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Finite Element Analysis of Influence of Non-homogenous Temperature Field on Designed Lifetime of Spatial Structural Elements under Creep Conditions

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Abstract

The techniques of modeling of continual fracture process for spatial bodies under long-term static force loading condition in non-homogenous temperature field are presented. The scalar damage parameter is used to describe the material continual fracture process. A stress-strain problem solution made with semianalytic finite element method (SFEM). Results of lifetime determination of responsible parts are presented.

Keywords: long-term loading condition, creep, damage, continual fracture, lifetime, spatial problem, semianalytic finite element method (SFEM).
AMS 2010 codes: 74C10, 74R99, 74S05, 74D10.

1 Introduction

Structural elements of responsible objects function often under long-term static or cyclic force loading. The process of creep or fatigue, accompanied by the gradual accumulation of scattered damage, the formation and growth of macroscopic defects (fracture zones) are occurs under such a loading conditions. This problem, similarly as well as other aspects of reliability analysis [1, 2, 5], is very important for a reliable determination of long-term strength and lifetime.

A description of above mentioned processes, which took the name "continual fracture", may be fulfilled efficiently using phenomenological scalar damage parameter, proposed in the works of V. Bolotin, L. Kachanov and Yu. Rabotnov [7, 13]. This approach is developed and implemented for different loading conditions in the publications of Ukrainian scientists M. Bobyr, V. Golub, G. L'vov, Yu. Shevchenko [5, 6, 9, 14] and foreign

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ones (Chen G., Hayhurst D., Lemaitre J., Murakami S., Otevrel I., etc.), particularly in [10–12, 15, 16]. It is shown that the damage accumulation process should be taking into account for correct final definition of part's operating time. However, finite element solution of this problem requires creation of effective finite element base and special algorithms for regarding of temperature deformation and damage accumulation process.

The purposes of this paper are:

1. To highlight the effective finite elements for decision of two-dimensional and spatial stress-strained state problem and the developed technique for modeling of continual fracture process under creep loading condition regarding non-homogenous temperature field.

2. To present the results of lifetime determination of responsible structural elements under creep loading condition and analysis of non-homogenous temperature field influence on a value of designed lifetime of responsible structural elements.

2 Initial Equations and Methods of Analysis

2.1 Continuum Fracture Mechanics Relations under Creep Conditions

The damage accumulation process is described via kinetic equations using phenomenological damage parameter (DP) ω, which changes in time from $\omega(t=0)=\omega_0=0$ to $\omega(t^*)=1$, where t^* is the time of the local loss of material bearing capacity.

In general, the expression for determination of creep strains rate intensity considering continual fracture is given by:

$$\xi_i^c = \frac{d\varepsilon_i^c}{dt} = \xi_i^c(\sigma_i, \vartheta_c, \omega, T), \tag{1}$$

where $\sigma_i = \sqrt{3s_{ij}s^{ij}/2}$ is the normal stresses intensity, $\vartheta = \int_{\varepsilon_{ij}^c} \sqrt{\frac{2}{3}\, d\varepsilon_{ij}^c\, d\varepsilon^{ij\,c}}$ is Odqvist's creep factor, ε_i^c is creep strains intensity, t denotes time, and $\omega \in [0,1]$ is the Kachanov-Rabotnov damage parameter.

If Rabotnov theory is used, then Eq. (1) takes the form:

$$\xi_i^c = \frac{B\sigma_i^n}{(1-\omega)^r}, \tag{2}$$

where B, n, r are the material constants, depend on temperature and are obtained from basic creep tests.

Relation among the components of tensor of creep strain rate ξ_{ij}^c and stresses are given as:

$$\xi_{ij}^c = \frac{d\varepsilon_{ij}^c}{dt} = \frac{3}{2}\xi_i^c \frac{s_{ij}}{\sigma_i}. \tag{3}$$

A DP value description under long-term static loading condition (under a creep process presence) can be conducted using the follow general expression [9]:

$$\frac{d\omega}{dt} = C\left[\frac{\sigma_e}{1-\omega^r}\right]^m \frac{1}{(1-\omega)^q}\omega^\beta, \tag{4}$$

where C, m, q, r, β are experimentally determined material constants, which are functions of temperature and σ_e are the equivalent stresses calculated according to the chosen strength criterion.

