Peristaltic slip flow of a Bingham fluid in an inclined porous conduit with Joule heating

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Abstract

Abstract: The present study deals with simultaneous effects of Joule heating and slip on peristaltic flow of a Bingham fluid in an inclined tapered porous channel with elastic walls. The closed form solutions for the stream function, the velocity and the temperature fields are obtained. The effects of the physical parameters on the flow characteristics are presented through graphs for both slip and no-slip cases. In addition, the performance of the temperature is studied with and without Joule heating effects. Moreover, the trapping phenomenon is analysed. The size of the trapped bolus increases with increasing values of the slip parameter and decreasing values of the magnetic, the permeability and the yield stress parameters. The present results are compared with the available results in the literature and our results agree well with the available results for some special cases.

Keywords: MHD peristaltic slip flow, Joule heating, Bingham fluid, inclined porous channel.

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1 Introduction

Peristaltic transport is a mechanism of a fluid flow produced by propagation of wave trains along the channel walls. This phenomenon has wide range of practical applications in physiology and biomedical engineering such as swallowing of foodstuff, blood movement in blood vessels, lymph drive in lymphatic vessels, urine transport through ureter, chyme movement in intestinal tract, ovum transport, bile flow in bile duct, etc. Initially Latham [1] and Shapiro et al. [2] investigated the mechanism of peristalsis. Later many investigators have

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The study of non-Newtonian fluid flow problems with heat transfer has many applications in chemical engineering and related industries; also, due to its tremendous applications in biomedical engineering, researchers have paid considerable attention on peristaltic flow of convective non-Newtonian fluids. The effects of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical channel with porous medium was analyzed by Vajravelu et al. [7]. Tripathi [8] presented a mathematical model for explaining the impact of heat transfer on swallowing of food bolus through the esophagus. Vajravelu et al. [9] made a theoretical model to study the peristaltic flow of a MHD phan-thien-tanner fluid in an asymmetric channel with heat transfer. Akram et al. [10] and Nadeem et al. [11] studied the impact of peristaltic flow of non-Newtonian fluids. Very recently the authors [12–18] have studied the peristalsis by considering different fluids and geometries.

Not much work has been reported on peristaltic flow by considering simultaneous effects of slip and magnetic field with heat transfer. Furthermore, the consideration of wall properties is essential to understand the behavior of physiological fluids. Srinivas et al. [27] examined the impact of slip conditions and heat transfer on MHD peristaltic transport. The effects of slip and wall properties on the peristaltic transport of a MHD Bingham fluid with heat transfer, was presented by Lakshminarayana et al. [28]. Satyanarayana et al. [29] presented a model to explain the effects of magnetohydrodynamics and heat transfer on peristaltic slip flow of Bingham fluid in porous channel with flexible walls. Moreover, the impact of wall properties is discussed by Srinivas and Kothandapani [30], Hayat et al. [31], Riaz et al. [32] and Sucharitha et al. [33, 34].

Present paper describes a mathematical model to investigate the impact of Joule heating and slip on magnetohydrodynamic peristaltic flow of a Bingham fluid in an inclined non-uniform porous channel with flexible walls. The expressions for the stream function, the velocity and the temperature fields are obtained. The effects of the physical parameters on the flow quantities are discussed in detail. The present study reveals many interesting results which could facilitate the further investigation in convective non-Newtonian fluid flow phenomenon.

2 Mathematical formulation

Consider the two dimensional flow of a MHD Bingham fluid in an inclined non-uniform channel with porous medium. Flow is due to sinusoidal wave trains propagating along the elastic walls of the channel with a constant speed \(c\) (see Fig. 1). The channel wall deformation is assumed as

\[
\frac{dp}{dx} = -\frac{\partial}{\partial y} \left( \tau_0 - \frac{\partial^2 \psi}{\partial y^2} \right) - N^2 \frac{\partial \psi}{\partial y} + \eta \sin \alpha \quad (2.1)
\]

where \(d(x) = d + \bar{m}x, \bar{m} << 1\).

