_1-\Omega_2)+A_{134}\cos(M_1+\omega_1+\omega_2+\Omega_1-\Omega_2)$$
$$+A_{135}\cos(2M_1+\omega_1+\omega_2+\Omega_1-\Omega_2)+A_{136}\cos(M_1-M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)$$
$$+A_{137}\cos(2M_1+M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)+A_{138}\cos(M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)$$
$$+A_{139}\cos(3M_1+M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)+A_{140}\cos(2M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)$$

$$V_{13} = +A_{141}\cos(M_1+2M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)+A_{142}\cos(2M_1+2M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)$$
$$+A_{143}\cos(M_1+3M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)+A_{144}\cos(2M_1+2M_2+\omega_1+\omega_2+\Omega_1-\Omega_2)$$
$$+A_{145}\cos(M_1-M_2+\omega_1-\omega_2-\Omega_1-\Omega_2)+A_{146}\cos(M_1-M_2-\omega_1-\omega_2+\Omega_1+\Omega_2)$$
$$+A_{147}\cos(M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{148}\cos(M_1+M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)$$
$$+A_{149}\cos(2M_1+M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{150}\cos(3M_1+M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)$$

$$V_{14} = A_{151}\cos(2M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{152}\cos(M_1+2M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)$$
$$+A_{153}\cos(2M_1+2M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{154}\cos(3M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)$$
$$+A_{155}\cos(M_1+3M_2-\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{156}\cos(\omega_1+\omega_2-\Omega_1+\Omega_2)$$
$$+A_{157}\cos(M_1+\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{158}\cos(2M_1+\omega_1+\omega_2-\Omega_1+\Omega_2)$$

$$V_{15} = +A_{159}\cos(M_2+\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{160}\cos(M_1+3M_2+\omega_1+\omega_2-\Omega_1+\Omega_2)$$
$$+A_{161}\cos(2M_1+M_2+\omega_1+\omega_2-\Omega_1+\Omega_2)+A_{162}\cos(M_1+2M_2+\omega_1+\omega_2-\Omega_1+\Omega_2)\ .$$
$$+A_{163}\cos(2M_1+2M_2+\omega_1+\omega_2-\Omega_1+\Omega_2)+)\ A_{164}\cos(M_1+M_2+\omega_1+\omega_2-\Omega_1+\Omega_2]$$

Where the coefficients A_s, $s = 0,1,...,164$ are functions of $(a_j\ ,e_j\ ,i_j)$, $j = 1,2$ are given in Appendix A.

4 Adding Perturbing Forces

We shall consider the effect of perturbation on the orbital elements due to the Earth Oblateness. So, the orbital elements of the two satellites can be written in the form:

$$\sigma_j(t) = \sigma_{jo} + (\Delta\sigma_j)_{obl} \qquad j = 1,2.$$

Where$(\Delta\sigma_j)_{obl}$ denote the first order perturbation in the orbital elements, and j = 1, 2 denotes satellites 1 and 2.

The expansion of the perturbed visibility function about some epoch time t_0 can be obtained by Taylor expansions about the osculating elements $(a_{0j}, e_{0j}, i_{0j}, \Omega_{0j}, \omega_{0j}, M_{0j})$ up to the first order as:

$$F(a_j,e_j,i_j,\Omega_j,\omega_j,M_j) = V(a_{0j},e_{0j},i_{0j},\Omega_{0j},\omega_{0j},M_{0j}) + \sum_{s=1}^{6}\left(\frac{\partial V}{\partial\sigma_s}\right)_0 \Delta\sigma_s$$

The symbols σ_s, represent any of the orbital elements. The summation ranges from s = 1 to 3 represents the elements $(\Omega_1, \omega_1, M_1)$ and from s = 4 to 6 represents $(\Omega_2, \omega_2, M_2)$ respectively. The quantities $\Delta\sigma_s$, s = 1, 2,..,6 represent the secular variations in the corresponding orbital elements due to the perturbation.

4.1 The effect of Earth Oblateness

A satellite under the influence of an inverse square gravitational law has truly constant orbital elements. In reality, however, there is a gradual change in the orbital elements due to the Earth's Oblateness. The principal effect of this is to introduce a short period oscillation of the orbital elements, which we can ignore in most cases. The argument of perigee ω, the longitude of the ascending node Ω , and the Mean anomaly M, however, experience a secular drift which significantly changes the long term prediction of maximum elevation angle. Using the method of variation of parameters to take proper account of all these secular variations due to earth oblateness up to J_2. The perturbation method is explained in many standard textbooks on [7].

The gravitational potential, U, of a satellite including the contribution of J_2, is given by [7]

$$U = \frac{\mu}{r}\left[1 + \frac{J_2 R_e^2}{2r^2}(1-3\sin^2\delta)\right]$$

Which may be expressed as:

$$U = U_0 + R$$

Where
U: the potential of the Earth,
U_0: the potential of purely spherical Earth,
R : the perturbing function,
$\mu = k^2 m$: the gravitational constant$\times$ mass of the Earth,

J_2:Coefficient of the 2^{th} harmonic,
δ: the satellite altitude.
Since the equation of the satellite orbit is

$$r = \frac{a\,(1-e^2)}{1+e\,\cos(f)}$$

From spherical trigonometry, we have the relation

$$\sin(\delta) = \sin(i)\,\sin(f+\omega)$$

,

Then the perturbing potential to the order of J_2 will take the form:

$$R = \frac{3}{2}\mu\frac{J_2R_e^2}{r^3}\left\{\frac{1}{3}-\frac{1}{2}\sin^2 i+\frac{1}{2}\sin^2 i\cos 2(f+\omega)\right\}$$

Where
a : is the semi-major axis of the orbit,
e : is the orbital eccentricity,
f : is the true anomaly,
i : is the orbit inclination,
ω: is the argument of perigee.
Since we consider only the secular variation, so we average the perturbative function, with respect to the mean anomaly M . The derivation and solution are given in many text books for example [8].

$$\begin{array}{l}
a(t) = a_0\ ,\ \ e(t) = e_0\ ,\ \ i(t) = i_0 \\
\bar{\Omega}(t) = \Omega_0 + \dot{\bar{\Omega}}\,(t-t_0)\ , \\
\bar{\omega}(t) = \omega_0 + \dot{\bar{\omega}}\,(t-t_0)\ , \\
\bar{M}(t) = M_0 + \bar{n}(t-t_0) + (\Delta\bar{n})\,(t-t_0)
\end{array}$$

