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A new stepsize change technique for Adams methods

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Abstract

In this paper a new technique for stepsize changing in the numerical solution of Initial Value Problems for ODEs by means of Adams type methods is considered. The computational cost of the new technique is equivalent to those of the well known interpolation technique (IT). It is seen that the new technique has better stability properties than the IT and moreover, its leading error term is smaller. These facts imply that the new technique can outperform the IT.

Keywords: stepsize change, linear multistep methods
AMS 2010 codes: 65107, CR517

1 Introduction

Consider the numerical solution of the initial value problem

$$\begin{cases} y'(t) = f(t, y(t)), & t \in [0, T] \\ y(0) = y_0 \end{cases} \tag{1}$$

where $T > 0$, and y_0 are given real numbers and $f : [0, T] \times \mathbf{R} \to \mathbf{R}$ is a continuously differentiable function in a neighborhood $\{(t, y) \mid t \in [0, T], |y - y(t)| \le a\}$ of the solution $(t, y(t))$, $t \in [0, T]$ of (1). Although the following results can be also applied to systems of ODE's, for simplicity we restrict our considerations to a single equation.

Assume that approximations y_j, y'_j, $j = 0, \ldots, n-1$, $n \ge k$ to the true solution and its derivative are known at the grid points t_j, $j = 0, \ldots, n-1$ and we want to advance the numerical solution computing new approximations

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at $t_n = t_{n-1} + h_n$ with a k-step Adams-Moulton method (AMk). Then, denoting by $P_n(t)$ the polynomial of degree $\leq k+1$ such that

$$P_n(t_n) = y_n, \quad P_n(t_{n-1}) = y_{n-1}, \quad P_n'(t_{n-j}) = f(t_{n-j}, P_{n-1}(\bar{t}_{n-j})), \quad j = 1, \ldots, k, \tag{2}$$

where $\bar{t}_{n-j}$ are back points that define the class of AMk method and the new approximation y_n is determined so that $P_n(t)$ satisfies the differential equation at t_n, i.e.,

$$P_n'(t_n) = f(t_n, y_n). \tag{3}$$

By substituting the Lagrange form of the interpolatory polynomial (2) into (3), one gets the difference equations of the AMk method. However, for our purposes it turns out more convenient to deal with the polynomial form of the AMk method. To define the method in this way, note that polynomials P_n and P_{n-1} associated to t_n and t_{n-1} respectively satisfy

$$P_n(t) = P_{n-1}(t) + \gamma q_n(t), \quad q_n(t) = \int_{t_{n-1}}^{t} (s - \bar{t}_{n-1}) \ldots (s - \bar{t}_{n-k})\, \mathrm{d}s \tag{4}$$

and γ is a real constant which may be determined from the two conditions

$$P_n(t_n) = y_n, \qquad P_n'(t_n) = f(t_n, y_n). \tag{5}$$

The equations (4) imply that P_n and P_{n-1} are such that their derivatives agree at the k back points $\bar{t}_{n-1}, \ldots, \bar{t}_{n-k}$ whereas y_n is chosen so that $P_n(t)$ satisfies the ODE at the advanced point t_n.

On a uniform grid we take as back points $\bar{t}_{n-j} = t_n - jh$ where h is the fixed stepsize of the grid. For non uniform grids there are many possibilities about the choice of back points. Firstly, taking as $\bar{t}_{n-j}$, $j = 1, \ldots, k$ the past grid points we get the AMk method with the so called variable coefficient technique for stepsize changing. On the other hand, taking the k equally spaced back points $\bar{t}_{n-j} = t_n - jh_n$, $j = 1, \ldots, k$, we have the AMk method with the interpolation technique for stepsize changing. Other techniques have been proposed by Skeel [14], Gupta and Wallace [8] and Jackson and Sack-Davis [10].

The above techniques are the most usual in practical codes for non-stiff IVP's. Thus, STEP written by Shampine and Gordon [12], DVDQ by Krogh [11] and EPISODE by Byrne and Hindmarsh [1] employ AM formulas with the variable coefficient technique while DIFSUB by Gear [5] and LSODE by Hindmarsh [9] use the interpolation technique.

The stability properties of these techniques have been studied by several authors (e.g. Gear and Tu [6], Crouzeix and Lisbona [4], Grigorieff [7], Skeel and Jackson [13] and Calvo, Lisbona and Montijano [2, 3]). From these papers it follows that the variable coefficient technique is more stable than the interpolation one. However, since the computational cost of the first is higher than the second one, for those problems with stability requirements not too strong, the interpolation technique can be advantageously used.

In this paper, a new technique for stepsize change in Adams methods is proposed. Its computational cost is similar to the interpolation technique but it has better stability properties and smaller leading error term. The paper is organized in the following manner: In section 2 the new technique is introduced in the frame of the polynomial formulation of AM methods. In section 3 the leading term of the local error is obtained. Finally, in section 4 the stability (0-stability) of this technique is studied and some comparisons with the interpolation technique are presented.

2 The new technique of stepsize change

As it was remarked earlier, when AMk methods are written in polynomial form, the difference between interpolation and variable coefficient techniques is the choice of the back points t_{n-j} on which agree the derivatives

$P_n' = P_{n-1}'$ of two consecutive polynomials. Moreover, when the back points are equally spaced, the calculations required to advance one step are considerably simplified.

