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AN ADAPTIVE OUTPUT FEEDBACK MOTION TRACKING CONTROLLER FOR ROBOT MANIPULATORS: UNIFORM GLOBAL ASYMPTOTIC STABILITY AND EXPERIMENTATION

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This paper deals with two important practical problems in motion control of robot manipulators: the measurement of joint velocities, which often results in noisy signals, and the uncertainty of parameters of the dynamic model. Adaptive output feedback controllers have been proposed in the literature in order to deal with these problems. In this paper, we prove for the first time that Uniform Global Asymptotic Stability (UGAS) can be obtained from an adaptive output feedback tracking controller, if the reference trajectory is selected in such a way that the regression matrix is persistently exciting. The new scheme has been experimentally implemented with the aim of confirming the theoretical results.

Keywords: adaptive control, robot control, nonlinear control, output feedback, Lyapunov UGAS stability.

1. Introduction

Measurement of joint velocities in a robot manipulator through tachometers may produce noisy signals so that its use in a controller may not be feasible (Daly and Schwarz, 2006). Output feedback controllers deal with this problem since they only require position measurements; therefore, joint velocities are substituted by signals produced by an observer or filter.

Several output feedback controllers have been proposed for the regulation case, obtaining global asymptotic stability results. Output feedback controllers with gravity compensation were proposed by Berghuis and Nijmeijer (1993), Burkov (1993) as well as Kelly (1993), while Arimoto *et al.* (1994) put forward an output feedback controller with desired gravity compensation. An output feedback controller which compensates gravity uncertainty effects was proposed by Ortega *et al.* (1995); however, the asymptotic stability result is local. An adaptive output-feedback controller with bounded inputs was put forward by Lopez-Araujo *et al.* (2012), achieving global convergence of position errors to zero.

Results obtained for output feedback controllers in the tracking case are, for the most part, local. Some output feedback tracking controllers proposed in the literature are those by Lim *et al.* (1996) or Nicosia and Tomei (1990). An output feedback controller with bounded inputs is proposed by Loria and Nijmeijer (1998), for which global asymptotic stability is obtained by Santibanez and Kelly (2001) in the presence of viscous friction and a proper bound of the desired joint speed. A proposed solution using a variable structure observer is reported by Abdessameud and Khelfi (2006). Other variations of this controller are proposed by Moreno-Valenzuela *et al.* (2008a; 2008b), who prove local asymptotic stability via singular perturbations theory. A generalization of the controller proposed by Santibanez and Kelly (2001) is designed by Zavala-Rio *et al.* (2011).

Uncertainty in robot parameters is another practical problem in robot manipulator control. Adaptive controllers can be used when some of the parameters of the robot dynamic model are unknown. In adaptive controllers, an estimate of the model parameters is computed through an update law (see, e.g., Witkowska and Śmierzchalski, 2012; Bańka *et al*, 2013).

Craig *et al.* (1987) proposed the first adaptive controller with a rigorous stability proof; however, the controller required knowledge of bounds on the robot parameters and measurement of joint accelerations. Other adaptive controllers were reported by Slotine and Li (1987), Sadegh and Horowitz (1987), Middleton and Goodwin (1988), as well as Kelly *et al.* (1989). An excellent tutorial is presented by Ortega and Spong (1989). An adaptive redesign of the PD with feedforward compensation is reported by Santibanez and Kelly (1999).

The only known proof of uniform global asymptotic stability for a full state feedback adaptive controller for the tracking case is presented by Loria *et al.* (2005).

As for adaptive output feedback controllers, only global convergence of tracking errors to zero has been reported. The first output feedback tracking controller was proposed by Zhang *et al.* (2000). A redesign of this controller is presented by Zergeroglu *et al.* (2000), which eliminates the need for a post-analysis transformation by considering only position measurements. An adaptive version of the output feedback controller reported by Loria and Nijmeijer (1998) was presented by Moreno-Valenzuela *et al.* (2010); global convergence is proved in the case of viscous friction large enough, while local exponential stability is proven when viscous friction is not large enough.

As far as the authors are aware, no proof of uniform global asymptotical stability has been presented for an adaptive output feedback tracking controller. So far in this paper, we prove for the first time that, for viscous friction large enough and if the reference trajectories are selected in such a way that the regression matrix is persistently exciting, uniform global asymptotic stability is achieved for the controller proposed by Moreno-Valenzuela *et al.* (2010). This paper extends the results presented in our earlier work (Yarza *et al.*, 2011), in the sense that experimental results are included, as well as further details about the conditions required for UGAS theoretical analysis.

The paper is structured as follows. Section 2 presents some preliminaries, including the robot dynamic model and its properties, the control objective and an important theorem on UGAS for a type of nonlinear system. Section 3 presents the main result of the paper, proving UGAS for an adaptive output feedback tracking controller, Section 4 presents experimental results and Section 5 concludes the paper.

Throughout this paper, we use the notation $\lambda_{\min}\{A(\boldsymbol{x})\}$ and $\lambda_{\max}\{A(\boldsymbol{x})\}$, to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in \mathbb{R}^n$. Also, we define $\lambda_{\min}\{A\}$ as the greatest lower bound (infimum) of $\lambda_{\min}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$. Similarly, we define $\lambda_{\max}\{A\}$ as the least upper bound (supremum) of $\lambda_{\max}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ and that of a matrix $A(\boldsymbol{x})$ is defined as the corresponding induced norm $\|A(\boldsymbol{x})\| = \sqrt{\lambda_{\max}\{A(\boldsymbol{x})^T A(\boldsymbol{x})\}}$. We denote by $\mathbb{R}_+$ the space of nonnegative real numbers. We denote by $\mathrm{col}[\boldsymbol{x}_1, \boldsymbol{x}_2]$ the vector $\begin{bmatrix}\boldsymbol{x}_1^T & \boldsymbol{x}_2^T\end{bmatrix}^T$.

2. Preliminaries

2.1. Robot dynamics. The dynamics of an n-link serial rigid robot manipulator, considering viscous friction, can be expressed as (Spong *et al.*, 2005)

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + F_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}, \tag{1}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint positions, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\ddot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint accelerations, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centrifugal and Coriolis torques, F_v is the $n \times n$ diagonal positive definite matrix of viscous friction coefficients, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied torques, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques, obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$, i.e.,

$$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}}. \tag{2}$$

We assume that the links are joined together with revolute joints. This assumption is instrumental in Properties 2–5.

2.2. Control objective. Assume that only the robot joint position vector $\boldsymbol{q}(t) \in \mathbb{R}^n$ is available for measurement and some of the robot parameters are unknown. Then, the adaptive output feedback tracking control problem consists in designing a control law to compute the employed torques vector $\boldsymbol{\tau} \in \mathbb{R}^n$ together with a parameter estimation update law so that the limit

$$\lim_{t\to\infty} \widetilde{\boldsymbol{q}}(t) = \boldsymbol{0} \tag{3}$$

is satisfied, where

$$\widetilde{\boldsymbol{q}}(t) = \boldsymbol{q}_d(t) - \boldsymbol{q}(t) \tag{4}$$

is the tracking error and $\boldsymbol{q}_d(t) \in \mathbb{R}^n$ is the desired joint position trajectories vector.

We assume that the desired time-varying trajectory $\boldsymbol{q}_d(t)$ is three times differentiable and bounded for all $t \geq 0$ in the sense that

$$\|\dot{\boldsymbol{q}}_d(t)\| \leq \mu_1, \tag{5}$$

$$\|\ddot{\boldsymbol{q}}_d(t)\| \leq \mu_2, \tag{6}$$

$$\|\dddot{\boldsymbol{q}}_d(t)\| \leq \mu_3, \tag{7}$$

where μ_1, μ_2 and μ_3 are known positive constants.