Finally, secular variations are associated with a steady non-oscillatory, continuous drift of an element from the adopted epoch value,

$$\begin{array}{l}
\Delta\bar{\Omega} = -\left[\frac{K}{a^2\eta^4}\left(1-2\gamma^2\right)\right]\bar{n}\,(t-t_0) \\
\Delta\bar{\omega} = \left[\frac{2K}{a^2\eta^4}\left(1-5\,\sigma^2\gamma^2\right)\right]\bar{n}\,(t-t_0) \\
\Delta\bar{M} = \left[1+\frac{K}{a^2\eta^3}\left(1-6\,\sigma^2\gamma^2\right)\right]\bar{n}\,(t-t_0)
\end{array} \tag{11}$$

Where,

$$\bar{n} = \sqrt{\mu/a^3}\ ,\ K = \frac{3}{2}J_2R_e^2\ ,\ \ \eta = \sqrt{1-e^2}$$

Adding Perturbations

According the Eq. 11, we can add the effects of two perturbations due to Oblateness on the six elements of orbits together as:

$$\begin{array}{l}
\omega(t) = \omega_0 + (\Delta\omega)_{obl} \\
\Omega(t) = \Omega_0 + (\Delta\Omega)_{obl} \\
M(t) = M_0 + (\Delta M)_{obl}
\end{array}$$

5 Numerical Examples

In what follows the visibility function were tested for some examples to obtain the mutual visibility between two Earth Satellites whatever the types of their orbits may be. Classical orbital elements for some satellites from the Center for Space Standards & Innovation were used as test data for this study, and are listed in Tables 1 and 2.

The visibility intervals are shown in the following Figures 2,4,6 without any perturbing force and with Oblateness force in Figures 3, 5, 7, calculated in Tables 1, 2 and 3, respectively.

Table 1 Norad Two - Line Element Sets For The Satellites AQUA, ARIRANG-2, HST and ODIN

Satellite Orbital Elements	1-AQUA	2-ARIRANG-2	3-HST	4-Odin
Equivalent altitude (Km)	699.588	682.6205	543.2687	540.5256
a (Km)	7077.725	7060.757	6921.405	6918.662
n (rev/min)	0.010408	0.010445	0.010779	0.010769
e	0.000286	0.001669	0.000256	0.001057
i (degree)	98.2031	98.0676	28.4705	97.591
Ω (degree)	121.6097	76.9906	17.611	200.4958
ω (degree)	54.081	258.4665	301.12	186.4076
M (degree)	125.1605	101.4671	170.9719	173.7019
$\rho\ (kg/km^3)$	3.63E-05	4.6E-05	0.000354	0.000369
$\rho_o\ (kg/km^3)$	0.000145	0.000145	0.000697	0.000697
h_0(Km) (kg/km^3)	600	600	500	500
H (Km)	71.835	71.835	63.822	63.822
Epoch Year & Julian Date	18180. 59770749	18180. 82019665	18182.935593	18182.93790454
time of data (min)	2018 06 29 13:31:30	2018 06 29 19:41:03.004	2018 07 01 21:57:32.134	2018 07 01 22:30:32.994

Table 2 Norad Two - Line Element Sets For The Satellites CFESAT and MTI

Satellite Orbital Elements	5-CFESAT	6-MTI
Equivalent altitude (Km)	468.8831	412.5092
a (Km)	6847.02	6790.646
n (rev/min)	0.010953	0.011074
e	0.000582	0.000812
i (degree)	35.4247	97.5789
Ω (degree)	203.043	17.7612
ω (degree)	183.8662	345.6071
M (degree)	176.2019	143.5229
$\rho\ (kg/km^3)$	0.001162	0.003008
$\rho_o\ (kg/km^3)$	0.001585	0.003725
h_0(Km) (kg/km^3)	450	400
H (Km)	60.828	58.515
Epoch Year & Julian Date	18182.5017322	18182.7746284
time of data (min)	2018 07 01 12:02:28.526	2018 07 01 18:02:08.608

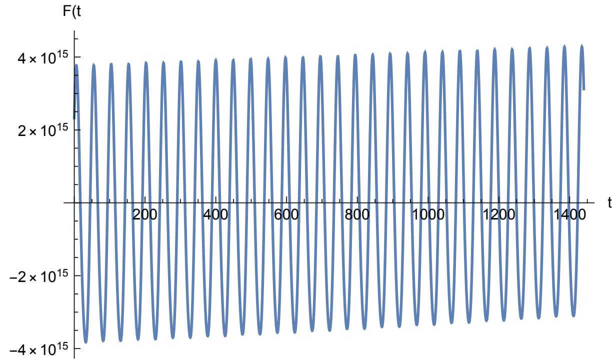

Fig. 2 Visibility Intervals Between AQUA and ARIRANG2 during 24-H

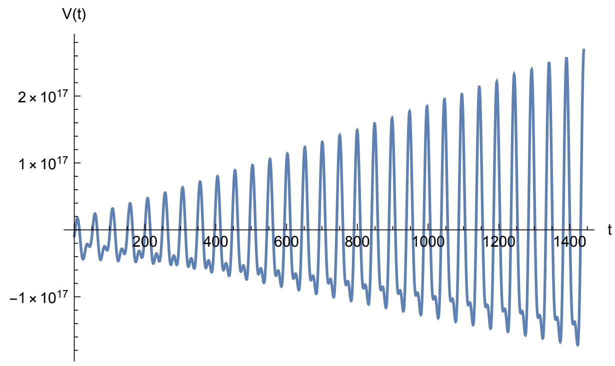

Fig. 3 Visibility Intervals Between AQUA and ARIRANG2 24-H with Oblateness Force

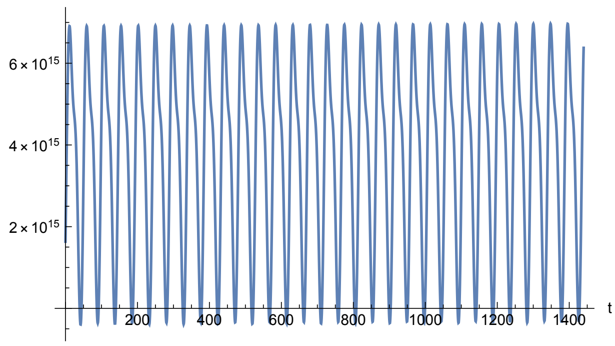

Fig. 4 Visibility Intervals Between HST and ODIN For 24-H

6 Conclusions

An analytical method for the rise -set time prediction for two satellites were derived through a visibility function in terms of classical orbital elements of the two satellites versus time. The secular variations of the orbital elements due to Earth Oblateness were taken into account in order to consider the changes in the nodal period of satellite and the changes in the long term prediction of maximum elevation angle.