Using long wavelength and small Reynolds number assumptions (see [28,30,34,35] for details), the simplified non-dimensional governing equations and corresponding boundary conditions for the present study can be written as

Momentum and energy equations are

\[
\frac{dp}{dx} = -\frac{\partial}{\partial y} \left( \tau_0 - \frac{\partial^2 \psi}{\partial y^2} \right) - N^2 \frac{\partial \psi}{\partial y} + \eta \sin \alpha \quad (2.2)
\]

where \(N^2 = M^2 + \sigma^2\)

\[
\frac{dp}{dy} = 0 \quad (2.3)
\]
\[ \frac{\partial^2 \theta}{\partial y^2} + Br \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + Br M^2 \left( \frac{\partial \psi}{\partial y} \right)^2 = 0 \]  

(2.4)

Fig. 1 Flow configuration.

The corresponding boundary conditions (see Ref. [28, 30]) are

\[ \frac{\partial^2 \psi}{\partial y^2} \left( -\tau_0 + \frac{\partial^2 \psi}{\partial y^2} \right) - N^2 \frac{\partial \psi}{\partial y} + \eta \sin \alpha = \left[ E_1 \frac{\partial^3 h}{\partial x^3} + E_2 \frac{\partial^3 h}{\partial x^2 \partial t} + E_3 \frac{\partial^2 h}{\partial x t} \right] \quad \text{at} \quad y = h, \]  

(2.5)

(Flexible boundary condition)

\[ \frac{\partial \psi}{\partial y} = -\beta \frac{\partial^2 \psi}{\partial y^2} \quad \text{at} \quad y = h, \]  

(2.6)

(Slip condition)

\[ \psi_p = 0; \; \psi_{yy} = \tau_0 \quad \text{at} \quad y = 0, \]  

\[ \psi = \psi_p \quad \text{at} \quad y = y_0 \]  

(2.7)

\[ \frac{\partial \theta}{\partial y} = 0 \quad \text{for} \quad 0 \leq y \leq y_0, \]  

\[ \theta = 1 \quad \text{at} \quad y = h \]  

(2.8)

The non-dimensional parameters and quantities used in the above governing equations are

\[
\begin{align*}
\chi &= \frac{x}{\lambda}, \quad y = \frac{y}{d}, \quad \Psi = \frac{\psi}{c^2d^2}, \quad \rho = \frac{\rho_d}{\mu c}, \quad \tau_0 = \frac{\tau_0}{\mu c}, \quad \tau_{yy} = \frac{\tau_{yy}}{\mu c}, \quad t = \frac{t}{N}, \quad m = \frac{m}{c}, \quad \delta = \frac{d}{\lambda}, \quad \varepsilon = \frac{a}{d} \\
\Re &= \frac{\rho c_d}{\mu}, \quad \Theta = \frac{(T - T_0)}{(T_1 - T_0)}, \quad M = \frac{\sqrt{\frac{\mu}{\sigma}} B_0}{\mu c}, \quad \sigma = \frac{d}{\sqrt{k}}, \quad \Pr = \frac{\nu_c}{k_0}, \quad Ec = \frac{c^2}{\xi(T_1 - T_0)}, \quad \eta = \frac{\rho c_d^2}{\mu c}, \quad E_1 = -\frac{\rho c_d}{\mu c}, \quad E_2 = \frac{m c_d}{c^2 d^2}, \quad E_3 = \frac{c_d}{c^2 d^2}, \quad \beta = \frac{\beta_0}{\mu c}, \quad h = \frac{y}{\pi} = 1 + mx + \varepsilon \sin 2\pi(x - t), \quad \xi = \frac{d}{\lambda}
\end{align*}
\]  

(2.9)

where \( u, \; v, \; p \) and \( \psi \) are the velocity components, pressure and stream functions respectively, \( \rho \) is the density, \( \mu \) is the viscosity, \( d \) is the mean width of the channel, \( a \) is the amplitude, \( \lambda \) is the wavelength, \( c \) is the wave speed, \( \xi \) is the specific heat, \( \nu \) is the kinematic viscosity, \( k_0 \) is the thermal conductivity, \( k \) is the permeability, \( \sigma_0 \) is the electrical conductivity, \( B_0 \) is the magnetic field, \( g \) is the acceleration due to gravity, \( T \) is the temperature,
τ₀ is the yield stress, Ec is the Eckert number, E₁, E₂ and E₃ are the elasticity parameters, m is the non-uniform parameter, σ is the permeability parameter, α is the inclination angle, β is the slip parameter, Br is the Brinkman number, M is the magnetic parameter, δ is the wave number, ε is the amplitude ratio, Pr is the Prandtl number and Re is the Reynolds number.