2.3. Properties of the dynamic model. Some important properties of the robot dynamics (1) include the following (Kelly *et al.*, 2005; Spong *et al.*, 2005)

Property 1. By using Christoffel's symbols, the matrix $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and the time derivative $\dot{M}(\boldsymbol{q})$ of the inertia matrix satisfy (Koditschek, 1984; Spong *et al.*, 2005)

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}})\right] \dot{\boldsymbol{q}} = 0, \quad \forall \boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n$$

and

$$\dot{M}(\boldsymbol{q}) = C(\boldsymbol{q}, \dot{\boldsymbol{q}}) + C(\boldsymbol{q}, \dot{\boldsymbol{q}})^T, \quad \forall \boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n.$$

Property 2. There exists a positive constant k_c such that for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$

$$\|C(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{z}\| \leq k_c \|\boldsymbol{y}\| \|\boldsymbol{z}\|.$$

Property 3. The gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ is bounded for all $\boldsymbol{q} \in \mathbb{R}^n$ (Craig *et al.*, 1987). This means that there exist constants $\gamma_i \geq 0$ such that

$$|g_i(\boldsymbol{q})| \leq \gamma_i, \quad i = 1, 2, \ldots, n,$$

for all $\boldsymbol{q} \in \mathbb{R}^n$, where $g_i(\boldsymbol{q})$ stands for the i-th element of vector $\boldsymbol{g}(\boldsymbol{q})$. Equivalently, there exists a positive constant k_1 such that

$$\|\boldsymbol{g}(\boldsymbol{q})\| \leq k_1, \quad \forall \boldsymbol{q} \in \mathbb{R}^n.$$

Property 4. There exists a positive constant k_g such that

$$\|\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{y})\| \leq k_g \|\boldsymbol{x} - \boldsymbol{y}\|$$

for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$.

Property 5. The so-called residual dynamics are defined by (Arimoto, 1995a;1995b; Kelly *et al.*, 2005)

$$\begin{aligned}\boldsymbol{h}(\widetilde{\boldsymbol{q}}, \dot{\widetilde{\boldsymbol{q}}}) &= [M(\boldsymbol{q}_d) - M(\boldsymbol{q}_d - \widetilde{\boldsymbol{q}})]\ddot{\boldsymbol{q}}_d \\ &\quad + [C(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d) - C(\boldsymbol{q}_d - \widetilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}_d - \dot{\widetilde{\boldsymbol{q}}})]\dot{\boldsymbol{q}}_d \\ &\quad + \boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q}_d - \widetilde{\boldsymbol{q}}).\end{aligned}$$

The residual dynamics satisfy the inequality

$$\|\boldsymbol{h}(\widetilde{\boldsymbol{q}}, \dot{\widetilde{\boldsymbol{q}}})\| \leq k_{c1}\mu_1 \|\dot{\widetilde{\boldsymbol{q}}}\| + \frac{s_2 s_1}{\tanh(s_2\sigma)} \|\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}})\|, \quad (8)$$

where $\sigma > 0$, the constant μ_1 in (5), and

$$s_1 = k_g + k_M\mu_2 + k_{c2}\mu_1^2, \quad (9)$$

$$s_2 = 2\frac{k_1 + k_2\mu_2 + k_{c1}\mu_1^2}{s_1}, \quad (10)$$

where

$$k_M \geq n^2 \left[\max_{i,j,k,\boldsymbol{q}} \left\|\frac{\partial M_{ij}(\boldsymbol{q})}{\partial q_k}\right\|\right], \quad (11)$$

$$k_{c2} \geq n^3 \left[\max_{i,j,k,\boldsymbol{q}} \left\|\frac{\partial c_{ijk}(\boldsymbol{q})}{\partial q_k}\right\|\right], \quad (12)$$

$$k_1 \geq \sup_{\boldsymbol{q} \in \mathbb{R}^n} \|\boldsymbol{g}(\boldsymbol{q})\|, \quad (13)$$

$$k_2 \geq \lambda_{\max}\{M(\boldsymbol{q})\}, \quad (14)$$

for all $\boldsymbol{q} \in \mathbb{R}^n$, where $M_{ij}(\boldsymbol{q})$ is the ij-element of matrix $M(\boldsymbol{q})$ and $c_{ijk}(\boldsymbol{q})$ is the ijk Christoffel symbol (Kelly *et al.*, 2005).

Property 6. The robot model (1) can be linearly parameterized as

$$\begin{aligned}&M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + F_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \\ &\quad = Y(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\theta} + M_0(\boldsymbol{q})\ddot{\boldsymbol{q}} \\ &\qquad + C_0(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + F_{v_0}\dot{\boldsymbol{q}} + \boldsymbol{g}_0(\boldsymbol{q})\end{aligned} \quad (15)$$

for all $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}} \in \mathbb{R}^n$, where $Y(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) \in \mathbb{R}^{n\times m}$ is the regression matrix and $\boldsymbol{\theta} \in \mathbb{R}^m$ is the vector of the unknown parameters of the robot, which are assumed to be constant. $M_0 \in \mathbb{R}^{n\times n}$, $C_0 \in \mathbb{R}^{n\times n}$, F_{v_0} and $\boldsymbol{g}_0$ include terms which depend only on known parameters.

Property 7. There exists a positive constant k_M such that for all $\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\omega} \in \mathbb{R}^n$

$$\left\|[M^{-1}(\boldsymbol{y}) - M^{-1}(\boldsymbol{z})]\boldsymbol{\omega}\right\| \leq k_M \|\boldsymbol{y} - \boldsymbol{z}\| \|\boldsymbol{\omega}\|. \quad (16)$$

Property 8. Under the conditions (5), (6) and (7), there exist positive constants k_y and k_{d_y} such that

$$\|Y(\boldsymbol{q}_d(t), \dot{\boldsymbol{q}}_d(t), \ddot{\boldsymbol{q}}_d(t))\| \leq k_y, \quad (17)$$

$$\left\|\dot{Y}(\boldsymbol{q}_d(t), \dot{\boldsymbol{q}}_d(t), \ddot{\boldsymbol{q}}_d(t))\right\| \leq k_{d_y}, \quad (18)$$

for all $t \geq 0$, with matrix Y defined in (15).

The proof of Property 7 is shown in Appendix A.

2.4. UGAS of a type of nonlinear systems. We start by recalling the definitions of PE and Uδ-PE functions given by Loria *et al.* (2002).

Definition 1. The locally integrable function $\Phi : \mathbb{R}_+ \to \mathbb{R}^{n\times m}$ is said to be *Persistently Exciting* (PE) if there exist $\mu > 0$ and $T > 0$ such that

$$\int_t^{t+T} \Phi(\tau)\Phi(\tau)^T \mathrm{d}\tau \geq \mu I, \quad \forall t \in \mathbb{R}_+. \quad (19)$$

Let $\boldsymbol{x} \in \mathbb{R}^n$ be partitioned as $\boldsymbol{x} = \mathrm{col}[\boldsymbol{x}_1, \boldsymbol{x}_2]$, where $\boldsymbol{x}_1 \in \mathbb{R}^{n_1}$ and $\boldsymbol{x}_2 \in \mathbb{R}^{n_2}$. Let the column vector $\boldsymbol{\phi} : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^m$ be such that $(t, \boldsymbol{x}) \mapsto \boldsymbol{\phi}(t, \boldsymbol{x})$ is locally integrable. Define also $\mathcal{D}_1 = \{\boldsymbol{x} \in \mathbb{R}^n : \boldsymbol{x}_1 \neq \boldsymbol{0}\}$.

Definition 2. The function ϕ is said to be *Uniformly δ-Persistently Exciting* (Uδ-PE) with respect to $\boldsymbol{x}_1$ if for each $\boldsymbol{x} \in \mathcal{D}_1$ there exist $\delta > 0$, $T > 0$ and $\mu > 0$ such that, for all $t \in \mathbb{R}_+$,

$$\|\boldsymbol{z} - \boldsymbol{x}\| \le \delta \implies \int_t^{t+T} \|\boldsymbol{\phi}(\tau, \boldsymbol{z})\| \,\mathrm{d}\tau \ge \mu. \tag{20}$$

The property of Uδ-PE defined above roughly means that for every *fixed* $\boldsymbol{x} \neq \boldsymbol{0}$ the function $\Phi(t) = \boldsymbol{\phi}(t, \boldsymbol{x})$ is PE in the sense of Definition 1 and μ and T are the same for all neighboring points of $\boldsymbol{x}$. For uniformly continuous functions, we do not need to check the condition on neighboring points. More precisely, we have the following.

Lemma 1. *If $\boldsymbol{\phi}(t, \boldsymbol{x})$ is continuous uniformly in t, then $\boldsymbol{\phi}(t, \boldsymbol{x})$ is Uδ-PE with respect to $\boldsymbol{x}_1$ if and only if for each $\boldsymbol{x} \in \mathcal{D}_1$ there exist $T > 0$ and $\mu > 0$ such that, for all $t \in \mathbb{R}_+$,*

$$\int_t^{t+T} \|\boldsymbol{\phi}(\tau, \boldsymbol{x})\| \,\mathrm{d}\tau \ge \mu. \tag{21}$$

In particular, a function of the form

$$\boldsymbol{\phi}(t, \boldsymbol{x}) = \Phi(t)^T \boldsymbol{x} \tag{22}$$

is Uδ-PE with respect to $\boldsymbol{x}$ if and only if Φ is PE (Loria *et al.*, 2005).