In the Table 3: The Visibility Intervals Between AQUA and ARIRANG 2, it is noticed from the first column of (The Function of Visibility without any perturbation) that the time of visibility periods oscillates in a periodic fashion till the ninth period then it decreases gradually. Also, from the second column we conclude that the effect of Earth Oblateness is the increasing the number of periods significantly. But the time intervals of the

Table 3 Visibility Intervals Between AQUA and ARIRANG 24 Houres

	Without Earth Oblateness Force				With Earth Oblateness Force			
	Rise	Set	visibility time		Rise	Set	visibility time	
			m	s			m	s
1	17.1074	44.8663	27	45.534	-	2.35384	-	-
2	66.4086	94.0482	27	38.376	14.4534	51.6719	37	13.1
3	115.669	143.318	27	38.94	64.843	100.955	36	6.72
4	164.973	192.499	27	31.56	114.976	150.222	35	14.76
5	214.236	241.768	27	31.92	164.951	199.507	34	33.36
6	263.542	290.948	27	24.36	214.755	248.754	33	59.94
7	312.807	340.216	27	24.54	264.522	298.042	33	31.2
8	362.115	389.396	27	16.86	314.144	347.279	33	8.1
9	411.382	438.663	27	16.86	364.522	396.57	32	2.88
10	460.692	487.842	27	8.94	413.301	445.8	32	29.94
11	509.962	537.108	27	8.76	462.871	495.093	32	13.32
12	559.276	586.286	27	6	512.306	544.32	32	0.84
13	608.547	635.552	27	0.3	561.825	593.615	31	47.4
14	657.863	684.729	26	51.96	611.207	642.839	31	37.92
15	707.135	733.994	26	51.54	660.689	692.135	31	26.76
16	756.453	783.171	26	43.08	710.033	741.357	31	19.44
17	805.728	832.435	26	42.42	759.488	790.655	31	10.02
18	855.048	881.611	26	33.78	808.804	839.876	31	4.32
19	904.325	930.874	26	32.94	858.238	889.174	30	56.16
20	953.647	980.049	26	24.12	907.532	938.394	30	51.72
21	1002.93	1029.31	26	22.8	965.95	987.694	30	44
22	1052.25	1078.49	26	14.4	1006.23	1036.91	30	40.8
23	1101.35	1127.75	26	13.2	1055.63	1086.21	30	34.8
24	1150.86	1176.92	26	3.6	1104.9	1135.43	30	31.8
25	1200.14	1226.18	26	2.4	1154.29	1184.73	30	26.4
26	1249.47	1275.36	25	53.4	1203.54	1233.95	30	24.6
27	1298.75	1324.62	25	52.2	1252.93	1283.25	30	19.2
28	1348.08	1373.79	25	42.6	1302.17	1332.47	30	18
29	1397.37	1423.05	25	40.8	1351.55	1381.77	30	13.2
30	-	-	-	-	1400.79	1430.99	30	12

visibility decreases gradually.

In the Table 4: The Visibility Intervals Between HST and ODIN, It is noticed from the first column of (The Function of Visibility without any perturbation) that the time of visibility periods oscillates in a periodic manner, and then it decreases gradually. It is also noticed that the second column explains the Earth Oblateness which affects the number of period's increases significantly and clearly. It is also observed that the time of periods of the visibility ascending increases till the ninth period then it increases and decreases in an oscillating manner and over time, stability occurs and the time period stabilizes.

In the Table 5: The Visibility Intervals Between CFESAT and MTI, It is noticed from the first column of (The Function of Visibility without any perturbation) that the time of visibility periods has gradually obvious increases. Also the periods' time of the visibility function increases significantly and clearly. The second column explains the Earth Oblateness which affects the number of periods increase significantly and clearly. It is also

Table 4 Visibility Intervals Between HST and ODIN 24 Houres

	Without Earth Oblateness Force				With Earth Oblateness Force			
	Rise	Set	visibility time		Rise	Set	visibility time	
			m	s			m	s
1	39.3255	44.4432	5	7.062	-	5.86362	-	-
2	87.2035	92.0758	4	52.338	14.1397	22.0401	7	54.024
3	134.82	139.913	5	5.52	28.8463	43.6288	14	46.59
4	182.699	187.545	4	50.76	57.927	91.3591	33	25.926
5	230.316	235.383	5	4.02	105.468	139.084	33	36.96
6	278.195	283.014	4	49.14	153.145	186.832	33	41.22
7	325.812	330.852	5	2.4	200.854	234.559	33	42.3
8	373.691	378.843	5	9.12	248.578	282.308	33	43.8
9	421.307	426.322	5	0.9	296.07	330.036	33	57.96
10	469.188	473.952	4	45.84	344.041	377.786	33	44.7
11	516.803	521.792	4	59.34	391.776	425.514	33	44.28
12	564.684	569.421	4	44.22	439.514	473.263	33	44.9
13	612.298	617.261	4	57.72	487.252	520.92	33	40.08
14	660.18	664.98	4	48	534.992	568.741	33	44.94
15	707.794	712.731	4	56.22	582.731	616.47	33	44.43
16	755.676	760.359	4	40.98	630.473	664.219	33	44.76
17	803.289	808.201	4	54.72	678.212	711.948	33	44.16
18	851.172	855.828	4	39.18	725.954	759.697	33	44.58
19	898.785	903.67	4	53.1	773.694	807.426	33	43.92
20	964.668	951.297	4	37.74	821.437	855.175	33	44.28
21	994.28	999.14	4	51.6	869.178	902.904	33	43.65
22	1042.16	1046.77	4	36.6	916.92	950.653	33	43.98
23	1089.78	1094.61	4	49.8	964.661	998.382	33	43.62
24	1137.66	1142.23	4	34.2	1012.4	1046.13	33	43.8
25	1185.27	1190.08	4	48.6	1060.15	1093.86	33	42.6
26	1233.16	1237.7	4	32.4	1107.89	1141.61	33	43.2
27	1280.77	1285.55	4	46.8	1155.63	1189.34	33	42.6
28	1328.65	1333.17	4	31.2	1203.37	1237.09	33	43.2
29	1376.26	1381.02	4	45.6	1251.11	1248.82	33	42.6
30	1424.15	1428.64	4	29.4	1298.86	1332.56	33	42
31	-	-	-	-	1346.6	1380.29	33	41.4
32	-	-	-	-	1394.34	1428.04	33	42