In view of these facts we will take the back points $\bar{t}_{n-j} = t_n - j\bar{h}, \quad j = 1, \dots, k$ equally spaced with a stepsize $\bar{h}$ adjusted to improve the stability and leading error term over the corresponding ones of the interpolation technique.

Denoting by $r_n = h_n/h_{n-1}$ the stepsize ratio, we have considered $\bar{h}$ given by

$$\bar{h} = h_n \phi(r_n)$$

where $\phi(r)$ is a bounded function for $0 \le r_* < r < r^*$ with $\phi(1) = 1$. In particular, the following functions have been studied:

$$\phi(r) = \alpha_k + (1-\alpha_k)r^{-1}, \tag{T1}$$

$$\phi(r) = \begin{cases} \alpha_k + (1-\alpha_k)r^{-1}, & \text{if } r > 1 \\ 1, & \text{if } r \le 1, \end{cases} \tag{T2}$$

$$\phi(r) = \begin{cases} \alpha_k, & \text{if } r > 1 \\ 1, & \text{if } r \le 1. \end{cases} \tag{T3}$$

where α_k is, for the AMk method, a constant that will be determined by imposing that the stability of the technique and the leading error term behave in an optimal way in a sense that will be made precise later.

Note that if $\alpha_k < 1$, then $\phi(r) < 1$ for all r for the T3 technique, and the same holds for the T1 and T2 techniques if $\alpha_k \in (0,1)$, for $r > 1$, i.e., when the current stepsize is decreased, and therefore $\bar{h} < h_n$. This means that our interpolation points $\bar{t}_{n-j}$ are closer to t_n than those used in the interpolation technique. The choice $\phi(r) = 1$ for $r \le 1$ in T2 and T3 is mainly due to the fact the stability in the interpolation technique is satisfactory for $r \le 1$.

On the other hand, in the techniques T1 and T2, the interpolation stepsize $\bar{h}$ is taken as a convex combination of the two last stepsizes h_{n-1} and h_n, i.e.

$$\bar{h} = \alpha_k h_n + (1-\alpha_k)h_{n-1}.$$

Next, to obtain the propagation matrices we write the AMK methods in matrix form by means of Nordsieck vectors (see [15]). Let

$$q_n(t) = \int_{\bar{t}_{n-1}}^{t} (s - \bar{t}_{n-1}) \dots (s - \bar{t}_{n-k})\, \mathrm{d}s. \tag{6}$$

the modifier polynomial of the new technique and $N_n(t,h)$ and $C_n(t,h)$ the $(k+2)$-Nordsieck vectors at the point t, with stepsize h of $P_n(t)$ and $q_n(t)$ respectively, given by

$$\begin{aligned} N_n(t,h) &= \left(P_n(t), hP_n'(t), \dots, \frac{h^{k+1}}{(k+1)!} P_n^{(k+1)}(t)\right)^T, \\ C_n(t,h) &= \left(q_n(t), hq_n'(t), \dots, \frac{h^{k+1}}{(k+1)!} q_n^{(k+1)}(t)\right)^T. \end{aligned} \tag{7}$$

With these notations, the first equation of (4) can be written equivalently in the form

$$N_n(t_n,h_n) = N_{n-1}(t_n,h_n) + \gamma C_n(t_n,h_n). \tag{8}$$

Now, using the Taylor formula to expand each component of $N_{n-1}(t_n,h_n)$ at the point t_{n-1} and substituting h_n by $r_n h_{n-1}$ we have

$$N_{n-1}(t_n,h_n) = PD(r_n)N_{n-1}(t_{n-1},h_{n-1}), \tag{9}$$

where $D(r) = \mathrm{diag}(1,\ldots,r^{k+1})$ and $P = (p_{ij})_{i,j=0}^{k+1} \in \mathbf{R}^{(k+2)\times(k+2)}$ is the Pascal matrix whose entries are

$$p_{ij} = \begin{cases} \binom{j}{i}, & \text{if } k+1 \geq j \geq i \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

On the other hand, putting in (6) $s = t_n + x\bar{h}$, the Nordsieck vector of $q_n(t)$ at t_n becomes

$$C_n(t_n,h_n) = \bar{h}^{k+1} D(\bar{r}_n)\, c, \tag{10}$$

where $c = (c_0, c_1, \ldots, c_{k+1})^T \in \mathbf{R}^{k+2}$ is a constant vector whose components are the coefficients of the polynomial

$$\Lambda(x) = \int_{-1}^{x} (s+1)\ldots(s+k)\,\mathrm{d}s = \sum_{j=0}^{k+1} c_j x^j, \tag{11}$$

and $\bar{r}_n = h_n/\bar{h} = 1/\phi(r)$.