We can now recall a useful theorem on the uniform global asymptotic stability of nonautonomous systems, presented by Loria *et al.* (2005). It applies to systems of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}) \tag{23}$$

with

$$\begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \end{bmatrix} = \boldsymbol{f}(t, \boldsymbol{x}) = \begin{bmatrix} \boldsymbol{f}_1(t, \boldsymbol{x}_1) + \boldsymbol{f}_2(t, \boldsymbol{x}) \\ \boldsymbol{f}_3(t, \boldsymbol{x}) \end{bmatrix}, \tag{24}$$

where $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1^T & \boldsymbol{x}_2^T \end{bmatrix}^T$, $\boldsymbol{x}_1 \in \mathbb{R}^{n_1}$, $\boldsymbol{x}_2 \in \mathbb{R}^{n_2}$, $\boldsymbol{f}_1 : \mathbb{R}_+ \times \mathbb{R}^{n_1} \to \mathbb{R}^{n_1}$, $\boldsymbol{f}_2 : \mathbb{R}_+ \times \mathbb{R}^{n} \to \mathbb{R}^{n_1}$, $\boldsymbol{f}_3 : \mathbb{R}_+ \times \mathbb{R}^{n} \to \mathbb{R}^{n_2}$, $n = n_1 + n_2$, and all functions vanish in the origin $\boldsymbol{x} = \boldsymbol{0}$. We define

$$\boldsymbol{f}_0(t, \boldsymbol{x}_2) = \boldsymbol{f}_2(t, \boldsymbol{x})|_{\boldsymbol{x}_1 = \boldsymbol{0}}, \tag{25}$$

and notice that, necessarily, $\boldsymbol{f}_0(t, \boldsymbol{0}) = \boldsymbol{0}$. Suppose the following assumptions are satisfied.

Assumption 1. There exists a continuously differentiable function $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$, which is positive definite, decrescent, radially unbounded and has a negative semidefinite time-derivative. More precisely, assume that there exist continuous, positive definite, radially unbounded functions $V_1, V_2 : \mathbb{R}^n \to \mathbb{R}_+$ and $U : \mathbb{R}^{n_1} \to \mathbb{R}_+$ continuous positive definite, such that

$$V_1(\boldsymbol{x}) \le V(t, \boldsymbol{x}) \le V_2(\boldsymbol{x}), \tag{26}$$

$$\dot{V}(t, \boldsymbol{x}) \le -U(\boldsymbol{x}_1), \tag{27}$$

for all $(t, \boldsymbol{x}) \in \mathbb{R}_+ \times \mathbb{R}^n$.

Assumption 2. The function $\boldsymbol{f}_2(t, \boldsymbol{x})$ is continuously differentiable and, moreover, it is uniformly bounded in t on each compact set of the state $\boldsymbol{x}_2$. More precisely, for each $r_2 > 0$ there exist $f_M > 0$ and continuous nondecreasing functions $p_i : \mathbb{R}_+ \to \mathbb{R}_+$ with $i = 1, 2$ such that $p_i(0) = 0$ and for all $(t, \boldsymbol{x}) \in \mathbb{R}_+ \times \mathbb{R}^n$

$$\max_{\|\boldsymbol{x}_2\| \le r_2} \left\{ \|\boldsymbol{f}_0(t, \boldsymbol{x}_2)\|, \left\| \frac{\partial \boldsymbol{f}_0}{\partial t} \right\|, \left\| \frac{\partial \boldsymbol{f}_0}{\partial \boldsymbol{x}_2} \right\| \right\} \le f_M, \tag{28}$$

$$\max_{\|\boldsymbol{x}_2\| \le r_2} \|\boldsymbol{f}_2(t, \boldsymbol{x}) - \boldsymbol{f}_0(t, \boldsymbol{x}_2)\| \le p_1(\|\boldsymbol{x}_1\|), \tag{29}$$

$$\max_{\|\boldsymbol{x}_2\| \le r_2} \{\|\boldsymbol{f}_1(t, \boldsymbol{x}_1)\|, \|\boldsymbol{f}_3(t, \boldsymbol{x})\|\} \le p_2(\|\boldsymbol{x}_1\|). \tag{30}$$

We are now ready to cite the theorem that we will employ to prove uniform global asymptotic stability of a nonlinear time-varying system of the form (24).

Theorem 1. (Loria *et al.*, 2002) *The system (23), (24) under Assumptions 1 and 2 is UGAS if and only if the function $\boldsymbol{f}_0(t, \boldsymbol{x}_2)$ is Uδ-PE with respect to $\boldsymbol{x}_2$.* ♦

Remark 1. In the work of Loria *et al.* (2002), the condition (26) is expressed as

$$\alpha_1(\|\boldsymbol{x}\|) \le V(t, \boldsymbol{x}) \le \alpha_2(\|\boldsymbol{x}\|), \tag{31}$$

with $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$. However, the condition (26) implies the existence of $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ such that (31) is fulfilled (see Khalil, 2002, Lemma 4.3).

3. Main result

The adaptive output feedback tracking controller, proposed by Moreno-Valenzuela *et al.* (2010), is given by

$$\begin{aligned} \boldsymbol{\tau} = {} & Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\hat{\boldsymbol{\theta}} + K_v \mathbf{tanh}(\widetilde{\boldsymbol{\vartheta}}) + F_{v0}\dot{\boldsymbol{q}}_d \\ & + K_p \mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}}) + M_0(\boldsymbol{q}_d)\ddot{\boldsymbol{q}}_d + C_0(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d)\dot{\boldsymbol{q}}_d \\ & + \boldsymbol{g}_0(\boldsymbol{q}_d), \end{aligned} \tag{32}$$

where $\widetilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the link position tracking error vector, K_p and K_v are $n \times n$ diagonal positive definite matrices, σ is a positive constant, M_0, C_0, F_{v0} and g_0 are defined in (15), and the reference trajectory $\boldsymbol{q}_d(t)$ is chosen such that the transpose of the regression matrix $Y(\boldsymbol{q}_d(t), \dot{\boldsymbol{q}}_d(t), \ddot{\boldsymbol{q}}_d(t))^T$, defined in Property 6, is PE in the sense of Definition 1.

The function $\mathbf{tanh}$ is defined as the hyperbolic tangent function in vectorial form, that is, $\mathbf{tanh}(\boldsymbol{y}) = [\tanh(y_1)\ \tanh(y_2)\ \cdots\ \tanh(y_n)]^T$, for all $\boldsymbol{y} \in \mathbb{R}^n$.

The signal $\widetilde{\boldsymbol{\vartheta}}(t)$ in (32) is obtained from the following nonlinear filter:

$$\dot{\boldsymbol{z}} = -A\mathbf{tanh}(\widetilde{\boldsymbol{\vartheta}}), \tag{33}$$

$$\widetilde{\boldsymbol{\vartheta}} = \boldsymbol{z} + B\widetilde{\boldsymbol{q}}, \tag{34}$$

with $\boldsymbol{z} \in \mathbb{R}^n$, A and B are $n \times n$ diagonal positive definite matrices.

The estimated parameter vector $\hat{\boldsymbol{\theta}}$ is computed through the update law

$$\begin{aligned}\hat{\boldsymbol{\theta}} = \Gamma_a[Y^T(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\widetilde{\boldsymbol{q}} - \int_0^t [\dot{Y}^T(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\widetilde{\boldsymbol{q}} \\ - \varepsilon Y^T(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}})]\, \mathrm{d}t],\end{aligned} \tag{35}$$

with Γ_a being a diagonal positive definite matrix and ε a positive constant suitably selected.

The system (1), (32), (33), (34), (35) is expressed by the closed loop equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\widetilde{\boldsymbol{q}} \\ \dot{\widetilde{\boldsymbol{q}}} \\ \widetilde{\boldsymbol{\vartheta}} \\ \widetilde{\boldsymbol{\theta}}\end{bmatrix} = \begin{bmatrix}\dot{\widetilde{\boldsymbol{q}}} \\ M(\boldsymbol{q})^{-1}[-C(\boldsymbol{q}, \boldsymbol{q}_d)\dot{\widetilde{\boldsymbol{q}}} - F_v\dot{\widetilde{\boldsymbol{q}}} \\ -K_v\mathbf{tanh}(\widetilde{\boldsymbol{\vartheta}}) - K_p\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}}) \\ -\boldsymbol{h}(\widetilde{\boldsymbol{q}}, \dot{\widetilde{\boldsymbol{q}}}) + Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\widetilde{\boldsymbol{\theta}}] \\ -A\mathbf{tanh}(\widetilde{\boldsymbol{\vartheta}}) + B\dot{\widetilde{\boldsymbol{q}}} \\ -\Gamma_a Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)^T\left[\dot{\widetilde{\boldsymbol{q}}} + \varepsilon\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}})\right]\end{bmatrix}, \tag{36}$$

where the origin is an equilibrium point, and $\boldsymbol{h}(\widetilde{\boldsymbol{q}}, \dot{\widetilde{\boldsymbol{q}}})$ represents the so-called residual dynamics defined in Property 5.