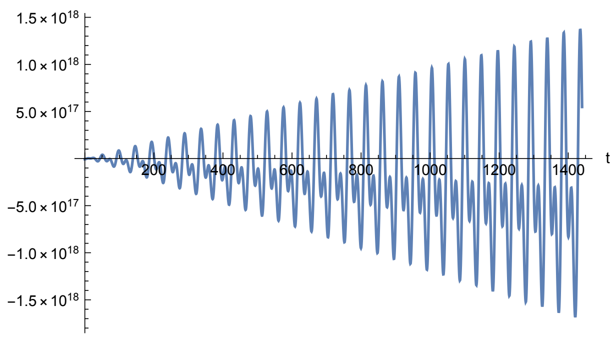

Fig. 5 Visibility Intervals Between HST and ODIN For 24-H with Oblateness Force

observed that the time of periods of the visibility increases and decreases in an oscillating manner.

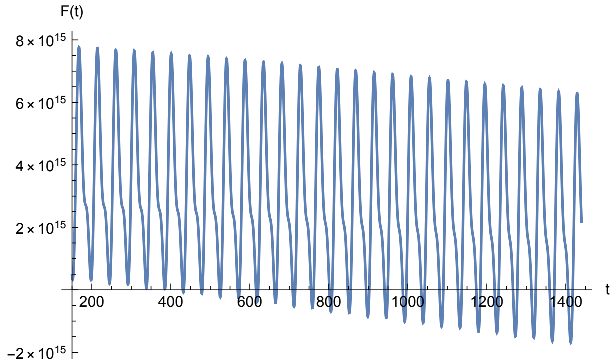

Fig. 6 Visibility Intervals Between CFESAT and MTI For 24-H

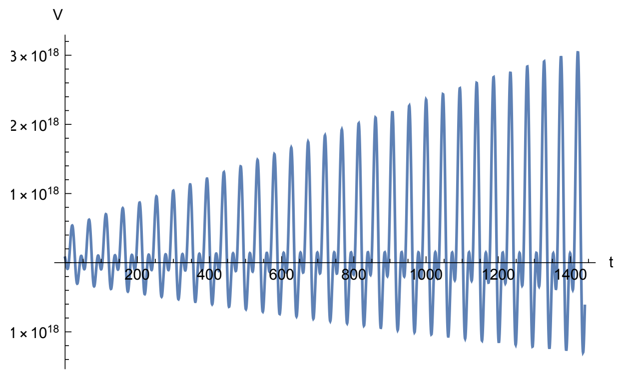

Fig. 7 Visibility Intervals Between CFESAT and MTI For 24-H with Oblateness Force

References

[1] Escobal,P.R ,October 1963, "Rise and set Times of a Satellite about an Oblate Planet," AIAA Journal Vol. I,No. 10, pp.2305-2310.
[2] Escobal,P.R. 1965,"Methods of orbit determination, Robert E. Krieger Publishing Company, Huntington, New York, 1965, pp. 26-29 and 360 - 369.
[3] Lawton, J. A. January-February, 1987."Numerical Method for Rapidly Determining Satellite - Satellite and Satellite-Ground Station In - View Periods, " Journal of guidance, Navigation, and Control, vol. 10,pp.32-36.
[4] Salvatore Alfano , David Negron, Jr., and Jennifer L. Moore, April-June 1992, "Rapid Determination of Satellite Visibility Periods", The Journal of the Astronautical Sciences, Vol.40, No.2, pp.281-295.
[5] P. L. Palmer & Yan Mai "A Fast Prediction Algorithm of Satellite Passes", 14th Annual AIAA/USU Conference on Small Satellite, Surrey Space Centre University of Surrey,Guildford,GU2 7XH, UK
[6] Brouwer, D. and Clemence, G. M.: 1955, Methods of Celestial Mechanics, Academic Press, New York.
[7] Goldstein, H. & Poole, C. & Safko, J , Celestial Mechanics, Barcelona University
[8] Michel Capderou, 2005, Satellites Orbits and Missions, Springer.

Appendix A

$$A_0 = -\frac{a_1^2 a_2^2}{8}\left[4 + 2e_1^2 + 2e_2^2 + \tfrac{1}{8}e_1^4 + \tfrac{1}{8}e_2^4 + 23\, e_1^2 e_2^2 - 8(\gamma_1^2 + \gamma_2^2) + 8(\gamma_1^4 + \gamma_2^4)\right.$$

$$\left. + 40\,\gamma_1^2\,\gamma_2^2 - 4(\gamma_1^2 + \gamma_2^2)\left(e_1^2 + e_2^2\right)\right] + R_e^2\left[a_1^2(1 + \tfrac{3}{2}e_1^2) + a_2^2(1 + \tfrac{3}{2}e_2^2)\right]$$

$$A_1 = a_1^2 a_2^2\left[3e_1 + \tfrac{1}{4}e_1^3 + \tfrac{5}{2}e_1 e_2^2\right] + \frac{R_e^2 a_1^2}{4}\left[e_1^3 - 8\, e_1\right]$$

$$A_2 = a_1^2 a_2^2\left[-\tfrac{1}{2}e_1^2 + \tfrac{3}{4}e_1^4 + \tfrac{13}{4}e_1^2 e_2^2 + e_1^2(\gamma_1^2 + \gamma_2^2)\right] + \frac{R_e^2}{12}\left[-6\,a_1^2 e_1^2 + 5\,a_1^2\, e_1^4\right]$$

$A_3 = \frac{a_1^2}{4}\left[5\,a_2^2\,e_1^3 - R_e^2\,e_1^3\right] \qquad A_4 = -\,a_1^2\,e_1^4\left[\frac{71}{64}\,a_2^2 + \frac{1}{6}R_e^2\right]$
$A_5 = a_1^2\,a_2^2\left[3e_2 + \frac{1}{4}e_2^3 + \frac{5}{2}e_1^2e_2\right] + \frac{R_e^2a_2^2}{4}\left[e_2^3 - 8e_2\right]$

$A_6 = a_1^2\,a_2^2\left[-\frac{1}{2}e_2^2 + \frac{3}{4}e_2^4 + \frac{13}{4}e_1^2e_2^2 + e_2^2(\gamma_1^2+\gamma_2^2)\right] + \frac{R_e^2a_2^2}{12}\left[-6\,e_2^2 + 5\,e_2^4\right]$