2.2 Finite element library

Finite elements (FE), which are used for discretization of two-dimentional problems of axially symmetrical and plane strain bodies, appear as arbitrarily-shaped quadrilaterals, see Fig. 1, a).

Each FE has a corresponding local coordinate system x^i, in such a manner that axes x^1 and x^2 are directed along the sides of the cross section of FE. In addition, in local coordinate system, the cross section of FE is mapped as a square with a unit side. Local coordinate system is used for determination of strains and stresses across FE.

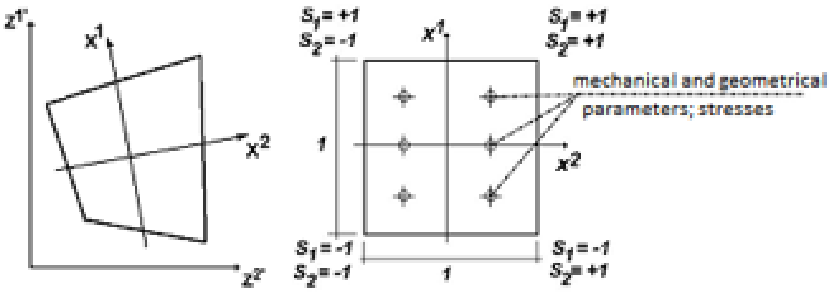

Fig. 1 Two-dimentional finite element in general (left) and local coordinate system.

No restrictions are required on the distribution pattern of mechanical and geometrical parameters across the area of cross section of FE and they are determined at several integration points (Fig. 1, b).

Unknowns are taken as components of nodal displacements of FE in basic coordinate system $u_{k'}$, where k' is a direction in the basic coordinate system.

$$u_{m'} = \sum_{S_1=\pm 1} \sum_{S_2=\pm 1} u_{m'(S_1 S_2)} \left(\frac{1}{2} S_1 x^1 + \frac{1}{2} S_2 x^2 + S_1 S_2 x^1 x^2 + \frac{1}{4} \right), \tag{5}$$

where $u_{m'(S_1 S_2)}$ are the values of the nodal displacements, which are given as components in the basic coordinate system, S_1 and S_2 are coordinates that determine the location of the nodes with respect to the center of the cross section of the element in the local coordinate system x^i.

Implementation of moment scheme of finite elements (MSFE) [3] allows to significantly increase efficiency of numerical analysis of complex spatial structures based on FEM. Besides that, MSFE eliminate strains during rigid-body displacement, as well eliminates the effect of “false shear”, which occurs during analysis of thin-walled structures using solid FE. Thus, the deformation values appearead in view of McLorens row:

$$\begin{aligned} \varepsilon_{\alpha\alpha} &= \mathring{\varepsilon}_{\alpha\alpha} + \mathring{\varepsilon}_{\alpha\alpha,3-\alpha} x^{3-\alpha} \\ \varepsilon_{12} &= \mathring{\varepsilon}_{12} \\ \varepsilon_{33} &= \mathring{\varepsilon}_{33} + \mathring{\varepsilon}_{33,\alpha} x^{\alpha}, \end{aligned} \tag{6}$$

where $\mathring{\varepsilon} = \varepsilon_{ij}\Big|_{x^\alpha=0}$, and $\mathring{\varepsilon}_{ij,\alpha} = \dfrac{\partial \varepsilon_{ij}}{\partial x^\alpha}\Big|_{x^\alpha=0}$.

The solution of evolutionary stress-strained state problems regarding nonlinear creep deformation and damage accumulation process of spatial bodies requires significant computational expenses. It is not always possible to solve these problems using traditional three-dimensional finite element problem definition.