Define the constants

$$\gamma_1 = \frac{s_1 s_2}{\tanh(s_2\sigma)}, \tag{37}$$

$$\gamma_2 = 2k_{c1}\mu_1 + \lambda_{\max}\{F_v\}, \tag{38}$$

$$\gamma_3 = k_{c1}\sqrt{n} + \sigma\lambda_{\max}\{M(\boldsymbol{q})\}\lambda_{\max}\{F_v\}. \tag{39}$$

Assumption 3. Assume that the damping introduced by the viscous friction coefficients F_v is large enough so that it satisfies

$$\lambda_{\min}\{F_v\} > k_{c1}\mu_1. \tag{40}$$

Assumption 4. The matrix of proportional gains K_p is large enough so that it achieves

$$\lambda_{\min}\{K_p\} > \gamma_1. \tag{41}$$

Assumption 5. The constant ε from the adaptive law (35) is selected such that it satisfies

$$\begin{aligned}&\frac{\gamma_1^2}{[1-\beta][\lambda_{\min}\{K_p\} - \gamma_1]\lambda_{\min}\{F_v\}} < \varepsilon \\ &< \min\left\{\frac{[\lambda_{\min}\{K_p\} - \gamma_1][\beta\lambda_{\min}\{F_v\} - k_{c1}\mu_1]}{[\lambda_{\min}\{K_p\} - \gamma_1]\gamma_3 + \gamma_2^2},\right. \\ &\quad \frac{2[\lambda_{\min}\{K_p\} - \gamma_1]\lambda_{\min}\{K_v B^{-1}A\}}{\lambda_{\max}^2\{K_v\}}, \\ &\quad \left.\frac{\sqrt{\sigma^{-1}\lambda_{\min}\{K_p\}\lambda_{\min}\{M(\boldsymbol{q})\}}}{\lambda_{\max}\{M(\boldsymbol{q})\}}\right\},\end{aligned}$$

where $\beta \in (0, 1)$, and A and B are the diagonal positive definite matrices employed in (33) and (34).

Remark 2. Assumption 3 is a condition that refers to the viscous friction matrix F_v and the bound on the time-derivative of the reference trajectory μ_1. Such a condition has already been proposed in the literature (e.g., Santibanez and Kelly, 2001; Moreno-Valenzuela *et al.*, 2010; Zavala-Rio *et al.*, 2011). As far as the authors are aware, all the saturated output feedback tracking controllers proposed in the literature so far require this condition in order to achieve globality. As for Assumption 4, it is a standard condition; it requires proportional gains to be large enough in order to overcome the torque effects of the inertia and Coriolis matrices and gravity vector (most saturated controllers employ similar conditions). On the other hand, Assumption 5 bounds the parameter ε in order to ensure positive definiteness of the Lyapunov function and negative definiteness of its time-derivative.

Our main stability result on the origin of (36) is summarized in the following proposition.

Proposition 1. *The origin* $\begin{bmatrix}\widetilde{\boldsymbol{q}}^T & \dot{\widetilde{\boldsymbol{q}}}^T & \widetilde{\boldsymbol{\vartheta}}^T & \widetilde{\boldsymbol{\theta}}\end{bmatrix} = \mathbf{0}$ *of (36), under Assumptions 3, 4 and 5, is UGAS if and only if the matrix* $Y(\boldsymbol{q}_d(t), \dot{\boldsymbol{q}}_d(t), \ddot{\boldsymbol{q}}_d(t))^T$ *is PE in the sense of Definition 1.*

3.1. Proof of Proposition 1.

If we define

$$\boldsymbol{x}_1 = \begin{bmatrix}\widetilde{\boldsymbol{q}} \\ \dot{\widetilde{\boldsymbol{q}}} \\ \widetilde{\boldsymbol{\vartheta}}\end{bmatrix}, \quad \boldsymbol{x}_2 = \widetilde{\boldsymbol{\theta}}, \quad \boldsymbol{x} = \begin{bmatrix}\boldsymbol{x}_1 \\ \boldsymbol{x}_2\end{bmatrix}, \tag{42}$$

then (36) can be expressed in the form (24) as follows:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix}\widetilde{\boldsymbol{q}} \\ \dot{\widetilde{\boldsymbol{q}}} \\ \widetilde{\boldsymbol{\vartheta}}\end{bmatrix}}_{\boldsymbol{x}_1} = \underbrace{\begin{bmatrix}\dot{\widetilde{\boldsymbol{q}}} \\ M(\boldsymbol{q})^{-1}[-C(\boldsymbol{q}, \boldsymbol{q}_d)\dot{\widetilde{\boldsymbol{q}}} - F_v\dot{\widetilde{\boldsymbol{q}}} \\ -K_v\mathbf{tanh}(\widetilde{\boldsymbol{\vartheta}}) - K_p\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}}) \\ -\boldsymbol{h}(\widetilde{\boldsymbol{q}}, \dot{\widetilde{\boldsymbol{q}}})] \\ -A\mathbf{tanh}(\widetilde{\boldsymbol{\vartheta}}) + B\dot{\widetilde{\boldsymbol{q}}}\end{bmatrix}}_{\boldsymbol{f}_1(t, \boldsymbol{x}_1)} \\ + \underbrace{\begin{bmatrix}\mathbf{0} \\ M(\boldsymbol{q})^{-1}Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\widetilde{\boldsymbol{\theta}} \\ \mathbf{0}\end{bmatrix}}_{\boldsymbol{f}_2(t, \boldsymbol{x})},\end{aligned} \tag{43}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\widetilde{\boldsymbol{\theta}}}_{\boldsymbol{x}_2} = \underbrace{-\Gamma_a Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)^T\left[\dot{\widetilde{\boldsymbol{q}}} + \varepsilon\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}})\right]}_{\boldsymbol{f}_3(t, \boldsymbol{x})}. \tag{44}$$

In order to prove the UGAS of the origin of the system (43), (44), we will use Theorem 1. The first step is to check that Assumption 1 is satisfied. Consider the

Lyapunov function

$$V(t,\boldsymbol{x}) = \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T M(\boldsymbol{q})\dot{\tilde{\boldsymbol{q}}} + \sum_{i=1}^{n} k_{v_i} b_i^{-1} \ln(\cosh(\tilde{\vartheta}_i)) + \sum_{i=1}^{n} k_{p_i}\sigma^{-1}\ln(\cosh(\sigma\tilde{q}_i)) + \varepsilon \mathbf{tanh}(\sigma\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\tilde{\boldsymbol{q}}} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T \Gamma_a^{-1}\tilde{\boldsymbol{\theta}}. \tag{45}$$

By bounding each of the terms of $V(t,\boldsymbol{x})$, upper and lower bounds are given by

$$V_1(\boldsymbol{x}) \le V(t,\boldsymbol{x}) \le V_2(\boldsymbol{x}), \tag{46}$$

where

$$V_1(\boldsymbol{x}) = \begin{bmatrix} \sqrt{\sum_{i=1}^{n} k_{p_i}\sigma^{-1}\ln(\cosh(\sigma\tilde{q}_i))} \\ \left\|\dot{\tilde{\boldsymbol{q}}}\right\| \end{bmatrix}^T P \times \begin{bmatrix} \sqrt{\sum_{i=1}^{n} k_{p_i}\sigma^{-1}\ln(\cosh(\sigma\tilde{q}_i))} \\ \left\|\dot{\tilde{\boldsymbol{q}}}\right\| \end{bmatrix} + \sum_{i=1}^{n} k_{v_i} b_i^{-1}\ln(\cosh(\tilde{\vartheta}_i)) + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\Gamma_a^{-1}\tilde{\boldsymbol{\theta}},$$

$$V_2(\boldsymbol{x}) = \frac{1}{2}\lambda_{\max}\{M\}\left\|\dot{\tilde{\boldsymbol{q}}}\right\|^2 + \sum_{i=1}^{n} k_{v_i} b_i^{-1}\ln(\cosh(\tilde{\vartheta}_i)) + \sum_{i=1}^{n} k_{p_i}\sigma^{-1}\ln(\cosh(\sigma\tilde{q}_i)) + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\Gamma_a^{-1}\tilde{\boldsymbol{\theta}} + \varepsilon\lambda_{\max}\{M\}\left\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\right\|\left\|\dot{\tilde{\boldsymbol{q}}}\right\|,$$

$$P = \begin{bmatrix} \sigma^{-1}\lambda_{\min}\{K_p\} & -\frac{\varepsilon}{2}\sqrt{2}\lambda_{\max}\{M\} \\ -\frac{\varepsilon}{2}\sqrt{2}\lambda_{\max}\{M\} & \frac{1}{2}\lambda_{\min}\{M\} \end{bmatrix}. \tag{47}$$

