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ANTIPLANE DEFORMATION OF A BIMATERIAL CONTAINING AN INTERFACIAL CRACK WITH THE ACCOUNT OF FRICTION I. SINGLE LOADING

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Abstract: The paper presents the exact analytic solution to the antiplane problem for a non-homogeneous bimaterial medium containing closed interfacial cracks, which faces can move relatively to each other with dry friction. The medium is subjected to the action of normal and arbitrary single loading in a longitudinal direction. Based on the discontinuity function method the problem is reduced to the solution of the system of singular integral-differential equations for stress and displacement discontinuities at the possible slippage zones. Influence of loading parameters and the effects of friction on the sizes of these zones is analyzed. The stress intensity factors, stress and displacement discontinuities, energy dissipation are determined for several characteristic types of external loading.

Keywords: Friction, Interfacial Crack, Energy Dissipation, Stress Intensity Factor, Antiplane Deformation, Longitudinal Shear, Bimaterial, Discontinuity Functions

1. INTRODUCTION

The study of the contact phenomena with the account of friction effect is one of the most actual problems in mechanical engineering (Goryacheva, 2001; Panasiuk et al., 1976; Sulym and Piskozub, 2004; Johnson, 1985; Hills et al.,1993; Herrmann and Loboda, 1999; Ulitko and Ostryk, 2006) etc. To a greater or lesser extent the contact phenomena both at macroscopic and microscopic levels are always accompanied with friction.

Fracture mechanics studies mainly problems for cracks with traction-free faces. Rarely the problems for cracks with uniformly distributed tractions (internal pressure) or concentrated forces applied at their faces are considered. The solution of the problems of first kind are important in the analysis of stress strain state near the oil or gas layers (Kit et al., 2003; Evtushenko and Sulym, 1981), mechanical influence of hydrogen, which due to high permeability and mobility migrates into the material cavities, thus, creating there high pressure and debond adhesive joints in the material, increasing defects size. The solution of the second kind has not only the applied value, but also serve as Green's functions, thus allows deriving the solutions of the problems with loadings arbitrarily applied at crack faces.

The problem of crack faces contact, which account for contact interaction, is less studied. Main achievements in this direction belong to the theory of interface cracks in a bimaterial within the framework of 2D elasticity, which uses different models of local crack face contact for elimination of physically incorrect oscillating behavior (Comninou, 1977, 1980; Schmueser, 1980; Aravas and Sharma, 1991; Herrmann and Loboda, 1999; Kharun and Loboda, 2001, 2003; Sulym and Piskozub, 2004).

Kundrat (2003) developed the two-phase model, when between the plastic bands (first phase of prefracture zone) at the continuation of the axis of a thin inclusion there develops a short zone of crumby material (the second phase), where the shear stress of adhesive interaction linearly decrease from zero at the tip to the value at the beginning of the plastic band, thus removing the singularity of the solution.

Among other directions of the account of friction in crack face interaction one can refer to the publication of Cherepanov (1966). The referred paper formulates two types of boundary conditions for overlapping faces of the mathematical cut (crack), at which discontinuities of normal stress, displacement and tangent traction are defined, and mechanical interaction of opposite faces of the cut can be arbitrary.

Sekine (1982) studied mechanics of deformation of inclusion-stringer in the infinite isotropic medium, which is stretched along the axis of inclusion. It was assumed, that the central zone of a perfect contact of materials was accompanied at both sides with two-phase zones of imperfect contact, moreover, the first one addressed smooth contact (without friction), and that at the tip of the inclusion (the second one) addressed friction (here normal stress was compressive, and the material of a matrix in the contact zone of inclusion did not depart from it in the normal direction).

The wide range of problems on the influence of friction on contact stress between half-planes with superficial smooth surface notches were studied by Martynyak et al. (2000, 2005, 2007).

The growth of delaminating crack (actually mutual slippage of the materials) at the interface of rigid fibrous inclusion was studied by Brussat and Westmann (1974) with the account of friction between components. In this relation it is necessary to pay attention to the works by Antipov (1995), Arkhipenko and Kryvyy (2008), Ulitko and Ostryk (2002, 2006), Weertman et al. (1983).