Semianalytic Finite Element Method (SFEM) is an effective instrument for numerical modeling of stress-strain state and deformation process of canonical form spatial bodies-inhomogeneous circle and prismatic bodies. The term “inhomogeneous” is used in the sense of the variability of the physical, mechanical properties, and geometrical dimensions of the body along the forming. Being based on SFEM, a discrete calculation model suggests the finite element mesh in the cross section of the examined object, and one finite element (FE) to be used in the orthogonal towards the cross sectional plane (along the forming, i.e., $z^{3'}$ coordinates). Thus, the FE size in the $z^{3'}$ direction is the same as the body one and the cross section of FE is appeared like as it is shown in Fig. 2.

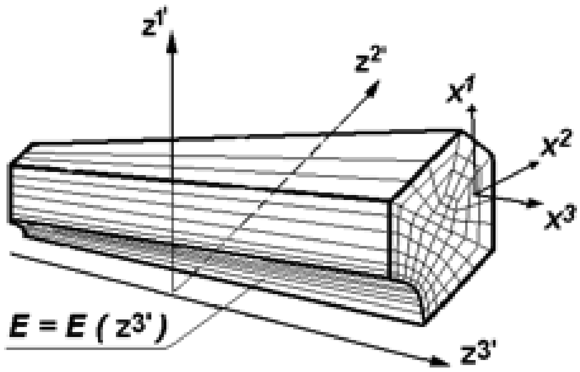

Fig. 2 SFEM discrete model of prismatic inhomogeneous body.

Derivation of unknown displacements in cross section of semianalytic finite element is given in Eq. (5). The unknown displacements values in the direction of generatrix (x^3) are approximated via decomposition by system of coordinate functions $\varphi^{(l)}$ by Lagrange ($l = 0,1$) and Mikhlin ($l = 2,\dots,L$) polynomials:

$$u_{m'} = \sum_{l=0}^{L} u_{m'}^{l} \varphi^{(l)}, \qquad u_{m',3} = \sum_{l=0}^{L} u_{m'}^{l} \varphi_{,3}^{(l)}, \qquad \text{where} \tag{7}$$

$$\varphi^{(0)} = \frac{1}{2}(1-x^3), \qquad \varphi^{(1)} = \frac{1}{2}(1+x^3).$$

$$\varphi^{(l)} = f^{(l)} p^{(l)} - f^{(l-2)} p^{(l-2)}, \qquad f^{(l)} = \sqrt{(4l^2-1)^{-1}}.$$

$$p^{(l)} = \sqrt{\frac{2l+1}{2}} \sum_{k=0}^{l} \frac{(-1)^k (l+k)!}{(l-k)!(k!)^2 2^{k+1}} \left[(1-x^3)^k + (-1)^l (1+x^3)^k\right].$$

Applied system of functions meets the conditions of completeness and linear independency, and enables simplified and effective formulation of various types of boundary conditions at buff-ends of the body by traditional FEM techniques, i.e., elimination of corresponding Eqs. (2).

Proposed finite elements are oriented on analysis of large class of prismatic bodies. They has to provide not only high accuracy of display of stress-strain state of complex structures, but also a high speed of convergence.

SFEM allows significantly reduce the computational expenses for solving of spatial problem, particularly on the stages of stiffness matrix calculating and FEM linear equations systems solving. The efficiency and accuracy of the method is shown for a wide range of linear and nonlinear problems of mechanics [3–5], where readers can also find a more detailed description of the method features, its implementation and links to additional author's publications.

3 Finite Element Algorithms for Continual Fracture Problem Solution

A problem of stress-strain state parameters determination under linear and nonlinear deformation process performed by the algorithm based on the use of the implicit integration over the time scheme with help of Newton-Kantorovich iterative procedure:

$$\{\Delta U_l\}_n^m = \{\Delta U_l\}_{n-1}^m + \beta\,[K_{ll}]^{-1}\left(\{Q_l\}^m - \{R_l\}_n^m\right), \tag{8}$$

where $1 \le \beta < 2$ is the relaxation parameter, $\{Q\}^m$ is the vector of full loads in nodes in step m, $[K_{ll}]$ is the FE stiffness matrix, and $\{R_l\}_n^m$ is the vector of nodes reactions in the iteration n of step m.