The matrix P is symmetric positive definite under Assumption 5. The time derivative of $V(t,\boldsymbol{x})$ is given by

$$\dot{V}(t,\boldsymbol{x}) = \varepsilon\mathbf{tanh}(\sigma\tilde{\boldsymbol{q}})^T[-F_v\dot{\tilde{\boldsymbol{q}}} - K_v\mathbf{tanh}(\tilde{\boldsymbol{\vartheta}}) - K_p\mathbf{tanh}(\sigma\tilde{\boldsymbol{q}}) + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{h}(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}})] + \varepsilon\sigma\dot{\tilde{\boldsymbol{q}}}^T M(\boldsymbol{q})\mathrm{Sech}^2(\sigma\tilde{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} - \dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{h}(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}}) - \dot{\tilde{\boldsymbol{q}}}^T F_v\dot{\tilde{\boldsymbol{q}}} - \mathbf{tanh}(\tilde{\boldsymbol{\vartheta}})^T K_v B^{-1} A\,\mathbf{tanh}(\tilde{\boldsymbol{\vartheta}}). \tag{48}$$

The time derivative (48) may be upper bounded by

$$\dot{V}(t,\boldsymbol{x}) \le -U(\boldsymbol{x}_1), \tag{49}$$

where

$$U(\boldsymbol{x}_1) = \begin{bmatrix} \|\mathbf{tanh}(\sigma\tilde{\boldsymbol{q}})\| \\ \left\|\dot{\tilde{\boldsymbol{q}}}\right\| \end{bmatrix}^T Q_1 \begin{bmatrix} \|\mathbf{tanh}(\sigma\tilde{\boldsymbol{q}})\| \\ \left\|\dot{\tilde{\boldsymbol{q}}}\right\| \end{bmatrix} + \begin{bmatrix} \|\mathbf{tanh}(\sigma\tilde{\boldsymbol{q}})\| \\ \left\|\mathbf{tanh}(\tilde{\boldsymbol{\vartheta}})\right\| \end{bmatrix}^T Q_2 \begin{bmatrix} \|\mathbf{tanh}(\sigma\tilde{\boldsymbol{q}})\| \\ \left\|\mathbf{tanh}(\tilde{\boldsymbol{\vartheta}})\right\| \end{bmatrix},$$

and

$$Q_1 = \begin{bmatrix} \frac{\varepsilon}{2}[\lambda_{\min}\{K_p\} - \gamma_1] & -\frac{1}{2}\gamma_1 - \frac{1}{2}\varepsilon\gamma_2 \\ -\frac{1}{2}\gamma_1 - \frac{1}{2}\varepsilon\gamma_2 & \lambda_{\min}\{F_v\} - k_{c1}\mu_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\varepsilon\gamma_3 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} \frac{\varepsilon}{2}[\lambda_{\min}\{K_p\} - \gamma_1] & -\frac{\varepsilon}{2}\lambda_{\max}\{K_v\} \\ -\frac{\varepsilon}{2}\lambda_{\max}\{K_v\} & \lambda_{\min}\{K_v B^{-1} A\} \end{bmatrix}.$$

Under Assumptions 3–5, Q_1 and Q_2 are positive definite matrices. Therefore, $U(\boldsymbol{x}_1)$ is a positive definite function, and Assumption 1 is satisfied.

We will now verify that Assumption 2 holds. To this end, notice that $\boldsymbol{f}_i$ for $i = 1,2,3$ have been defined in (43) and (44). It is clear from (25) and (43) that

$$\boldsymbol{f}_0(t,\boldsymbol{x}_2) = \begin{bmatrix} \mathbf{0} \\ M(\boldsymbol{q}_d)^{-1}Y(\boldsymbol{q}_d,\dot{\boldsymbol{q}}_d,\ddot{\boldsymbol{q}}_d)\tilde{\boldsymbol{\theta}} \\ \mathbf{0} \end{bmatrix}. \tag{50}$$

In Appendix B it is proven that (28)–(30) are satisfied for functions $\boldsymbol{f}_1$, $\boldsymbol{f}_2$, $\boldsymbol{f}_3$ and $\boldsymbol{f}_0$.

Since Assumption 2 is already satisfied, it only remains to show that $\boldsymbol{f}_0(t,\boldsymbol{x}_2)$ is Uδ-PE with respect to $\boldsymbol{x}_2$. Hence, we need to show that for each $\boldsymbol{x}_2 \neq \mathbf{0}$ there exist $\mu > 0$ and $T > 0$ such that, for all $t \ge 0$,

$$\int_t^{t+T} \boldsymbol{x}_2^T Y(\boldsymbol{q}_d(\tau),\dot{\boldsymbol{q}}_d(\tau),\ddot{\boldsymbol{q}}_d(\tau))^T M(\boldsymbol{q}_d(\tau))^{-1} \times M(\boldsymbol{q}_d(\tau))^{-1} Y(\boldsymbol{q}_d(\tau),\dot{\boldsymbol{q}}_d(\tau),\ddot{\boldsymbol{q}}_d(\tau))\boldsymbol{x}_2\,\mathrm{d}\tau \ge \mu. \tag{51}$$

Since $M(\boldsymbol{q}_d(\tau))^{-1}$ is full rank, then (51) holds if and only if the function $\boldsymbol{\phi}(t,\boldsymbol{x}) = Y(\boldsymbol{q}_d(t),\dot{\boldsymbol{q}}_d(t),\ddot{\boldsymbol{q}}_d(t))\boldsymbol{x}_2$ is Uδ-PE. Since $\boldsymbol{\phi}(t,\boldsymbol{x})$ is in the form (22), with $\Phi(t)^T = Y(\boldsymbol{q}_d(t),\dot{\boldsymbol{q}}_d(t),\ddot{\boldsymbol{q}}_d(t))$, it is U$\delta$-PE if and only if $Y(\boldsymbol{q}_d(t),\dot{\boldsymbol{q}}_d(t),\ddot{\boldsymbol{q}}_d(t))^T$ is PE in the sense of Definition 1. Hence, from Theorem 1, the origin of (36) is UGAS.

4. Experimental results

Experimental results were carried out to show the performance of the adaptive output feedback tracking controller (32) and to confirm the theoretical analysis. The control scheme was proved in a two degrees of freedom planar arm prototype moving in the vertical plane, whose links are connected through revolute joints. This is the direct drive robot manipulator used by Reyes and Kelly (2001), built at the CICESE Research Center and located at the Laguna Institute of Technology, Mexico. Two tests were performed in order to observe the performance of the proposed controller for two different desired trajectories. Robot parameters are shown in Table 1.

Table 1. Robot parameters.

Description	Notation	Value	Units
Mass of link 1	m_1	23.902	kg
Mass of link 2	m_2	3.88	kg
Length of link 1	l_1	0.45	m
Length of link 2	l_2	0.45	m
Distance to the center of mass 1	l_{c_1}	0.091	m
Distance to the center of mass 2	l_{c_2}	0.048	m
Inertia relative to center of mass 1	I_1	1.266	kg m^2
Inertia relative to center of mass 2	I_2	0.093	kg m^2
Gravity acceleration	g	9.81	m/s^2
Coefficient of viscous friction 1	f_{v_1}	2.288	N m s/rad
Coefficient of viscous friction 2	f_{v_2}	0.175	N m s/rad

For Test 1, the desired trajectory was selected as

$$q_{d_1} = c_1(1 - e^{-at^3}) + c_2(1 - e^{-at^3})\sin(\omega_1 t), \quad (52)$$

$$q_{d_2} = c_3(1 - e^{-bt^3}) + c_4(1 - e^{-bt^3})\sin(\omega_2 t). \quad (53)$$

For Test 2, the desired trajectory was selected as

$$q_{d_1} = k_1(1 - e^{-at^3}) + k_2\sin(\omega_3 t) + k_3\sin(\omega_4 t), \quad (54)$$

$$q_{d_2} = k_4\sin(\omega_5 t) + k_5\sin(\omega_6 t). \quad (55)$$

Parameters of the desired trajectories are shown in Table 2. Control parameters used for the experimental tests are shown in Table 3.