3 Solution of the problem

By differentiating Eq. (2.2) with respect to y we obtain

\[ \frac{\partial^2}{\partial y^2} \left( -\tau_0 + \frac{\partial^2 \psi}{\partial y^2} \right) - N^2 \frac{\partial^3 \psi}{\partial y^3} = 0, \] (3.1)

By solving Eq. (3.1) with boundary conditions (2.5), (2.6) and (2.7) we obtain the stream functions and
corresponding velocities in the plug and non-plug flow regions as

\[
\psi_p = \frac{A_0}{N} \left[ (\sinh Ny_0 - A_1 \cosh Ny_0) - \frac{A_0}{N^2} \right] y,
\]

(3.2)

\[
u_p = \frac{A_0}{N} \left[ (\sinh Ny_0 - A_1 \cosh Ny_0) - \frac{A_0}{N^2} \right],
\]

(3.3)

\[
\psi = \frac{A_0}{N^2} (\cosh Ny - A_1 \sinh Ny) - \frac{(A_0 + A_2 + \eta)}{N^2},
\]

(3.4)

\[
u = \frac{A_0}{N} (\sinh Ny - A_1 \cosh Ny) - \frac{A_0}{N^2},
\]

(3.5)

where

\[
y_0 = \frac{N}{A_1} \tanh^{-1} \left( \frac{1}{A_1} \right), A_0 = -8 \pi^3 \left[ (E_1 + E_2) \cos 2\pi(x - t) - \frac{E_1}{2\pi} \sin 2\pi(x - t) \right] - \eta \sin \alpha,
\]

A_1 = \frac{\sinh Ny + \beta N \cosh Ny - A_0 \sinh Nh}{\cosh Nh + \beta N \sinh Nh}, A_2 = \tau_0 \left( (Ny_0 A_1 + 1) \cosh Ny_0 - (Ny_0 + A_1) \sinh Ny_0 - 1 \right).

4 Results and discussion

To find the impact of physical parameters, we have plotted the velocity, temperature and heat transfer coefficient profiles in figures 2–22 with the fixed values of \(x = 0.2, \alpha = 0.24, \sigma = 1, m = 0.2, M = 1, \tau_0 = 0.4, \eta = 1, \alpha = \pi/4, E_1 = 0.2, E_2 = 0.2, E_3 = 0.1, Br = 0.2 \) (for temperature \(E_1 = 0.005, E_2 = 0.005, E_3 = 0.001 \)). It is observed that the velocity is higher in slip flow when compared with the nonslip flow whereas the temperature exhibits the opposite behavior. In fact, the growth in slip diminishes the friction between the wall and the fluid. This may be the cause to increase in the velocity and reduction in the internal heat production. The impacts of magnetic parameter \(M \) and permeability parameter \(\sigma \) on the velocity and the temperature fields are presented in figures 2–5. We found that an increase in \(M \) and \(\sigma \) reduces the velocity profiles. This is due to the influence of drag force (Lorentz force) which opposes the flow. We also noticed an enhancement in the temperature field. This may be due to the Joule heating impact. Similar behavior has been noticed in the study of Hayat et al. [23].

From figures 6–9, it is observed that an increase in the yield stress \(\tau_0 \) is to reduce the velocity field and to increase the temperature field. We noticed that the higher values of the non-uniform parameter \(m \) increase both the velocity and the temperature profiles. The effects of the wall flexibility parameters are described through

\[
Nu = -\left. \frac{d\theta}{dy} \right|_{y=h}.
\]

(3.7)
Fig. 8 Temperature profiles for \( \tau_0 \).