$A_7 = \frac{a_2^2}{4}\left[5\,a_1^2\,e_2^3 - R_e^2\,e_2^3\right] \qquad A_8 = -\,a_2^2\,e_2^4\left[\frac{71}{64}\,a_1^2 + \frac{1}{6}R_e^2\right]$

$A_9 = a_1^2a_2^2\left[-2\,e_1e_2 - \frac{15}{8}e_1^3e_2 - \frac{15}{8}e_1e_2^3 + 2e_1e_2\left(\gamma_1^2+\gamma_2^2\right)\right]\; A_{10} = \frac{5}{4}a_1^2a_2^2\,e_1e_2^2$

$A_{11} = -\frac{19}{8}a_1^2a_2^2\,e_1e_2^3,\quad A_{12} = \frac{5}{4}a_1^2a_2^2\,e_1^2e_2,\quad A_{13} = -\frac{19}{8}a_1^2a_2^2\,e_1^3e_2,$

$A_{14} = -\frac{101}{32}a_1^2a_2^2\,e_1^2e_2^2\,, A_{15} = a_1^2a_2^2\left[-4\,e_1e_2 + \frac{1}{8}e_1^3e_2 - \frac{1}{8}e_1e_2^3 - 3e_1e_2\left(\gamma_1^2+\gamma_2^2\right)\right],$

$A_{16} = \frac{5}{4}a_1^2a_2^2\,e_1^2e_2\,, A_{17} = \frac{13}{8}a_1^2a_2^2\,e_1^3e_2, A_{18} = \frac{7}{4}a_1^2a_2^2\,e_1e_2^2\,, A_{19} = \frac{9}{32}a_1^2a_2^2\,e_1^2e_2^2,$

$A_{20} = \frac{9}{8}a_1^2a_2^2\,e_1e_2^3, A_{21} = \frac{1}{8}a_1^2a_2^2\,e_1^2\gamma_1^2\,, A_{22} = -\frac{9}{8}a_1^2a_2^2\,e_2^2\gamma_2^2\,, A_{23} = 2a_1^2a_2^2\,e_1\gamma_1^2,$

$A_{24} = -\,a_1^2a_2^2\left[\gamma_1^2 + \frac{1}{2}e_1^2\gamma_1^2 + \frac{1}{2}e_2^2\gamma_1^2 - \gamma_1^4 - 5\gamma_1^2\gamma_2^2\right]\,, A_{25} = -\frac{3}{8}a_1^2a_2^2\,e_1^2\gamma_1^2,$

$A_{26} = -\frac{3}{8}a_1^2a_2^2\,e_2^2\gamma_1^2\,, A_{27} = -2a_1^2a_2^2\,e_1e_2\gamma_1^2\,, A_{28} = -3\,a_1^2a_2^2\,e_1e_2\gamma_1^2\,,$

$A_{29} = a_1^2a_2^2\left[2e_2\,\gamma_1^2 - \frac{9}{8}e_2^2\,\gamma_1^2\right], A_{30} = -\frac{271}{64}a_1^2a_2^2\,e_1^2e_2^2\,, A_{31} = \frac{9}{2}a_1^2a_2^2\,e_1e_1^2,$

$A_{32} = a_1^2a_2^2\left[-\frac{3}{2}\,e_2^2 + \frac{19}{8}e_1^2e_2^2 + \frac{5}{16}e_2^4 + \frac{9}{8}\gamma_1^2e_2^2 + \frac{9}{8}\gamma_2^2e_2^2\right]\,, A_{33} = -\frac{7}{4}a_1^2a_2^2\,e_1e_1^2,$

$A_{34} = -\frac{5}{64}a_1^2a_2^2\,e_1^2e_2^2\,, A_{35} = \frac{11}{128}a_1^2a_2^2\,e_2^4\,, A_{36} = -\frac{27}{16}a_1^2a_2^2\,e_1e_1^3, A_{37} = \frac{1}{16}a_1^2a_2^2\,e_1e_1^3,$

$A_{38} = a_1^2a_2^2e_1e_2^2\,, A_{39} = \frac{1}{4}a_1^2a_2^2e_1e_2^2\,, A_{40} = -\frac{19}{64}a_1^2a_2^2\,e_1^2e_2^2$

$A_{41} = a_1^2a_2^2\left[-\frac{3}{4}\,e_1e_2^2 - \frac{1}{2}e_1^2e_2^2 - \frac{9}{16}e_2^4 + \frac{3}{8}e_2^2\gamma_1^2 + \frac{3}{8}e_2^2\gamma_2^2\right],$

$A_{42} = a_1^2a_2^2\left[-\frac{1}{2}\,e_1e_2 + \frac{1}{2}e_1^3e_2 - \frac{11}{16}e_1e_2^3 + 2e_1e_2\gamma_1^2 + 2e_1e_2\gamma_2^2\right]$

$A_{43} = a_1^2a_2^2\left[-\frac{1}{2}\,e_2 + \frac{1}{4}e_1^2e_2 + \frac{3}{4}e_2^3\right]\,, A_{44} = a_1^2a_2^2\left[-\frac{1}{2}\,e_1e_2 + \frac{31}{16}e_1^3e_2 - \frac{19}{16}e_1e_2^3\right]$

$A_{45} = \frac{1}{4}a_1^2a_2^2\,e_1^2e_2\,, A_{46} = \frac{1}{4}a_1^2a_2^2\,e_1^3e_2,$

$A_{47} = a_1^2a_2^2\left[-\frac{1}{2} + \frac{3}{4}e_1^2 - \frac{7}{64}e_1^4 + \frac{3}{4}\,e_2^2 + e_1^2e_2^2 - \frac{7}{64}e_2^4 + \frac{1}{2}\,\gamma_1^2 + \frac{1}{2}\,\gamma_2^2\right]$

$A_{48} = a_1^2a_2^2\left[-\frac{1}{2}\,e_1 + \frac{1}{4}e_1e_2^2 + \frac{3}{4}e_1^3\right]\,, A_{49} = \frac{1}{4}a_1^2a_2^2\,e_1^3\,, A_{50} = \frac{1}{4}a_1^2a_2^2\,e_1^4$

$A_{51} = a_1^2a_2^2\left[-\,\frac{3}{4}e_1^2 - \frac{9}{16}e_1^4 - \frac{1}{2}e_1^2e_2^2 + \frac{3}{8}e_1^2\,\gamma_1^2 + \frac{3}{8}e_1^2\,\gamma_2^2\right],$