The desired trajectory (52)–(53) for Test 1 has the feature that its initial positions, velocities, and accelerations are zero and evolve smoothly, which prevents torque values from saturating the actuators and the required velocities do not surpass the permitted velocity motor limits. As for the desired trajectory (54)–(55) used for Test 2, it is a harder one, since its initial velocities are different from zero, which demands greater initial torques. Besides, the trajectory for Test 2 includes an additional sinusoidal term assuring so the persistency of excitation.

Table 2. Parameters of the desired joint trajectory $\boldsymbol{q}_d(t)$ for Tests 1 and 2.

Desired trajectory parameters	Value	Unit
c_1	0.7854	rad
c_2	0.1745	rad
c_3	1	rad
c_4	0.5	rad
ω_1	7.5	rad/s
ω_2	1.75	rad/s
a	2	1/s^3
b	1.8	1/s^3
k_1	1.5707	rad
k_2	0.1745	rad
k_3	0.1745	rad
k_4	0.25	rad
k_5	0.25	rad
ω_3	6	rad/s
ω_4	4	rad/s
ω_5	0.5	rad/s
ω_6	1.5	rad/s

The robot dynamics are linearly parameterized as in (15) in the following manner:

$$Y(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \end{bmatrix}, \quad (56)$$

$$M_0(\boldsymbol{q}) = \begin{bmatrix} m_1 l_{c_1}^2 + I_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad (57)$$

$$C_0(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (58)$$

$$F_{v_0} = \begin{bmatrix} f_{v_1} & 0 \\ 0 & f_{v_2} \end{bmatrix}, \quad (59)$$

$$\boldsymbol{g}_0(\boldsymbol{q}) = \begin{bmatrix} m_1 l_{c_1} g \sin(q_1) \\ 0 \end{bmatrix}, \quad (60)$$

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} m_2 \\ m_2 l_{c_2} \\ m_2 l_{c_2}^2 + I_2 \end{bmatrix}, \quad (61)$$

where

$$y_{11} = l_1^2 \ddot{q}_1 + l_1 g \sin(q_1), \quad (62)$$

$$y_{12} = 2l_1\cos(q_2)\ddot{q}_1 + l_1\cos(q_2)\ddot{q}_2 - l_1\sin(q_2)\dot{q}_2\dot{q}_1 - l_1\sin(q_2)(\dot{q}_1 + \dot{q}_2)\dot{q}_2 + g\sin(q_1 + q_2), \quad (63)$$

$$y_{13} = \ddot{q}_1 + \ddot{q}_2, \quad (64)$$

$$y_{21} = 0, \quad (65)$$

$$y_{22} = l_1\cos(q_2)\ddot{q}_1 + l_1\sin(q_2)\dot{q}_1^2 + g\sin(q_1 + q_2), \quad (66)$$

$$y_{23} = \ddot{q}_1 + \ddot{q}_2. \quad (67)$$

From Table 1, it is possible to observe that

$$\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix} = \begin{bmatrix} m_2 \\ m_2 l_{c_2} \\ m_2 l_{c_2}^2 + I_2 \end{bmatrix} = \begin{bmatrix} 3.88 \\ 0.18624 \\ 0.1019 \end{bmatrix}. \tag{68}$$

We have numerically verified that the regression matrix $Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)$, given by (56), is persistently exciting according to Definition 1, where $\Phi(t)^T = Y(\boldsymbol{q}_d(t), \dot{\boldsymbol{q}}_d(t), \ddot{\boldsymbol{q}}_d(t))$, for the desired trajectories (52)–(53) and (54)–(55). For the reference trajectory (52)–(53), the condition

$$\int_t^{t+T} \Phi(\tau)\Phi(\tau)^T \,\mathrm{d}\tau \geq \mu I, \quad \forall t \in \mathbb{R}_+, \tag{69}$$

where $\Phi(t)^T = Y(\boldsymbol{q}_d(t), \dot{\boldsymbol{q}}_d(t), \ddot{\boldsymbol{q}}_d(t))$, is satisfied with $\mu = 51$ and $T = 2.5$. For the reference trajectory (54)–(55), the condition (69) is satisfied with $\mu = 6.8$ and $T = 2.5$.

For Test 1, Figs. 1 and 2 show the tracking errors $\widetilde{q}_1(t)$ and $\widetilde{q}_2(t)$ for Joints 1 and 2, respectively. Figures 3 and 4 show the employed torques $\tau_1(t)$ and $\tau_2(t)$, and Figs. 5–7 show the estimated parameters $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, and $\hat{\theta}_3(t)$, respectively. We have computed the Root Mean Square (RMS) index, for the steady state position errors of Test 1, given by

$$RMS = \sqrt{\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \|\widetilde{\boldsymbol{q}}\|^2(t)\,\mathrm{d}t}, \tag{70}$$

with $T_1 = 10$ s and $T_2 = 60$ s, obtaining a value of 0.0078 [rad].

For Test 2, Figs. 8 and 9 show tracking errors $\widetilde{q}_1(t)$ and $\widetilde{q}_2(t)$ for Joints 1 and 2, respectively. Figures 10 and 11 show the employed torques $\tau_1(t)$ and $\tau_2(t)$, and Figs. 12–14 show the estimated parameters $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, and $\hat{\theta}_3(t)$, respectively. The root mean square index for the steady state position error of Test 2 is 0.0079 [rad].

By taking into account that maximum torques that the actuators can deliver are $\tau_1^{\max} = 150$ [Nm] and $\tau_2^{\max} = 15$ [Nm], from Figs. 3 and 4 for Test 1 and Figs. 10 and 11 for Test 2 one can observe that torques evolve inside of the permitted limits.

On the other hand, we can observe that position errors do not converge to zero in both tests, showing an oscillatory behaviour, which is present mainly because of uncompensated friction, unmodeled high frequency dynamics and discretization errors due to the fact of digitally implementing the robot control system. For Test 2, we observe greater position errors at the beginning; this is explained because, as mentioned above, reference 2 is harder since its initial velocities are different from zero. For both cases we can observe that the RMS position error index is better than or similar to that of other adaptive control systems (see, e.g., Loria *et al.*, 2005; Kelly *et al.*, 2005). It can also be observed that parameter estimators converge to values which are very close to the real ones. So, the experimental results confirm the theoretical stability analysis which claims uniform global convergence to zero for all state variables: position, velocity and parameter errors.

Table 3. Parameters used in the proposed control law.

Controller parameters	Value	Unit
k_{p_1}	60	N m
k_{p_2}	7	N m
k_{v_1}	10	N m
k_{v_2}	5	N m
ε	3	1/s
σ	50	
a_1	100	$1/s^2$
a_2	100	$1/s^2$
b_1	100	1/s
b_2	100	1/s
γ_1	0.64	
γ_2	0.08	
γ_3	0.05	

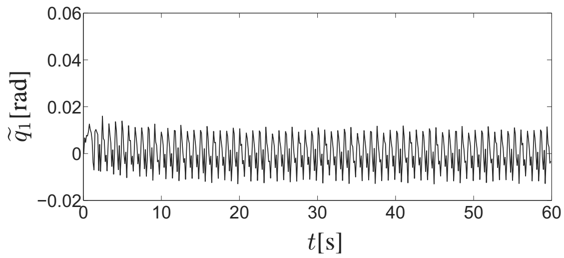

Fig. 1. Time evolution of the tracking error for Joint 1 denoted as $\widetilde{q}_1(t)$ for Test 1.

5. Conclusions

In this paper, the adaptive output feedback tracking controller proposed by Moreno-Valenzuela *et al.* (2010) was revised. Uniform global asymptotic stability of the

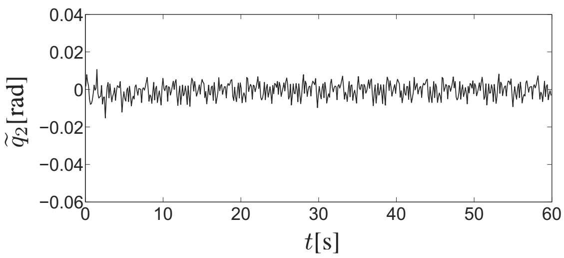

Fig. 2. Time evolution of the tracking error for Joint 2 denoted by $\widetilde{q}_2(t)$ for Test 1.