For isotropic material, temperature components of strain tensor components are obtained by Eq. 3:

$$\varepsilon_{ij}^T = \alpha_T \Delta T g_{ij}, \tag{9}$$

where $\alpha_T = \alpha_T\left(z^{k'}, T\right)$ is the linear expansion coefficient of the material, $\Delta T = T - T_0$ is an increase of temperature in a covered point of the body relative to the initial state $T = T_0$.

During step-by-step representation of deformation process expansion coefficient of stresses increments in the step m are determined by values of increments of total $\Delta\varepsilon_{ij}$, and temperature $(\Delta\varepsilon_{ij})^T$ strains:

$$(\Delta\sigma_{ij})_m = C_{ijkl}\left(\{\Delta\varepsilon_{kl}\}_m - \left\{(\Delta\varepsilon_{kl})^T\right\}_m\right). \tag{10}$$

Creep problem solution considering damage accumulation is being executed by means of step-by-step algorithm on the parameter of time. When starting each iteration n of a step m, stress values σ_{ij} are calculated considering creep deformation process by the formula:

$$(\sigma_{ij})_n^m = \frac{1}{3}\delta^{ij}(\sigma_{ij})_n^m + \left(s^{ij}\right)_n^m. \tag{11}$$

Components of stress tensor σ_{ij} are defined in compliance with the Hook's law considering an increment of total and temperature strains:

$$(\sigma_{ij})_n^m = (\sigma_{ij})_{n-1}^m + (\Delta\sigma_{ij})_n^m, \tag{12}$$

while the components of stress deviator $\left(s^{ij}\right)_n^m$ relate to an increment in creep deformation $\Delta\varepsilon_{ij}^c$:

$$\left(\Delta\varepsilon_{ij}^c\right)_n^m = \left(\xi_{ij}^c\right)_n^m \Delta t_m, \qquad \left(s^{ij}\right)_n^m = \left(s^{ij}\right)_n^m - G_1\left(\Delta\varepsilon_{ij}^c\right)_n^m, \tag{13}$$

where $\left(\xi_{ij}^c\right)_n^m = \dfrac{3}{2}[\xi_i^c]_m^n \dfrac{(s_{ij})_n^m}{(\sigma_i)_m^n}$ components of creep deformation rate tensor, $G_1 = E/(1-2\mu)$ are elastic constants, and Δt_m is the time interval value.

The DP values addition $(\Delta\omega)_m$ and accumulated DP values ω_m in a time interval m calculated with next relation:

$$\omega_m = \omega_{m-1} + (\Delta\omega)_m = \omega_{m-1} + \left(\frac{d\omega}{dt}\right)_m \Delta t_m. \tag{14}$$

The criterion of local loss of the material bearing capacity is $\omega(t^*) > \omega^*$, where $\omega^* \approx 1$ is the critical DP value. Its fulfillment in some point of studied object K with coordinates $\left(z^{i'}\right)_K = z^{i'*} = \left\{z^{1'*}, z^{2'*}, z^{3'*}\right\}$, indicates the transition of the scattered damages accumulation process, which accounted integrally using DP, to occurrence of macroscopic defects – initial areas of continual fracture. These points in time determining the value of the designed lifetime of studied object.

4 Results of Finite Element Modeling of Continual Fracture and Lifetime Definition

The allowed approaches have been applied to solving of practical problems regarding consideration of influence of non-homogenous temperature field on designed lifetime value of responsible structure element – the blade of a gas turbine under creep. Analysis has been carried out separately for root and blade; this is because of specifics of semianalytic finite element model-building.

Based on results of elastic deformation modeling of blade, based on three-dimensional FEM, a dangerous cross section R_0 was chosen. This section is characterized with combination of average stress σ_0 and average temperature T_0, which leads to the most intensive creep process and damage accumulation. Listed values (σ_0 and T_0) are used further to describe the design scheme and the results of the problem solving.

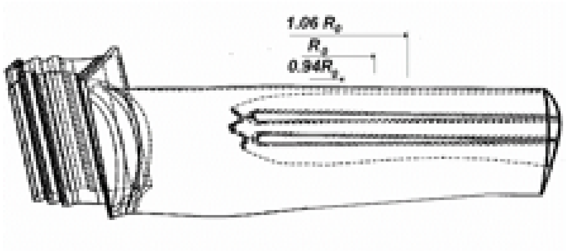

Fig. 3 Gas turbine blade.