Substituting (9) and (10) into (8) and putting $N_j = N_j(t_j,h_j)$ we have

$$N_n = PD(r_n)N_{n-1} + \gamma \bar{h}^{k+1} D(\bar{r}_n)c. \tag{12}$$

Finally, the value of constant γ is determined from the second component of (12) and the equation (5). Thus, we arrive at the matrix equation of the AMk method with the new technique of stepsize change

$$N_n = \Omega(r_n)N_{n-1} + \frac{h_n}{c_1 \bar{r}_n} f(t_n,y_n) D(\bar{r}_n)c \tag{13}$$

where the propagation matrix $\Omega(r)$ is given by

$$\begin{aligned} &\Omega(r_n) = (I - \omega(\bar{r}_n)^{-1} D(\bar{r}_n) c e_2^T) P D(r_n) \\ &\omega(\bar{r}_n) = e_2^T D(\bar{r}_n) c = k!\,\bar{r}_n, \quad e_2 = (0,1,0,\ldots)^T \in \mathbf{R}^{k+2}, \\ &\bar{r}_n = 1/\phi(r_n). \end{aligned} \tag{14}$$

3 The local truncation error

Although the matrix form (13)-(14) of the AMk method will be useful to study the stability of the methods, for practical purposes it is more convenient to write the equations in the equivalent form

$$\begin{aligned} &N_{n,0} = PD(r_n)N_{n-1} = (y_{n,0}, h_n y'_{n,0}, \ldots, \frac{h_n^{k+1}}{(k+1)!} y_{n,0}^{(k+1)})^T \\ &y_n = y_{n,0} + \frac{1}{\ell_1 \bar{r}_n}\left(hf(t_n,y_n) - hy'_{n,0}\right) \\ &N_n = N_{n,0} + (y_n - y_{n,0}) D(\bar{r}_n)\ell \end{aligned} \tag{15}$$

where ℓ is the $(k+2)$-vector defined by $\ell = \frac{1}{c_0}c = (1,\ell_1,\ldots,\ell_{k+1})^T$. The first equation of (15) is the prediction stage in which the Nordsieck vector at t_n is predicted from the corresponding one at t_{n-1}. The second equation of (15) is a nonlinear algebraic equation which must solved by some iterative method (e.g. fixed point) to get y_n. In the last equation (15) the predicted $N_{n,0}$ is corrected to the final value N_n.

To define the truncation error $EL(t_n)$ at the grid point t_n, we assume that the true solution $y(t)$ is sufficiently differentiable and then

$$EL(t_n) = y(t_n) - y_n^*$$

where y_n^* is the solution computed by using (15) with an exact Nordsieck vector at t_{n-1}, i.e.

$$N_{n-1}^* = \left(y(t_{n-1}), h_{n-1}y'(t_{n-1}), \dots \frac{h_{n-1}^{k+1}}{(k+1)!} y^{(k+1)}(t_{n-1})\right)^T.$$

Next, let us calculate the leading term of $EL(t_n)$ when it is expanded in powers of h_n. We write it in the form

$$EL(t_n) = (y(t_n) - y_{n,0}) + (y_{n,0} - y_n^*). \tag{16}$$

Taking into account the first equation of (15), the first term in the right hand side of (16) is

$$y(t_n) - y_{n,0} = \frac{h_n^{k+2}}{(k+2)!} y^{(k+2)}(t_{n-1}) + \mathscr{O}(h_n^{k+3}).$$

Similarly, from the second equation of (15) we get

$$y_n^* - y_{n,0} = \frac{1}{\ell_1 \bar{r}_n} \frac{h_n^{k+2}}{(k+1)!} y^{(k+2)}(t_{n-1}) + \mathscr{O}(h_n^{k+3}),$$

and therefore,

$$EL(t_n) = C_{k+2} h_n^{k+2} y^{(k+2)}(t_{n-1}) + \mathscr{O}(h_n^{k+3}),$$

with

$$C_{k+2} = \frac{1}{(k+2)!}\left(1 - \frac{k+2}{\bar{r}_n \ell_1}\right).$$

The values of $(k+2)/\ell_1$, for $2 \le k \le 8$ are given in Table 1.

Table 1

k	2	3	4	5	6	7	8
$\frac{k+2}{\ell_1}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{251}{120}$	$\frac{665}{228}$	$\frac{19087}{7560}$	$\frac{5257}{1920}$	$\frac{1070017}{362880}$

Since all these values are greater than unity, it follows that

$$|C_{k+2}(1)| > |C_{k+2}(\bar{r})|$$

whenever $1 < \bar{r} < \dfrac{(k+2)/\ell_1}{2-(k+2)/\ell_1}$, i.e., when $1 > \phi(r) > \dfrac{2\ell_1 - k - 2}{k+2}$. This means that for both T2 and T3 techniques with $\alpha_k \in (0,1)$ the coefficient of $EL(t_n)$ is smaller than the corresponding to the IT. On the other hand, for the T1 technique, this coefficient is smaller only if $r_n > 1$.

4 The 0-stability of the new techniques

In this section we study the 0-stability of AMk methods with the techniques T1 and T2 for the stepsize changing introduced in section 2. Taking into account that $f(t,y)$ satisfies a uniform Lipschitz condition in a neighborhood of the true solution and assuming that the stepsize ratios are bounded, it may be proved (e.g. [4,7]) that the stability of (13) is independent of the non-linear term and therefore the method (13) is stable if and only if there exist numbers $K, r^*, h^* > 0$ such that for any grid $\{t_n\} \subset [0,T]$ with $h_n = t_n - t_{n-1} \leq h^*$, and $r_n \leq r^*$, the inequality

$$\|\Omega_k(r_n)\Omega_k(r_{n-1})\dots\Omega_k(r_m)\| < K < \infty \tag{17}$$

holds for all $m \leq n$

Since the propagation matrix $\Omega_k(r)$ depends only on the stepsize ratio and on function $\phi(r)$, following the ideas of [2] we introduce

Definition 1.