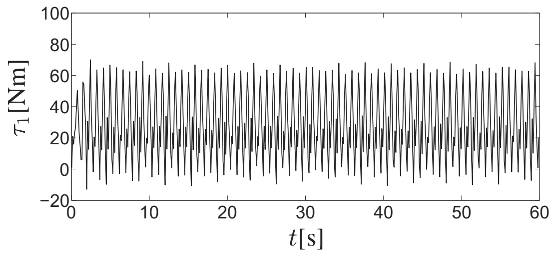

Fig. 3. Time evolution of the applied torque for Joint 1 denoted by $\tau_1(t)$ for Test 1.

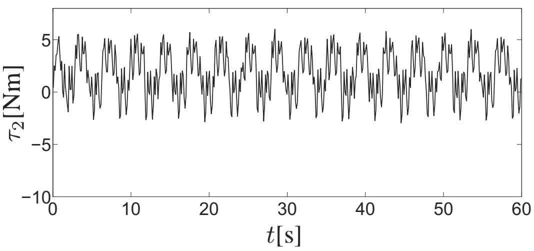

Fig. 4. Time evolution of the torque applied for Joint 2 denoted by $\tau_2(t)$ for Test 1.

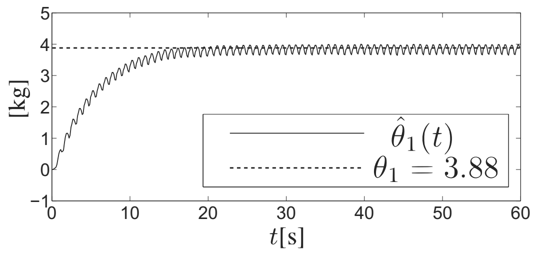

Fig. 5. Time evolution of the estimated parameter $\hat{\theta}_1(t)$ for Test 1.

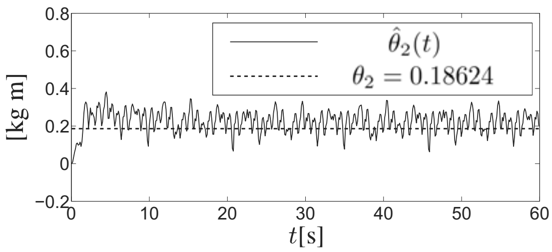

Fig. 6. Time evolution of the estimated parameter $\hat{\theta}_2(t)$ for Test 1.

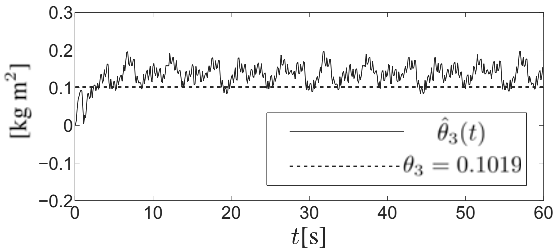

Fig. 7. Time evolution of the estimated parameter $\hat{\theta}_3(t)$ for Test 1.

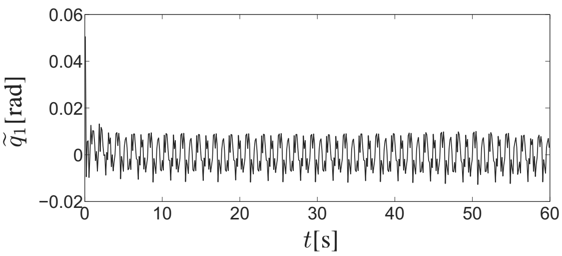

Fig. 8. Time evolution of the tracking error for Joint 1 denoted by $\widetilde{q}_1(t)$ for Test 2.

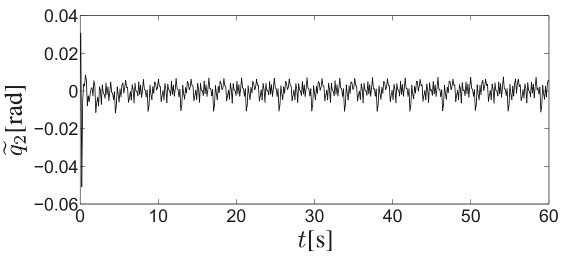

Fig. 9. Time evolution of the tracking error for Joint 2 denoted by $\widetilde{q}_2(t)$ for Test 2.

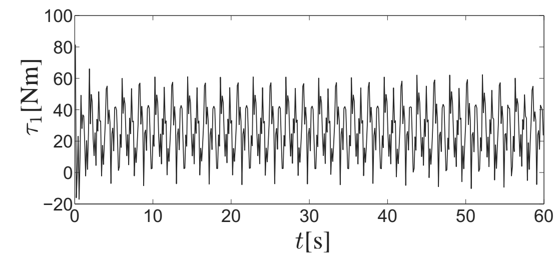

Fig. 10. Time evolution of the torque applied for Joint 1 denoted by $\tau_1(t)$ for Test 2.

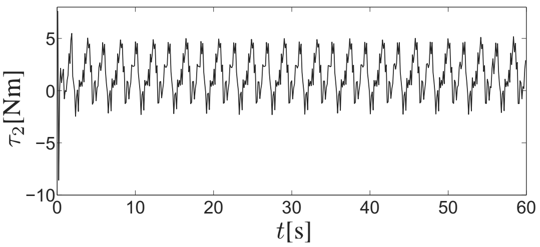

Fig. 11. Time evolution of the torque applied for Joint 2 denoted by $\tau_2(t)$ for Test 2.

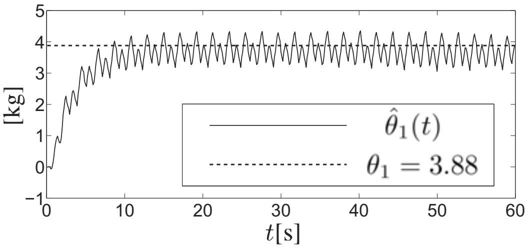

Fig. 12. Time evolution of the estimated parameter $\hat{\theta}_1(t)$ for Test 2.

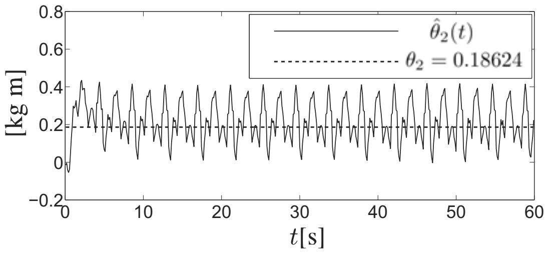

Fig. 13. Time evolution of the estimated parameter $\hat{\theta}_2(t)$ for Test 2.

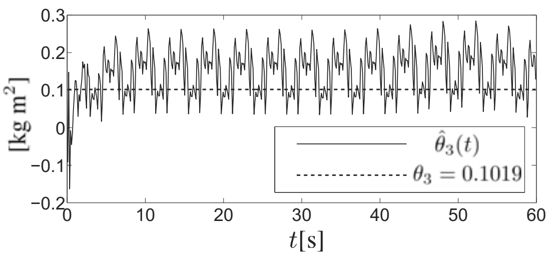

Fig. 14. Time evolution of the estimated parameter $\hat{\theta}_3(t)$ for Test 2.

controller was proved. As far as the authors are aware, this is the first proof of uniform global asymptotic stability of an adaptive output feedback tracking controller. The stability analysis was carried out via Lyapunov theory, complemented by a theorem proposed by Loria *et al.* (2002) on the uniform global asymptotic stability of a certain type of nonlinear systems. Experimental results were presented in order to show the performance of the controller and to confirm the theoretical proposal.

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Appendix A

Here we present a proof for Property 7. The inverse of the inertia matrix $M(\boldsymbol{q})$ may be expressed as

$$M^{-1}(\boldsymbol{q}) = \frac{1}{\det(M(\boldsymbol{q}))} A(\boldsymbol{q}), \tag{A1}$$

where $A(\boldsymbol{q})$ is the adjoint matrix of $M(\boldsymbol{q})$. Based on (A1), we may obtain the following expression for the partial derivatives of the elements of matrix $M^{-1}(\boldsymbol{q})$:

$$\begin{aligned}\frac{\partial M_{ij}^{-1}(\boldsymbol{q})}{\partial q_k} = \frac{1}{\det^2(M(\boldsymbol{q}))} \Bigg[&\det(M(\boldsymbol{q})) \frac{\partial a_{ij}(\boldsymbol{q})}{\partial q_k} \\ &- a_{ij}(\boldsymbol{q}) \frac{\partial \det(M(\boldsymbol{q}))}{\partial q_k} \Bigg],\end{aligned} \tag{A2}$$

for $i, j, k = 1, 2, \ldots, n$, where $M_{ij}^{-1}(\boldsymbol{q})$ is the ij-element of matrix $M^{-1}(\boldsymbol{q})$ and a_{ij} is the ij-element of matrix $A(\boldsymbol{q})$. Since we are assuming robot manipulators whose links are joined together with revolute joints, elements a_{ij} and the determinant $\det(M(\boldsymbol{q}))$ contain only sinusoidal functions of q_k, and also their partial derivatives. Therefore, such expressions are bounded, and there exists a positive constant k_{M_1} such that

$$\left| \frac{\partial M_{ij}^{-1}(\boldsymbol{q})}{\partial q_k} \right| \le k_{M_1}, \tag{A3}$$

for all $i, j, k = 1, 2, \ldots, n$. Therefore (see Kelly *et al.*, 2005, Corollary A.1, p. 385)

$$\left\| [M^{-1}(\boldsymbol{y}) - M^{-1}(\boldsymbol{z})]\boldsymbol{\omega} \right\| \le n^2 k_{M_1} \left\| \boldsymbol{y} - \boldsymbol{z} \right\| \left\| \boldsymbol{\omega} \right\|. \tag{A4}$$