The description of creep and damage accumulation processes is carried out with the following equation:

$$\frac{d\varepsilon_c}{dt} = \frac{B\sigma^n}{(1-\omega)^r}, \qquad \frac{d\omega}{dt} = C\left(\frac{\sigma}{1-\omega}\right)^m \frac{1}{(1-\omega)^q}, \tag{15}$$

where $B = B(T), C = C(T), m = m(T), n = n(T), r = r(T), q = q(\sigma, T)$ are the material constants, and T is the temperature.

The root of gas turbine blade (Fig. 4, left) is a prismatic body that carries heat and force loading. Interaction between blade an root has been modeled by nonuniformly distributed load q. Teeth of the root bear on corresponding slots in wheel rim, which are being deformed. Thus, boundary conditions in the form of elastic supports employed alongside of surfaces of connection between root's teeth and wheel rim.

Finite element model is obtained for one-half of the detail due to symmetry relative to vertical axis. Boundary conditions are obtained from symmetry conditions and constraint in radial (z^1) direction. Analysis of convergence of obtained results depending on number of unknowns in finite element model has been carried out using comparison of distribution of dimensionless stress intensity [10] for meshes with 1074, 3344, and 9596 unknowns. Inaccuracy of stresses determination under elastic deformation reaches around 2%. Thus, in the future analysis the mesh with 3344 unknowns (Fig. 4, right) was used further for creep process modeling.

In terms of analysis, evaluation of influence of non-homogenous temperature field on evolution of stress-strain state parameters and designed lifetime of root has been performed. Linear law is describing temperature change across the height of root's cross section. Maximum and minimum values of temperature lie in $\pm 0.5\%$ range from temperature in section that located at a distance of $0.9345R_0$ from rotation axis of wheel rim.

During analysis, the provision has been made that due to small size of the root throughout its height the influence of temperature strains on stress-strain state is insignificant. At the same time, dependence of constants in Eq. (15) on temperature leads to significant differences in the flow of creep process with consideration of non-homogenous distribution of temperature, which leads to change of designed lifetime.

For analysis of influence of temperature field, an examination of differences in distribution of dimensionless stress intensity (Fig. 5, left) and damage (Fig. 5, right) throughout the whole running time has to be performed. These values have been taken at the point, where maximum stress occurs with ($T \neq$ const) and without ($T =$ const) consideration of non-homogenous temperature field.

For such temperature distribution the value of designed lifetime is $t^*(T) = 0.91t^*$, where t^* is a lifetime value obtained under constant temperature in whole root volume. Thus, regard of non-homogenous temperature field allows adjusting of designed lifetime of blade root on 10% approximately.

Stationary gas turbine blade is a spatial body of complicated shape. The blade is swirled at the vertical axis, has a variable at height cross-sectional area. It is influenced by centrifugal forces in a heterogeneous, both in