- A sequence of real positive numbers $\{r_j\}$ is called a stable sequence of stepsize ratios if inequality (17) holds for all $m \leq n$ with a positive constant K independent of n and m.
- A set of real positive numbers J_k is called a stability set for (13) if every sequence $\{r_j\}$ with $r_j \in J_k$ is a stable sequence.

To simplify the condition (17), let us remark that the propagation matrices $\Omega(r_n)$ have the special form

$$\Omega_k(r_n) = \left(\begin{array}{c|c|c} 1 & b_n & \overline{b}_n^T \\ \hline 0 & 0 & 0 \\ \hline 0 & \overline{a}_n & \overline{\Omega}_k(r_n) \end{array}\right)$$

where $b_n \in \mathbf{R}$, $\overline{a}_n, \overline{b}_n \in \mathbf{R}^k$, $\overline{\Omega}_k(r_n) \in \mathbf{R}^{k\times k}$. Consequently, any product of propagation matrices $\Omega_k(r_n)\dots\Omega_k(r_m)$ can be written as

$$\Omega_k(r_n)\dots\Omega_k(r_m) = \left(\begin{array}{c|c|c} 1 & b_{n,m} & \overline{b}_{n,m}^T \\ \hline 0 & 0 & 0 \\ \hline 0 & \overline{a}_{n,m} & \overline{\Omega}_{n,m} \end{array}\right)$$

with

$$\begin{aligned} \overline{\Omega}_{n,m} &= \overline{\Omega}_k(r_n)\overline{\Omega}_k(r_{n-1})\dots\overline{\Omega}_k(r_m), & \overline{a}_{n,m} &= \overline{\Omega}_{n,m+1}\overline{a}_m \\ b_{n,m} &= b_m + \sum_{i=m+1}^{n} \overline{b}_i^T \overline{\Omega}_{i-1,m}\overline{a}_m, & \overline{b}_{n,m}^T &= \overline{b}_m^T + \sum_{i=m+1}^{n} \overline{b}_i^T \overline{\Omega}_{i-1,m} \end{aligned}$$

and $\overline{\Omega}_{i,j} = I$ si $i < j$.

We are now in position to state:

Theorem 1. *The method (13)–(14) is stable if and only if there exist positive constants K_1, K_2 such that the inequalities*

$$\begin{aligned} &\|\overline{\Omega}_{n,m}\| = \|\overline{\Omega}_k(r_n)\overline{\Omega}_k(r_{n-1})\dots\overline{\Omega}_k(r_m)\| \leq K_1 \\ &\|\sum_{i=m+1}^{n} \overline{b}_i^T \overline{\Omega}_{i-1,m}\| \leq K_2. \end{aligned}$$

for all $n \geq m \geq k-1$.

One important stability result on interpolatory AMk methods given by Gear and Tu [6] (see also [13]) is that the stability can be ascertained if the stepsize remains fixed for al least k steps before a stepsize change. Such a property was generalized by Calvo, Lisbona and Montijano [3], showing that even with a finite number of stepsize changes between k consecutive equal stepsizes, the AMk method remains stable. Using the same reasoning as in [3], it can be proved the following properties can be proved:

Theorem 2.

i) If there is a positive integer p, such that in any set of $p+2(k+1)$ consecutive steps there at least $(k+1)$ consecutive steps with constant size, then the AMk method is stable.

ii) If there is a norm $\|.\|$ such that $\|\overline{\Omega}_k(r)\| \le K_1 < 1$ for all $r \le 1$, then the AMk method remains stable with the following strategy of stepsize variation: Arbitrary decreases of stepsizes are allowed while the size of a step can be increased only after $k+1$ consecutive steps with the same length.

Next, we turn our attention to the choice of the α-parameter in the new techniques. We consider first the two steps AM method. In this case, after some elementary calculations it is found that $\overline{\Omega}_2(r)$ is a 2×2 matrix given by

$$\overline{\Omega}_2(r) = r^2 \begin{pmatrix} 1-\frac{3}{2}\bar{r} & (3-\frac{9}{4}\bar{r})r \\ -\frac{1}{3}\bar{r}^2 & (1-\frac{1}{2}\bar{r}^2)r \end{pmatrix}$$

where $\bar{r} = 1/\phi(r)$. By using Schur's criteria, it follows that the spectral radius $\rho(\overline{\Omega}_2(r)) < 1$ if and only if the following inequalities hold

$$\begin{aligned} &r^5|(\bar{r}-1)(\bar{r}-2)| < 2 \\ &r^2|(2-3\bar{r})+r(2-\bar{r}^2)| < |2+r^5(\bar{r}-1)(\bar{r}-2)|. \end{aligned} \tag{18}$$