$A_{52} = a_1^2a_2^2\left[-\frac{9}{2}\,e_1e_2 + e_1^3e_2 + e_1e_2^3 + 4e_1e_2\gamma_1^2 + 4e_1e_2\gamma_2^2\right]\,,$

$A_{53} = a_1^2a_2^2\left[+\frac{3}{2}\,e_2 - \frac{11}{4}e_1^2e_2 - e_2^3 - 2e_2\gamma_1^2 - 2e_2\gamma_2^2\right],$

$A_{54} = a_1^2a_2^2\left[\frac{3}{2}\,e_1e_2 + \frac{7}{8}e_1^3e_2 + \frac{13}{6}\,e_1e_2^3\right], A_{55} = a_1^2a_2^2e_1^2e_2, A_{56} = -\frac{7}{8}a_1^2a_2^2\,e_1^3e_2,$

$A_{57} = -\frac{23}{16} a_1^2 a_2^2 e_1 e_2^3$, $A_{58} = \frac{1}{2} a_1^2 a_2^2 e_2^3$,$A_{59} = \frac{17}{16} a_1^2 a_2^2 e_1 e_2^3$, $A_{60} = \frac{3}{128} a_1^2 a_2^2 e_2^4$,

$A_{61} = -\frac{9}{8} a_1^2 a_2^2 \left(\gamma_1^2 e_2^2 + \gamma_2^2 e_1^2\right)$, $A_{62} = -\frac{3}{8} a_1^2 a_2^2 e_1^2 \gamma_1^2$,$A_{63} = -4R_e^2 a_1 a_2 \gamma_1 \gamma_2 \sigma_1 \sigma_2$

$A_{64} = 4R^2 a_1 a_2 \gamma_1 \gamma_2 \sigma_1 \sigma_2$, $A_{65} = \frac{9}{2} a_1^2 a_2^2 e_1^2 e_2$, $A_{66} = -\frac{11}{8} a_1^2 a_2^2 e_1^3 e_2$,

$A_{67} = \frac{1}{8} a_1^2 a_2^2 e_1^3 e_2$, $A_{68} = \frac{5}{8} a_1^2 a_2^2 e_1^3$, $A_{69} = \frac{3}{128} a_1^2 a_2^2 e_1^4$ $A_{70} = \frac{9}{4} a_1^2 a_2^2 e_1^2 e_2$,

$A_{71} = -\frac{133}{64} a_1^2 a_2^2 e_1^2 e_2^2$, $A_{72} = 2 a_1^2 a_2^2 e_2 \gamma_1^2$, $A_{73} = -4 a_1^2 a_2^2 e_1 e_2 \gamma_1^2$,

$A_{74} = a_1^2 a_2^2 [-\frac{3}{2} e_1^2 + \frac{5}{16} e_1^4 - \frac{45}{8} e_1^2 e_2^2 + \frac{9}{8} e_1^2 \gamma_1^2 + \frac{9}{8} e_1^2 \gamma_2^2]$

$A_{75} = a_1^2 a_2^2 [-\gamma_1^2 - \frac{1}{2} e_1^2 \gamma_1^2 - \frac{1}{2} e_2^2 \gamma_1^2 + \gamma_1^4 + \gamma_1^2 \gamma_2^2]$ $A_{76} = 2 a_1^2 a_2^2 e_1 \gamma_1^2$,

$A_{77} = -\frac{9}{8} a_1^2 a_2^2 e_1^2 \gamma_1^2$, $A_{78} = -2 a_1^2 a_2^2 e_1 e_2 \gamma_1^2$, $A_{79} = -\frac{3}{8} a_1^2 a_2^2 e_2^2 \gamma_1^2$

$A_{80} = -\frac{1}{2} a_1^2 a_2^2 \gamma_2^4$, $A_{81} = 2 a_1^2 a_2^2 \gamma_1 \gamma_2^3$, $A_{82} = -2 a_1^2 a_2^2 \gamma_1 \gamma_2^3$, $A_{83} = 2 a_1^2 a_2^2 e_1 \gamma_2^2$

$A_{84} = a_1^2 a_2^2 [-\gamma_2^2 + \gamma_1^2 \gamma_2^2 + \gamma_2^4 - \frac{1}{2} e_1^2 \gamma_2^2 - \frac{1}{2} e_2^2 \gamma_2^2]$, $A_{85} = -\frac{3}{8} a_1^2 a_2^2 e_1^2 \gamma_2^2$,

$A_{86} = -\frac{3}{8} a_1^2 a_2^2 e_2^2 \gamma_2^2$, $A_{87} = -2 a_1^2 a_2^2 e_1 e_2 \gamma_2^2$, $A_{88} = -4 a_1^2 a_2^2 e_1 e_2 \gamma_2^2$,

$A_{89} = 2 a_1^2 a_2^2 e_2 \gamma_2^2$, $A_{90} = 2 a_1^2 a_2^2 e_2 \gamma_2^2$, $A_{91} = -4 a_1^2 a_2^2 e_1 e_2 \gamma_2^2$,

$A_{92} = a_1^2 a_2^2 [-\gamma_2^2 + 5\gamma_1^2 \gamma_2^2 + \gamma_2^4 - \frac{1}{2} e_1^2 \gamma_2^2 - \frac{1}{2} e_2^2 \gamma_2^2]$, $A_{93} = 2 a_1^2 a_2^2 e_1 \gamma_2^2$

$A_{94} = -\frac{9}{8} a_1^2 a_2^2 e_1^2 \gamma_2^2$, $A_{95} = 2 a_1^2 a_2^2 e_1 e_2 \gamma_2^2$, $A_{96} = -\frac{3}{8} a_1^2 a_2^2 e_2^2 \gamma_2^2$

$A_{97} = -3 a_1^2 a_2^2 \gamma_1^2 \gamma_2^2$, $A_{98} = \frac{9}{4} a_1^2 a_2^2 e_1^2 \gamma_1 \gamma_2$, $A_{99} = -4 a_1^2 a_2^2 e_1 \gamma_1 \gamma_2$,

$A_{100} = a_1^2 a_2^2 \gamma_1 \gamma_2 \left[2 + e_1^2 + e_2^2 - 2\gamma_1^2 - 4\gamma_2^2\right]$ $A_{101} = \frac{3}{4} a_1^2 a_2^2 e_1^2 \gamma_1 \gamma_2$,

$A_{102} = \frac{3}{4} a_1^2 a_2^2 e_2^2 \gamma_1 \gamma_2$, $A_{103} = 4 a_1^2 a_2^2 e_1 e_2 \gamma_1 \gamma_2$, $A_{104} = 8 a_1^2 a_2^2 e_1 e_2 \gamma_1 \gamma_2$,