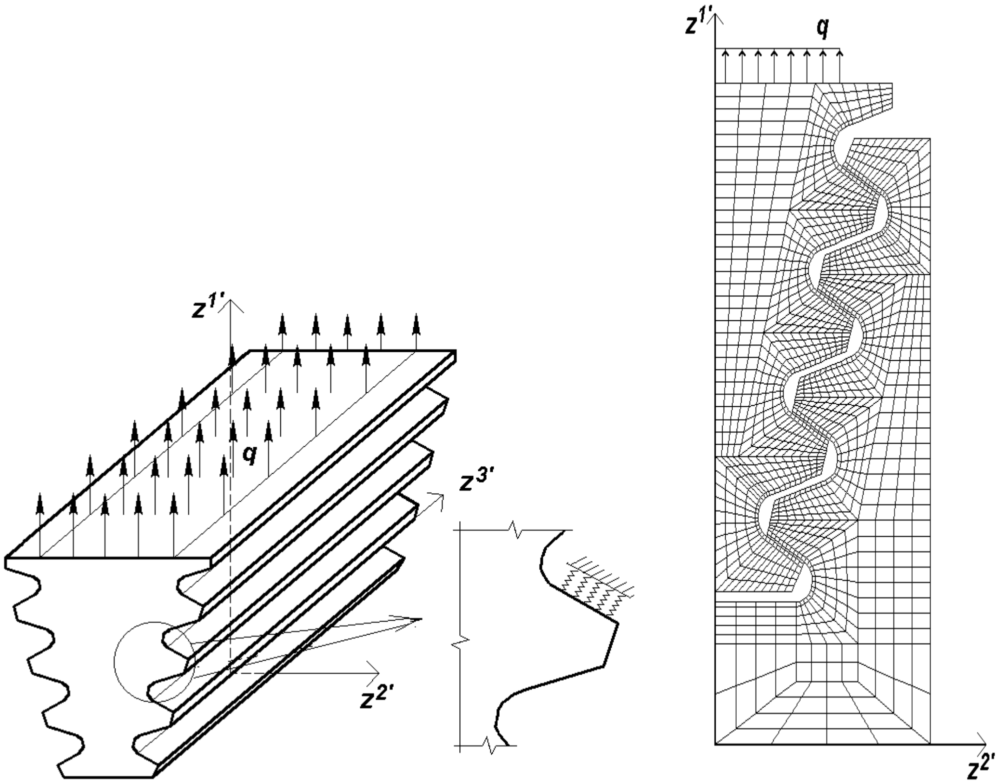

Fig. 4 Root of gas turbine blade: design scheme (left) and SFEM discrete model.

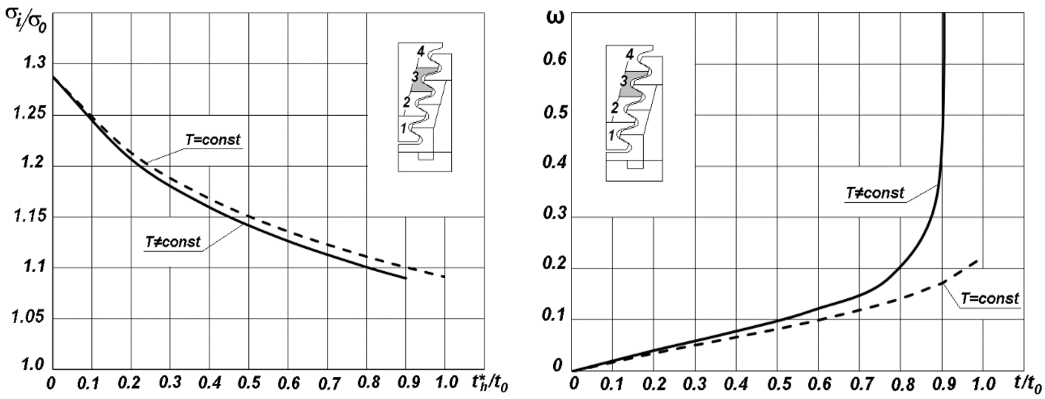

Fig. 5 Change in time of dimensionless stress intensity (left), and damage parameter.

height and in cross-section, temperature field (Fig. 6, left). Creep deformation process modeling is made for a blade fragment with size $0.94R_0 < R < 1.06R_0$.

Fragment is loaded with its own centrifugal load p. The simulation of the upper part of blade in section $R = 1.06R_0$ implemented with unevenly distributed load $q = q(z^{1'}, z^{2'})$ that meets stress values, been applied in this cross-section.

To account for the impact of blade swirling at the vertical axis has been applied a change of the original material density on a cross-section plane. For a central part of cross section (in the vicinity of the middle channel), for which swirling leads to increased stress, the output material density has been increased, while for peripheral parts of the section it was reduced. The averaged material density in whole cross section remained unchanged.

The distribution of dimensionless stress intensity along the height of blade fragment obtained based on this approach is identical with the one results obtained from the three-dimensional FEM solution under elastic deformation.

Temperature distribution in blade fragment and finite element model of gas turbine blade are shown at Fig. 6, right.