Thus, starting with the technique T1, for each value $\alpha = \alpha_2$ we may found the set of all $r \ge 0$ such that $\rho(\overline{\Omega}_2(r)) < 1$ which is given by (18). This set has the form $[0, \varphi_1(\alpha))$. Figure 1 shows the function $\varphi_1(\alpha)$, and a remarkable fact is that it attains a maximum for $\alpha = \alpha_2^* = 0.7677$. In view of this, a reasonable choice for the AM2 method with the T1 technique would be $\alpha_2 = 0.7677$. Note that in the interpolation technique $\alpha_2 = 1$ and then the spectral radius $\rho(\overline{\Omega}_2(r)) < 1$ only for $r \in [0, 1.695)$ while for the new technique, the same condition is satisfied for $r \in [0, 1.803)$. A similar study can be carried out for the remaining techniques as well as for $k > 3$. In Table 2 the optimal values of the α-parameters for the T1 and T3 techniques and $k \le 7$ are given. Moreover, Table 2 includes the greatest value of r_k such that $\rho(\overline{\Omega}_k(r)) < 1$ for $r \in [0, r_k)$ for the T1, T3 and IT techniques respectively. In all cases, it is clear that the stability of the new techniques is better than that of the IT.

Table 2

	T1		T3		interp.
k	α_k^*	r_k	α_k^*	r_k	r_k
2	0.7677	1.803	0.8987	1.803	1.695
3	0.7374	1.491	0.9161	1.489	1.439
4	0.7172	1.321	0.9322	1.321	1.297
5	0.7272	1.251	0.9524	1.250	1.233
6	0.7373	1.196	0.9685	1.194	1.187
7	0.8989	1.162	0.9846	1.163	—

Although the sets $I_k = \{r \mid \rho(\overline{\Omega}_k(r)) < 1\}$, see Figures 2-7, are good guides to select the optimal values of the parameters, it is clear that I_k is not in general a stability set (see e.g. Th. 4.1 [3]). One approach to get stability sets consist in finding a suitable matrix H such that the set M_k defined by

$$M_k = \{r \in R^+ \ ; \ ||H^{-1}\overline{\Omega}_k(r)H||_1 < 1\} \subset I_k \tag{19}$$

be as large as possible. Moreover, it is convenient for practical applications for M_k to have the form $[0, d_k)$ with $d_k > 1$. It has been proved in [3], that all closed set of M_k is a stability set for the AMk method.

Finally, we briefly outline the choice of H for the new techniques and the corresponding stability sets. The approach is purely numerical and it follows essentially the same ideas used in [3]. For a value $r \in I_k$ such that $\overline{\Omega}_k(r)$ possesses a complete set of eigenvectors, we take these vectors as columns of $H(r)$. Then, we get the corresponding set $M_k(H(r))$. By a numerical search we obtain an optimal $r_k \in I_k$. In Table 3 the optimal values of r_k for $k \leq 6$ are given. It must be pointed that for the T1 technique $M_k = I_k$, $k = 2, \ldots, 5$ and for the T3 technique $M_k = I_k$ only for $k = 2$ and $k = 4$. Therefore, optimal norms have obtained in these cases.

Table 3

	T1			T3			interp.	
k	r_k	$M_k(r_k)$	α_k	r_k	$M_k(r_k)$	α_k	r_k	$M_k(r_k)$
2	1.803	$[0, 1.803]$	0.8122	1.803	$[0, 1.803]$	0.9184	1.695	$[0, 1.695]$
3	1.462	$[0, 1.491]$	0.8000	1.422	$[0, 1.461]$	0.9898	1.419	$[0, 1.437]$
4	1.315	$[0, 1.321]$	0.8000	1.296	$[0, 1.321]$	1.0020	1.297	$[0, 1.297]$
5	1.221	$[0, 1.251]$	0.9000	1.094	$[0, 1.123]$	0.9796	0.905	$[0, 1.056]$
6	1.120	$[0, 1.172]$	0.7489	1.017	$[0, 1.096]$	1.0351	1.170	$[0, 0.257] \cup [.971, 1.181]$