Fig. 9 Temperature profiles for \( m \).

Fig. 10 Velocity profiles for \( E_1 \) and \( E_2 \).

Fig. 11 Velocity profiles for \( E_3 \).

Fig. 12 Temperature profiles for \( E_1 \) and \( E_2 \).

Fig. 13 Temperature profiles for \( E_3 \).
figures 10–13. We notice that the velocity increases with increasing $E_1$ and $E_2$ (due to the wall tension and mass characterization property); but the velocity drop is noticed for increasing values of $E_3$ (due to the damping force). The influence of the elasticity parameters on the temperature has similar effects as in the case of the velocity. This agrees with the results of Hayat et al. [21].

From figures 14 and 15 we see that for increasing values of the inclination angle $\alpha$ both the velocity and temperature fields increase. The impact of the Brinkman number $Br$ is shown in figure 16. An increase in $Br$ enhances the temperature field.

Figures 17 and 18 are drawn to validate the present results with the published available results in the literature. It is perceived that for $\alpha = 0$ and in the absence of Joule heating our results reduce to those of Satyanarayana et al. [19]. Further, we notice an increase in the values of the velocity and temperature fields when compared with the results in [19], due to the presence of Joule heating. The deviations in heat transfer coefficient are shown in figures 19–22. It can be seen that the absolute value of the Nusselt number is higher in the slip flow compared to the nonslip flow. Figures 19 and 20 reveal that the heat transfer coefficient increases for large values of $M$ (due to the existence of Joule heating) whereas it is a decreasing function of $\sigma$. From figures 21 and 22, we observe an increase in the Nusselt number with the increasing $Br$ and decreasing $\alpha$.

5 Trapping phenomenon

The process of the formation of fluid bolus by the closed streamlines in the fluid flow is called trapping phenomenon and the trapped bolus moves forward with the peristaltic wave. The behaviour of streamlines is presented in figures 23–26. The effect of the slip parameter $\beta$ is shown in figure 23. An increase in $\beta$ increases the size of the bolus. From figures 24–26, we notice that an increase in the magnetic parameter $M$, the
Fig. 17 Velocity profiles for validation.

Fig. 18 Temperature profiles for validation.

Fig. 19 Variation of Nusselt number for $M$.

Fig. 20 Variation of Nusselt number for $\sigma$.

Fig. 21 Variation of Nusselt number for $Br$.

Fig. 22 Variation of Nusselt number for $\alpha$. 
Fig. 23 Stream lines for $\beta$.

Fig. 24 Stream lines for $M$. 

Peristaltic slip flow of a Bingham fluid in an inclined porous conduit with Joule heating
Fig. 25 Stream lines for $\sigma$.

Fig. 26 Stream lines for $\tau_0$. 
permeability parameter \(\sigma\) and the yield stress \(\tau_0\) reduces the size of the trapped bolus.

6 Conclusions

The effects of the wall slip and the Joule heating on MHD peristaltic flow of a Bingham fluid in a porous channel are studied. Long wavelength and small Reynolds number approximations are used to obtain the exact solutions of the problem. The results have applications in biomedical engineering and oil industries. Some of the interesting results are summarised as follows:

1. Velocity is an increasing function of the slip parameter \(\beta\) whereas it is a decreasing function of the magnetic parameter \(M\), the yield stress \(\tau_0\) and the permeability parameter \(\sigma\).
2. The effect of the Joule heating is to increase the temperature.
3. Slip parameter \(\beta\) reduces the temperature while it is an increasing function of the magnetic parameter \(M\) and the permeability parameter \(\sigma\).
4. Inclination angle \(\alpha\) and and the non-uniform parameter \(m\) increase both the velocity and the temperature fields.
5. Increase in both the velocity and the temperature is identified due to increase in the wall parameters \(E_1\) and \(E_2\), while reduction is noticed in the case of other wall parameter \(E_2\).
6. Nusselt number is an increasing function of the slip parameter \(\beta\) and the magnetic parameter \(M\).
7. Size of the trapped bolus is reduced for the increasing values of \(M\) and \(\sigma\) while it increases with increasing \(\sigma\).

References