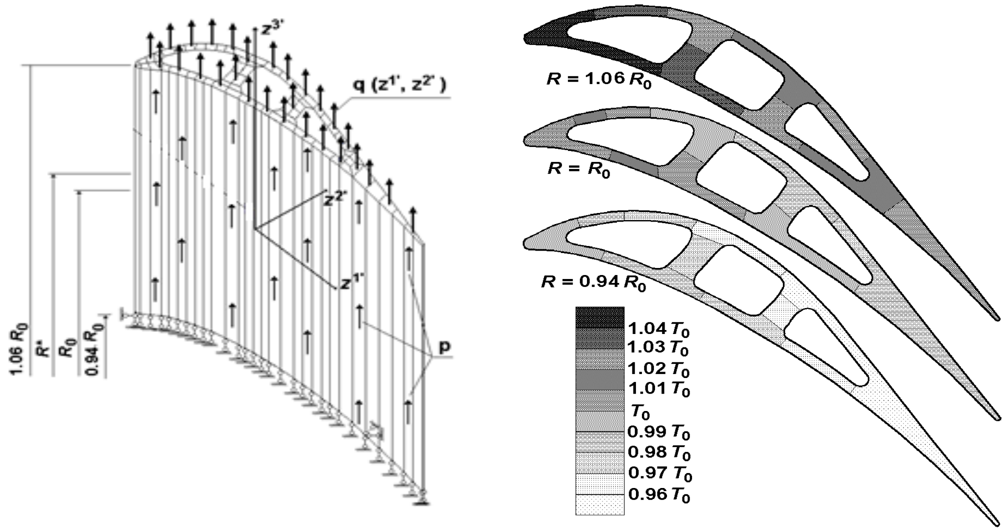

Fig. 6 Gas turbine blade: finite element model (left), and temperature distribution in blade fragment.

Solving of given problem using SFEM showed that consideration of non-homogenous distribution of temperature across cross section does not have a significant influence on stress-strain state and value of lifetime. Under non-homogenous distribution of temperature along the height of the blade, the maximum difference in stresses intensity values (around 10%) is observed at vicinity of point 2, where damage value is insignificant. Changing of stresses across the height of the blade is illustrated in Fig. 7, left. But obtained time-dependent damage parameter values at point 1 show (Fig. 7, right), that even such small reduction of stresses leads to adjustment of main designed lifetime of the blade up to 9% ($t^*(T) = 1.09t^*$).

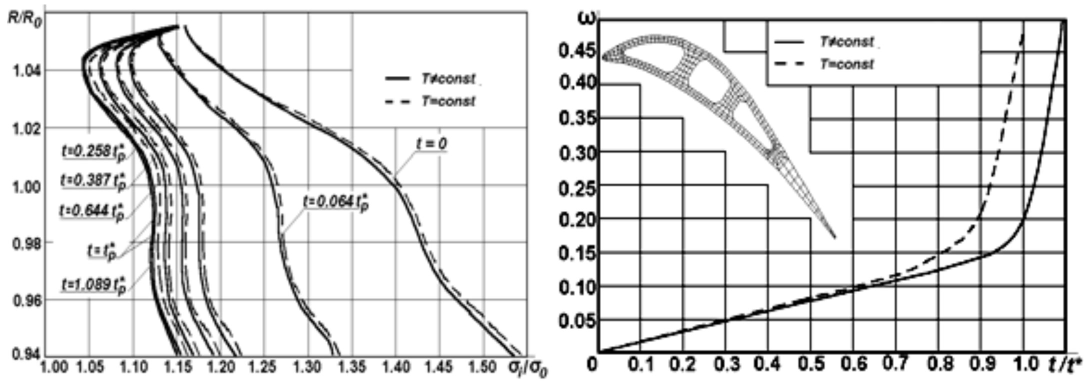

Fig. 7 Changing of stresses across the height of the blade (left), time-dependent damage parameter values.

5 Conclusions

Developed in this paper methods for modeling of continual fracture process allows to determine of designed lifetime values for responsible structural elements that work under long-term static loading condition. Consideration of non-homogenous temperature distribution in studied object allows to adjust the value of designed lifetime in the range of 10% approximately.

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