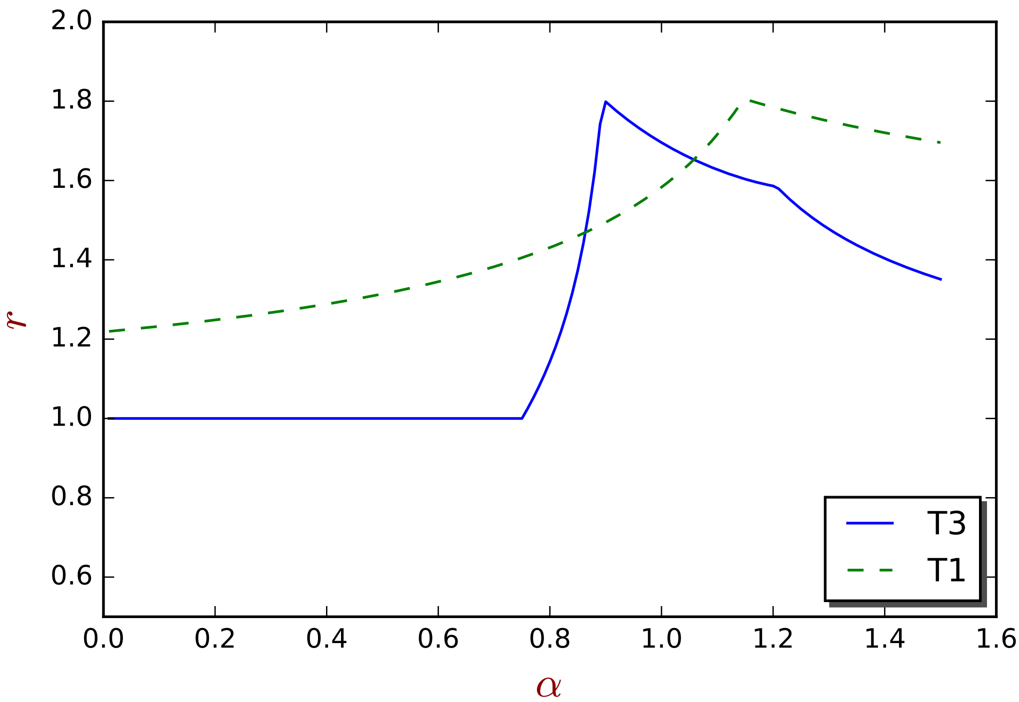

Fig. 1 Plot of the $\varphi_1(\alpha))$ for AM2 methods for T1 and T3 techniques

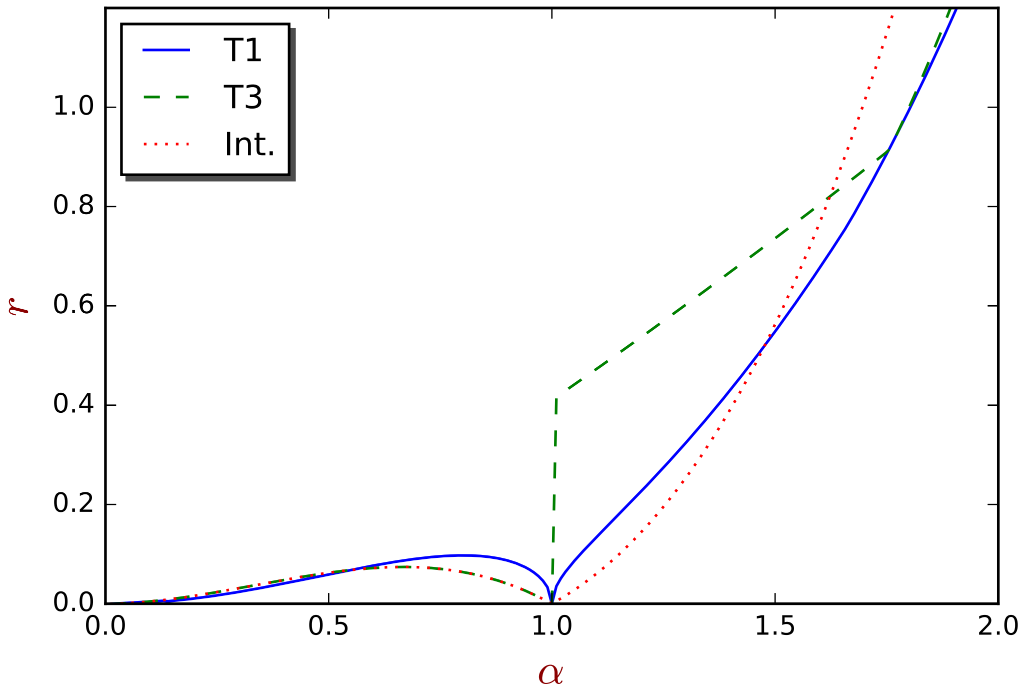

Fig. 2 Spectral radius of the AM2 methods for T1, T3 and interpolation techniques