$A_{105} = -4 a_1^2 a_2^2 e_2 \gamma_1 \gamma_2$, $A_{106} = \frac{9}{4} a_1^2 a_2^2 e_2^2 \gamma_1 \gamma_2$,

$A_{107} = a_1^2 a_2^2 \gamma_1 \gamma_2 \left[-2 - e_1^2 - e_2^2 + 4\gamma_1^2 + 4\gamma_2^2\right]$,$A_{108} = -\frac{3}{4} a_1^2 a_2^2 e_1^2 \gamma_1 \gamma_2$,

$A_{109} = 5 a_1^2 a_2^2 e_2 \gamma_1 \gamma_2$, $A_{110} = -\frac{9}{4} a_1^2 a_2^2 e_2^2 \gamma_1 \gamma_2$, $A_{111} = \frac{9}{2} R_e^2 a_1 a_2 e_1 e_2 \gamma_2^2$,

$A_{112} = R_e^2 a_1 a_2 \left[3e_2 - \frac{3}{2} e_2 e_1^2 - 6 e_2 \gamma_1^2\right]$, $A_{113} = R_e^2 a_1 a_2 \left[\frac{3}{2} e_1 e_2 - \frac{9}{8} e_1^3 e_2 - \frac{3}{2} e_1 e_2 \gamma_1^2\right]$,

$A_{114} = \frac{9}{8} R_e^2 a_1 a_2 e_1^2 e_2$, $A_{115} = -\frac{2}{3} R_e^2 a_1 a_2 e_2^4$,

$A_{116} = R_e^2 a_1 a_2 \left[-\frac{3}{4} e_2^2 + \frac{3}{8} e_1^2 e_2^2 + \frac{3}{4} e_2^4 + \frac{3}{4} e_2^2 \gamma_1^2 + \frac{3}{4} e_2^2 \gamma_2^2\right]$

$A_{117} = -\frac{3}{8} R_e^2 a_1 a_2 e_1 e_2^2$, $A_{118} = -\frac{9}{8} R_e^2 a_1 a_2 e_1^2 e_2^2$,

$A_{119} = R_e^2 a_1 a_2 \left[-e_2 + \frac{1}{2} e_1^2 e_2 - \frac{3}{4} e_2^3 + e_2 \gamma_1^2 + e_2 \gamma_2^2\right], A_{120} = -\frac{3}{8} R_e^2 a_1 a_2 e_1^2 e_2$

$A_{121} = R_e^2 a_1 a_2 \left[-\frac{1}{2} e_1 e_2 + \frac{3}{8} e_1^3 e_2 + \frac{3}{8} e_1 e_2^3 + \frac{1}{2} e_1 e_2 \gamma_1^2 + \frac{1}{2} e_1 e_2 \gamma_2^2\right],$

$A_{122} = R_e^2 a_1 a_2 \left[-2 + 2 e_1^2 + 2e_2^2 + \frac{1}{16} e_1^4 + \frac{1}{16} e_2^4 - \frac{1}{2} e_1^2 e_2^2 + 4 \gamma_1^2 + 4 \gamma_2^2 - 4 \gamma_1^2 \gamma_2^2\right],$

$A_{123} = R_e^2 a_1 a_2 \left[-e_1 + \frac{3}{4} e_1^3 + \frac{1}{2} e_1 e_2^2 + e_1 \gamma_1^2 + e_1 \gamma_2^2\right]$

$A_{124} = R_2^2 a_1 a_2 \left[-\frac{3}{4} e_1^2 + \frac{3}{4} e_1^4 + \frac{3}{8} e_1^2 e_2^2 + \frac{3}{4} e_1^2 \gamma_1^2 + \frac{3}{4} e_1^2 \gamma_2^2\right], A_{125} = -\frac{2}{3} R_e^2 a_1 a_2 e_1^4$

$A_{126} = R_e^2 a_1 a_2 \left[-\frac{1}{4} e_2^2 + \frac{1}{8} e_1^2 e_2^2 - \frac{1}{12} e_2^4 + \frac{1}{4} e_2^2 \gamma_1^2 - 2\gamma_2^2 + \frac{1}{4} e_1^2 \gamma_2^2 + \frac{5}{4} e_2^2 \gamma_2^2 + 2\gamma_1^2 \gamma_2^2\right]$

$A_{127} = -\frac{1}{8} R_e^2 a_1 a_2 e_1 e_2^2, A_{128} = -\frac{3}{32} R_e^2 a_1 a_2 e_1^2 e_2^2, A_{129} = \frac{1}{8} R_e^2 a_1 a_2 e_1 e_2^3$

$A_{130} = -\frac{1}{12} R_e^2 a_1 a_2 e_2^3, A_{131} = -\frac{1}{24} R_e^2 a_1 a_2 e_1 e_2^3, A_{132} = -\frac{3}{64} R_e^2 a_1 a_2 e_2^4,$

$A_{133} = -\frac{9}{2} R_e^2 a_1 a_2 e_1 e_2 \gamma_2^2, A_{134} = 3 R_e^2 a_1 a_2 e_2 \gamma_2^2, A_{135} = \frac{3}{2} R_e^2 a_1 a_2 e_1 e_2 \gamma_2^2,$

$A_{136} = -\frac{1}{4} R_e^2 a_1 a_2 e_2^2 \gamma_2^2, A_{137} = -R_e^2 a_1 a_2 e_1 \gamma_2^2, A_{138} = 3 R_e^2 a_1 a_2 e_1 \gamma_2^2,$

$A_{139} = -\frac{3}{4} R_e^2 a_1 a_2 e_1^2 \gamma_2^2, A_{140} = \frac{3}{2} R_e^2 a_1 a_2 e_1 e_2 \gamma_2^2, A_{141} = -R_e^2 a_1 a_2 e_2 \gamma_2^2,$

$A_{142} = -\frac{1}{2} R_e^2 a_1 a_2 e_1 e_2 \gamma_2^2, A_{143} = -\frac{3}{4} R_e^2 a_1 a_2 e_2^2 \gamma_2^2, A_{144} = \frac{3}{8} R_e^2 a_1 a_2 e_1^2 e_2$

$A_{145} = R_e^2 a_1 a_2 \left[-\frac{1}{4} e_1^2 \gamma_1^2 - \frac{1}{4} e_1^2 \gamma_2^2 - 2 \gamma_1^2 \gamma_2^2\right], A_{146} = -\frac{1}{32} R_e^2 a_1 a_2 e_1^2 e_2^2,$