The problem of crack faces contact accounting for friction

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is less studied in the case of antiplane shear (Sulym, 2007; Pasternak et al., 2010).

This paper presents the technique for studying the influence of friction during the out-of-plane deformation (antiplane problem) of a solid with a closed crack on the formation of slippage zones and energy dissipation in the case of quasi-static (slow) application of single monotonously increasing loading. In the general case of= loading type and position with respect to closed cracks one can assume that the loading at the beginning is low enough to cause slippage. Then it is enough to cause slippage, but the slippage zones do not reach the size of the cracks, and thus the stresses are finite at their ends. And finally for enough big loading the cracks' sizes bound the growth of slippage zones, and thus, at one or both of its tips stress singularity arises.

The discontinuity function method (Sulym et al., 2007, 2008, 2010) and the singular integral-differential equation approach are the base of the proposed technique. It is assumed that the loading applied to the piecewise-homogeneous medium with interface strip-like (tunnel) crack can be divided into two types: the first one satisfies the plane strain conditions, and the second one performs the out-of-plane deformation of a medium. Thus one can formulate two problems, one of which (the antiplane one) is further called the primary problem, and the second (the plane one) is auxiliary as it allows to determine the distribution of compressive traction at the contact surfaces of half-spaces and layers. Thus, due to the independent separation of in-plane and out-of-plane problems in the linear elasticity of isotropic materials, the solution of the primary problem does not influence the solution of auxiliary one.

2. PROBLEM STATEMENT

Consider the infinite isotropic matrix consisting of two half-spaces with the elastic constants $E_k,\ \nu_k,\ G_k\quad (k=1{,}2)$, which interface L contains N coaxial strip-like cracks. In this extent, it is possible to consider contact defects with different rheological properties. Everything depends on the models used for simulating their behavior (Sulym et al. 2004, 2007, 2008, 2010).

The fixed reference Cartesian coordinate system $Oxyz$ is considered, which xOz plane coincides with the material interface, and Oz axis is directed along the longitudinal axes of cracks.

Consider the stress strain state of the solid's cross-section with the xOy plane, which is perpendicular to the shear direction z. The cross-sections of the bimaterial perpendicular to the interface form two half-planes $S_k\ (k=1{,}2)$, and the material interface correspond to the abscissa $L{\sim}x$. The latter contains median lines of cracks' sections, which mold a line $L'=\bigcup_{n=1}^{N} L'_n=\bigcup_{n=1}^{N}[b_n^-;\ b_n^+]$ (Fig. 1). The application of similar traditional notation for an axis z and a complex variable $z=x+iy$ should not cause misunderstanding in the solution of the problem.

Contact between the bimaterial medium components along a line $L''=L\backslash L'$ is supposed to be mechanically perfect, and the contact along defects' (cracks') faces L' is assumed to be performed according to the laws of tangential mechanic contact, at which bodies contact mechanically perfect until the moment, when relative sliding of crack surfaces may start in some areas $\gamma_n\subset L'_n$ at the material interface (Johnson, 1985; Goryacheva, 2001; Sulym and Piskozub, 2004). Outside the linesγ_n the value of tangential traction at the places of the slippage absence does not exceed the level of maximal admissible traction, and the mutual displacement of crack faces is not observed (the displacement discontinuity $[w]_n$ is zero). The sign (an action direction) of tangent traction is chosen depending on a sign of a difference of displacement at both sides of a cut γ_n at the considered point. The following notations are used hereinafter: $[\phi]=\phi(x,-0)-\phi(x,+0)$, $\langle\phi\rangle=\phi(x,-0)+\phi(x,+0)$.