Property 7 is then satisfied with

$$k_M \ge n^2 k_{M_1}. \tag{A5}$$

Appendix B

Here, we will show how the conditions (28), (29) and (30) are satisfied. From the definition of $\boldsymbol{f}_0$ in (50), we have that

$$\begin{aligned}\frac{\partial \boldsymbol{f}_0}{\partial t} &= \begin{bmatrix} \boldsymbol{0} \\ \frac{\mathrm{d}}{\mathrm{d}t}[M(\boldsymbol{q}_d)^{-1}] Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\widetilde{\boldsymbol{\theta}} \\ \boldsymbol{0} \end{bmatrix} \\ &\quad + \begin{bmatrix} \boldsymbol{0} \\ M(\boldsymbol{q}_d)^{-1} \dot{Y}(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\widetilde{\boldsymbol{\theta}} \\ \boldsymbol{0} \end{bmatrix},\end{aligned} \tag{B1}$$

$$\frac{\partial \boldsymbol{f}_0}{\partial \boldsymbol{x}_2} = \begin{bmatrix} \boldsymbol{0} \\ M(\boldsymbol{q}_d)^{-1} Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d) \\ \boldsymbol{0} \end{bmatrix}, \tag{B2}$$

where $\boldsymbol{x}_2 = \widetilde{\boldsymbol{\theta}}$ as is defined in (42).

The time derivative of matrix $M(\boldsymbol{q}_d)^{-1}$ may be expressed in the following manner:

$$\frac{\mathrm{d}}{\mathrm{d}t}[M(\boldsymbol{q}_d)^{-1}] = -M(\boldsymbol{q}_d)^{-1} \dot{M}(\boldsymbol{q}_d) M(\boldsymbol{q}_d)^{-1}. \tag{B3}$$

It can be observed from Eqn. (A1) that matrix $M(\boldsymbol{q}_d)^{-1}$ is bounded. On the other hand, the matrix $\dot{M}(\boldsymbol{q}_d)$ is bounded under the condition (5). Therefore, $\frac{\mathrm{d}}{\mathrm{d}t}[M(\boldsymbol{q}_d)^{-1}]$ is bounded, and under the conditions (17) and (18), matrix $[\frac{\mathrm{d}}{\mathrm{d}t}[M(\boldsymbol{q}_d)^{-1}] Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d) + M(\boldsymbol{q}_d)^{-1} \dot{Y}(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)]$ is also bounded. Therefore, if $\|x_2\| \le r_2$, or equivalently, $\left\|\widetilde{\boldsymbol{\theta}}\right\| \le r_2$, then $\partial \boldsymbol{f}_0 / \partial t$ is bounded. On the other hand, the boundedness of $\partial \boldsymbol{f}_0 / \partial \boldsymbol{x}_2$ is straightforward from the boundedness of matrices $M(\boldsymbol{q}_d)^{-1}$ and $Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)$. Therefore, the condition (28) is satisfied.

Notice, from the definition of function $\boldsymbol{f}_1$ in (43) that

$$\begin{aligned}\|\boldsymbol{f}_1\|^2 &= \|\dot{\widetilde{\boldsymbol{q}}}\|^2 + \|M(\boldsymbol{q})^{-1}[-C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\widetilde{\boldsymbol{q}}} - F_v \dot{\widetilde{\boldsymbol{q}}} \\ &\quad - K_v \mathbf{tanh}(\boldsymbol{\vartheta}) - K_p \mathbf{tanh}(\sigma \widetilde{\boldsymbol{q}}) \\ &\quad - \boldsymbol{h}(\widetilde{\boldsymbol{q}}, \dot{\widetilde{\boldsymbol{q}}})]\|^2 \\ &\quad + \| - A \mathbf{tanh}(\boldsymbol{\vartheta}) + B \dot{\widetilde{\boldsymbol{q}}}\|^2 \\ &\le \|\boldsymbol{x}_1\|^2 \left[1 + \frac{1}{\lambda_{\min}\{M\}} (k_{c_1} \|\boldsymbol{x}_1\| + \zeta_1)^2 \right] \\ &\quad + \|\boldsymbol{x}_1\|^2 [\lambda_{\max}\{A\} + \lambda_{\max}\{B\}]^2,\end{aligned} \tag{B4}$$

where

$$\begin{aligned}\zeta_1 &= 2 k_{c_1} \mu_1 + \lambda_{\min}\{F_v\} + \lambda_{\max}\{K_v\} \\ &\quad + \sigma \lambda_{\max}\{K_p\} + \frac{s_1 \sigma^2}{\tanh(s_2 \sigma)}.\end{aligned} \tag{B5}$$

Therefore,

$$\|\boldsymbol{f}_1\| \le \|\boldsymbol{x}_1\| \sqrt{\frac{1}{\lambda_{\min}^2\{M\}} (k_{c_1} \|\boldsymbol{x}_1\| + \zeta_1)^2 + \zeta_2}, \tag{B6}$$

where

$$\zeta_2 = 1 + (\lambda_{\max}\{A\} + \lambda_{\max}\{B\})^2. \tag{B7}$$

Also, notice that

$$\begin{aligned}\|\boldsymbol{f}_3\| &= \|\Gamma_a Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)^T[\dot{\widetilde{\boldsymbol{q}}} + \varepsilon\,\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}})]\| \\ &\le \lambda_{\max}\{\Gamma_a\}k_y\left[\|\dot{\widetilde{\boldsymbol{q}}}\| + \varepsilon\sigma\,\|\widetilde{\boldsymbol{q}}\|\right] \\ &\le \lambda_{\max}\{\Gamma_a\}k_y(1+\varepsilon\sigma)\,\|\boldsymbol{x}_1\|\,,\end{aligned} \tag{B8}$$

where we used the fact that $\|\mathbf{tanh}(\sigma\widetilde{\boldsymbol{q}})\| \le \|\sigma\widetilde{\boldsymbol{q}}\|$ (Kelly *et al.*, 2005). Therefore, the condition (30) is satisfied with

$$p_2(\|\boldsymbol{x}_1\|) = \max\{p_{21}(\|\boldsymbol{x}_1\|), p_{22}(\|\boldsymbol{x}_1\|)\}\,, \tag{B9}$$

where

$$p_{21}(\|\boldsymbol{x}_1\|) = \sqrt{\frac{1}{\lambda^2_{\min}\{M\}}(k_{c_1}\,\|\boldsymbol{x}_1\| + \zeta)^2 + \zeta_2}\,\|\boldsymbol{x}_1\|\,, \tag{B10}$$

$$p_{22}(\|\boldsymbol{x}_1\|) = \lambda_{\max}\{\Gamma_a\}k_y(1+\varepsilon\sigma)\,\|\boldsymbol{x}_1\|\,. \tag{B11}$$

On the other hand, we have that, for all $\|\boldsymbol{x}_2\| \le r_2$, with r_2 defined in Assumption 2,

$$\begin{aligned}\|\boldsymbol{f}_2 - \boldsymbol{f}_0\| &= \big\|[M(\boldsymbol{q})^{-1} - M(\boldsymbol{q}_d)^{-1}]\times \\ &\quad\times Y(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\boldsymbol{x}_2\| \\ &\le k_M k_y r_2\,\|\widetilde{\boldsymbol{q}}\| \\ &\le k_M k_y r_2\,\|\boldsymbol{x}_1\|\,,\end{aligned} \tag{B12}$$

with $\boldsymbol{f}_2$ and $\boldsymbol{f}_0$ defined in (43) and (50), respectively, and Property 7 applied. Therefore, the condition (29) is satisfied with

$$p_1(\|\boldsymbol{x}_1\|) = k_M k_y r_2\,\|\boldsymbol{x}_1\|\,. \tag{B13}$$

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