$A_{147} = R_e^2 a_1 a_2 \left[3e_1 - \frac{3}{2} e_1 e_2^2 - 3 e_1 \gamma_1^2 - 3 e_1 \gamma_2^2\right],$

$A_{148} = R_e^2 a_1 a_2 \left[-\frac{1}{4} e_1^2 + \frac{1}{8} e_1^2 e_2^2 - \frac{1}{12} e_1^4 + \frac{1}{4} e_1^2 \gamma_1^2 + \frac{1}{4} e_1^2 \gamma_2^2\right], A_{149} = -\frac{1}{12} R_e^2 a_1 a_2 e_1^3,$

$A_{150} = -\frac{3}{64} R_e^2 a_1 a_2 e_1^4, A_{151} = R_e^2 a_1 a_2 \left[\frac{3}{2} e_1 e_2 + \frac{1}{8} e_1^3 e_2 - \frac{9}{8} e_1 e_2^3 - \frac{3}{2} e_1 e_2 \gamma_1^2 - \frac{3}{2} e_1 e_2 \gamma_2^2\right],$

$A_{152} = -\frac{1}{8} R_e^2 a_1 a_2 e_1^2 e_2, A_{153} = -\frac{1}{24} R_e^2 a_1 a_2 e_1^3 e_2, A_{154} = \frac{9}{8} R_e^2 a_1 a_2 e_1^2 e_2,$

$A_{155} = -\frac{3}{32} R_e^2 a_1 a_2 e_1^2 e_2^2, A_{156} = -\frac{9}{2} R_e^2 a_1 a_2 e_1 e_2 \gamma_1^2, A_{157} = 3 R_e^2 a_1 a_2 e_2 \gamma_1^2,$

$A_{158} = \frac{3}{2} R_e^2 a_1 a_2 e_1 e_2 \gamma_1^2, A_{159} = 3 R_e^2 a_1 a_2 e_1 \gamma_1^2,$

$A_{160} = -\frac{3}{4} R_e^2 a_1 a_2 \gamma_1^2 e_2^2, A_{161} = -R_e^2 a_1 a_2 e_1 \gamma_1^2, A_{162} = -R_e^2 a_1 a_2 e_2 \gamma_1^2$

$A_{163} = -R_e^2 a_1 a_2 e_1 e_2 \gamma_1^2, A_{164} = R_e^2 a_1 a_2 \left[-2\gamma_1^2 + e_1^2 \gamma_1^2 + e_2^2 \gamma_1^2 + 2 \gamma_1^2 \gamma_2^2\right]$

Table 5 Visibility Intervals Between CFESAT and MTI 24 Houres

	Without Earth Oblateness Force				With Earth Oblateness Force			
	Rise	Set	visibility time		Rise	Set	visibility time	
			m	s			m	s
1	430.519	432.975	2	27.36	2.48729	10.1979	7	42.6366
2	476.988	479.867	2	52.74	27.8353	40.9145	13	4.752
3	523.051	527.147	4	5.76	49.141	56.6622	7	31.26
4	569.583	573.973	4	23.4	74.605	87.7845	13	10.77
5	615.801	621.095	5	17.64	95.8712	103.138	7	16.008
6	662.353	667.899	5	32.76	121.338	134.611	13	16.38
7	708.642	714.947	6	18.3	142.528	149.665	7	8.22
8	755.203	761.74	6	32.22	168.1	181.437	13	20.22
9	801.535	808.742	7	12.42	189.261	196.185	6	55.44
10	848.101	855.526	7	25.5	214.827	228.227	13	24
11	894.462	902.496	8	2.04	235.913	242.757	6	50.64
12	941.03	949.275	8	14.7	261.583	275.027	13	26.64
13	987.413	996.22	8	48.42	282.643	289.308	6	39.9
14	1033.98	1042.99	9	0.6	308.306	321.796	13	29.4
15	1080.38	1089.92	9	32.4	329.285	335.916	6	37.86
16	1126.95	1136.69	9	44.4	355.057	368.578	13	31.26
17	1173.37	1183..6	10	13.8	376.01	382.491	6	28.86
18	1219.93	1230.37	10	26.4	401.777	429.13	13	29.52
19	1266.36	1277.26	10	54	422.638	429.13	6	42.6
20	1312.93	1324.02	11	5.4	448.525	462.103	13	43.68
21	13.5937	1370.9	11	31.8	469.357	475.726	6	22.14
22	1405.93	1417.66	11	43.8	495.241	508.85	13	36.54
23	-	-	-	-	515.968	522.395	6	25.62
24	-	-	-	-	541.985	555.61	13	37.5
25	-	-	-	-	562.68	569.009	6	19.74
26	-	-	-	-	588.7	602.351	13	39.06
27	-	-	-	-	609.273	615.705	6	25.92
28	-	-	-	-	635.441	649.103	13	39.72
29	-	-	-	-	655.978	662.335	6	21.42
30	-	-	-	-	682.152	695.45	13	17.88
31	-	-	-	-	702.554	709.056	6	30.12
32	-	-	-	-	728.89	742.586	13	41.76
33	-	-	-	-	749.251	755.7	6	26.94
34	-	-	-	-	775.6	789.321	13	43.26
35	-	-	-	-	795.811	802.441	6	37.98
36	-	-	-	-	822.335	836.06	13	43.5
37	-	-	-	-	842.502	849.102	6	36
38	-	-	-	-	869.042	882.795	13	45.18
39	-	-	-	-	889.048	895.865	6	49.02
40	-	-	-	-	915.775	929.528	13	45.18
41	-	-	-	-	953.733	942.536	6	48.18
42	-	-	-	-	962.48	976.261	13	46.86
43	-	-	-	-	982.266	989.316	7	33
44	-	-	-	-	1009.21	1022.99	13	46.8
45	-	-	-	-	1028.95	1036	7	3
46	-	-	-	-	1055.91	1069.72	13	48.6
47	-	-	-	-	1075.47	1082.79	7	19.2
48	-	-	-	-	1102.64	1116.44	13	48
49	-	-	-	-	1122.15	1129.84	7	19.8
50	-	-	-	-	1149.34	1163.18	13	50.4
51	-	-	-	-	1168.66	1176.29	7	37.8
52	-	-	-	-	1196.07	1209.89	13	49.2
53	-	-	-	-	1215.34	1222.99	7	39
54	-	-	-	-	1242.76	1256.63	13	52.2