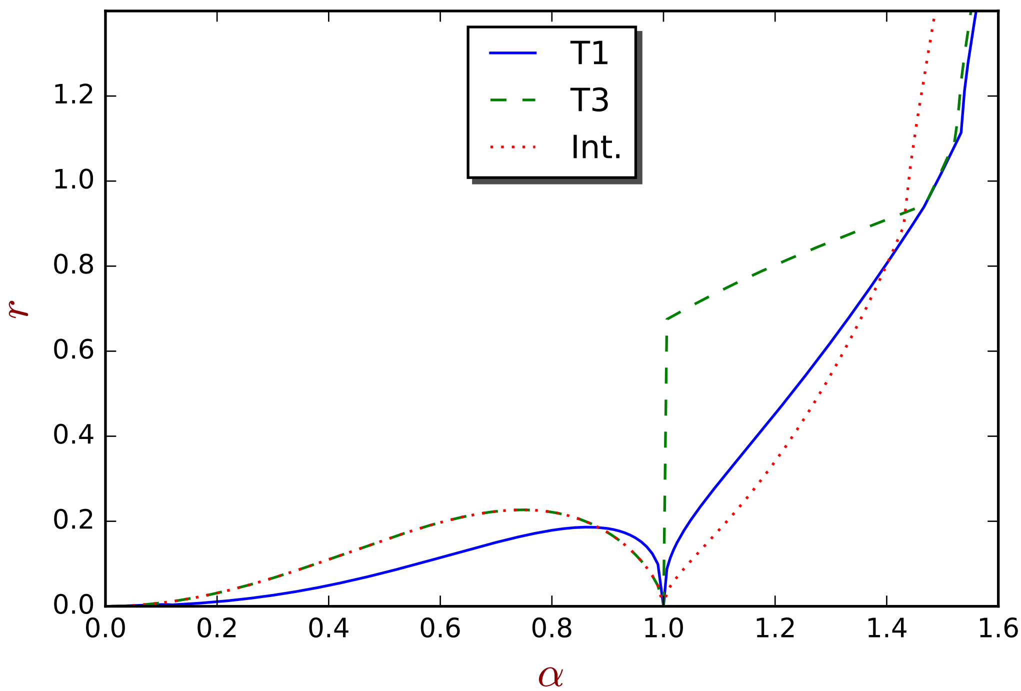

Fig. 3 Spectral radius of the AM3 methods for T1, T3 and interpolation techniques

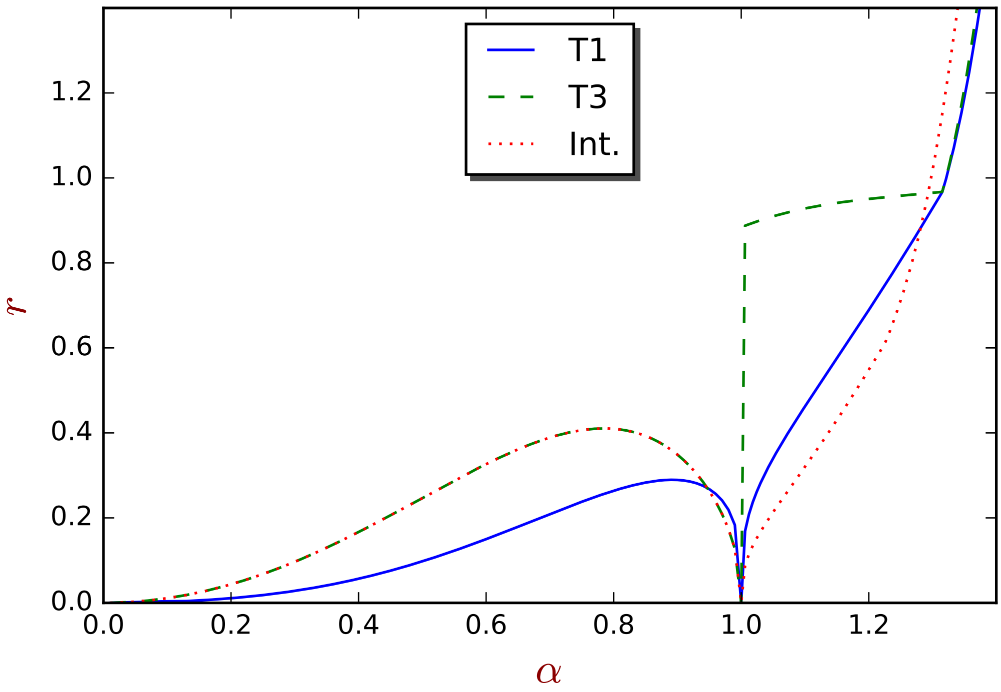

Fig. 4 Spectral radius of the AM4 methods for T1, T3 and interpolation techniques

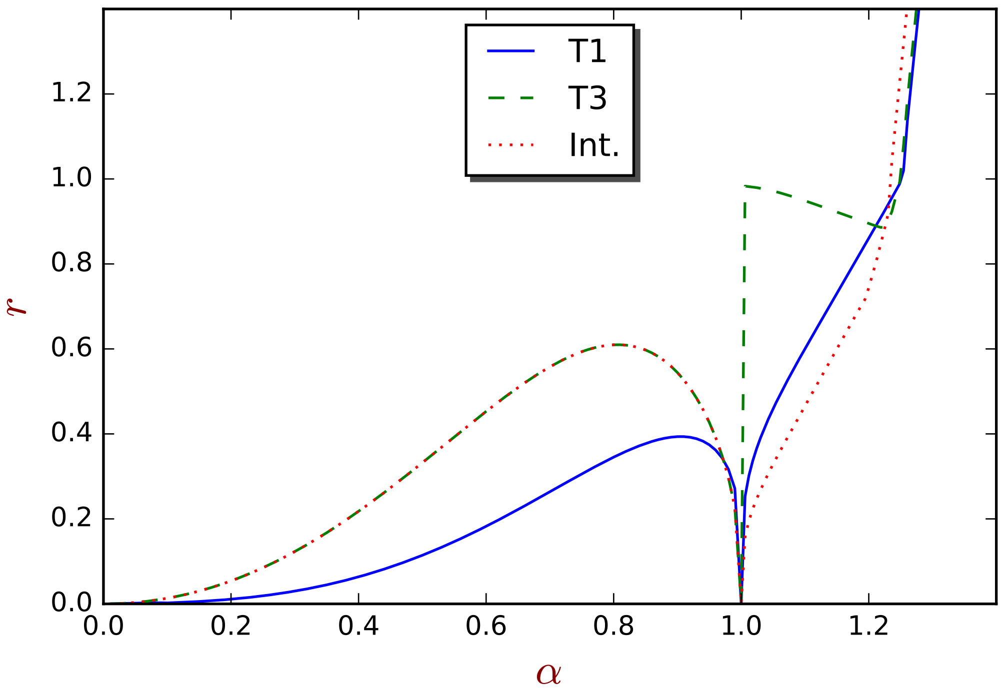

Fig. 5 Spectral radius of the AM5 methods for T1, T3 and interpolation techniques

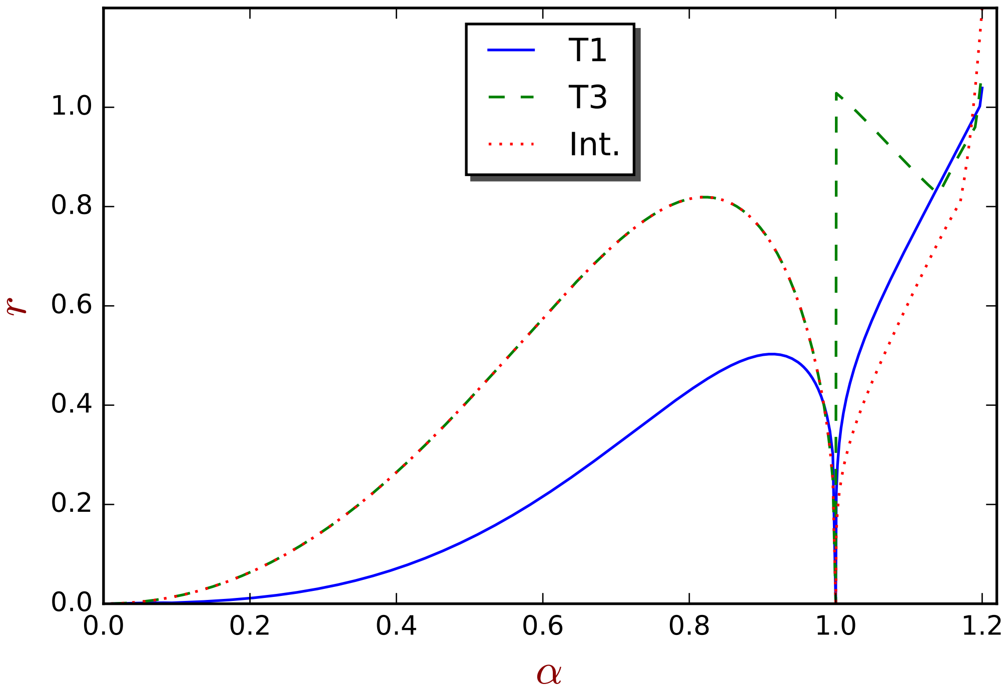

Fig. 6 Spectral radius of the AM6 methods for T1, T3 and interpolation techniques

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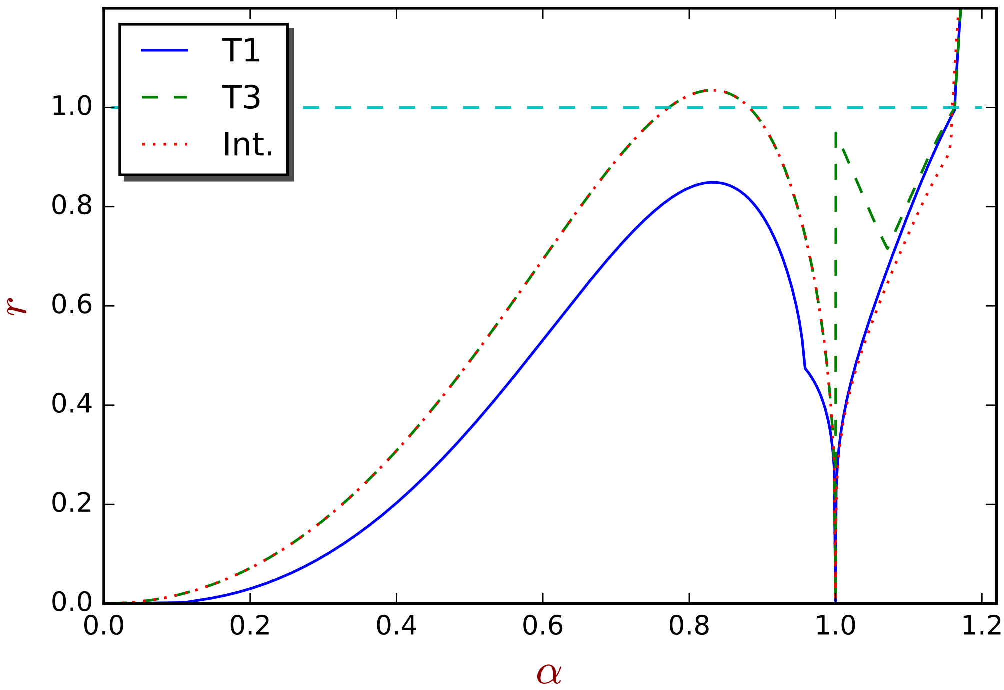

Fig. 7 Spectral radius of the AM7 methods for T1, T3 and interpolation techniques. Note that there exists an interval near 0.8 where the spectral radius of T1 and interpolation techniques is greater than one.

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