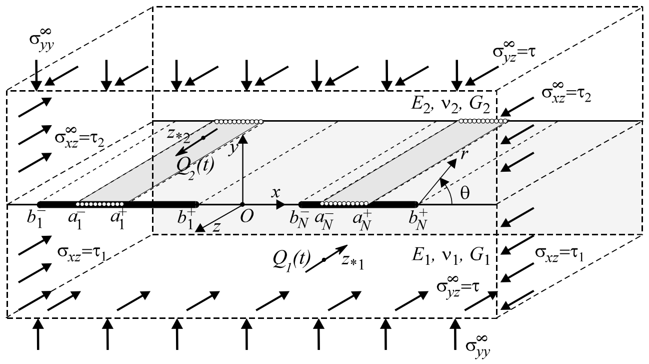

Fig. 1. The loading and geometric scheme of the problem

The medium is loaded with the mechanical load (stress at the infinity, the concentrated forces, etc.) such that their action causes the quasi-static stress strain state in a bimaterial solid. Simultaneously with the main loading, which defines the out-of-plane deformation, the medium is subjected to the additional in-plane (xOy) compressing loading, which causes the plane strain deformation. Its influence on the solution of the primary problem of longitudinal shear occurs only in the case when at the contact lines $\gamma_n\subset L'_n$ of crack faces contact the perfect mechanical contact is violated, where the normal stress σ_{yyk} is negative and the pressed surfaces of contacting materials move relatively to each other in the z direction.

The contact conditions with the possibility of slippage with friction at the closed crack provide that at the achievement by tangent traction σ_{yz} at the lines γ_n of a certain critical value $\tau_{yz}^{\max}$ the slippage occurs, and the tangent traction cannot exceed this threshold. Thus, within the classical Amontons' law of friction (Goryacheva, 2001; Johnson, 1985; Hills et al., 1993; Comninou, 1977, 1980), consider a variant of a contact problem according to which the tangent traction (friction traction) is constant along the lines γ_n:

$$\begin{aligned}&\sigma_{yzn}^{\pm}=-\operatorname{sgn}([w]_n)\tau_{yz}^{\max},\\&\tau_{yz}^{\max}=-\alpha\sigma_{yy}\ \left(\sigma_{yy}<0,\quad |w^- - w^+|\neq 0\right)'\end{aligned}\tag{1}$$

where α is a coefficient of dry friction. Outside the lines γ_n the tangent traction at the crack points without slippage does not exceed the possible admissible level

$$\left|\sigma_{yzn}\right|\le\tau_{yz}^{\max}\quad\left(\sigma_{yy}<0,\quad [w]_n=0\right)\tag{2}$$

and the mutual crack face displacement (displacement discontinuity) is absent. The sign (an action direction) of tangent traction is chosen depending on a sign of the difference of displacements $[w]_n$ at a considered point of γ_n. At those points of L'_n, where $\sigma_{yyk}\ge 0$, the classical conditions of a traction-free crack are formulated:

$$\sigma_{yzk}=0.\tag{3}$$

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Application of the law of friction in the form of (1) allows to simplify the boundary conditions for the primary problem, however, a choice of more difficult models of friction incorporating Eqs (1), (2) (Goryacheva, 2001; Sulym and Piskozub, 2004; Pasternak et al., 2010; Popov, 1966; Johnson, 1985; Hills et al., 1993), which account for wear and thermal emissions (Bogdanovich and Tkachuk, 2009; Pyrjev et al., 2012; Datsyshyn and Kadyra, 2006; Goryacheva et al., 2001), will not essentially complicate the solution process.

Following the approach of Refs (Panasyuk et al., 1976; Piskozub and Sulim, 2008; Sulym, 2007), consider that the influence of defects of contact is possible to model with the stress and displacement discontinuities at L'_n:

$$[\Xi]_{L'_n} \equiv \Xi^- - \Xi^+ = \mathbf{f}^n, \tag{4}$$

where $\Xi(z,t) = \left\{\sigma_{yy}, \sigma_{xy}, \sigma_{yz}, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial w}{\partial x}\right\}(z,t)$ is a state vector; $\mathbf{f}^n(x,t) = \{f_1^n, f_2^n, f_3^n, f_4^n, f_5^n, f_6^n\}(x,t)$ is a discontinuity vector; *t* is time as a formal monotonously increasing parameter related with the convertible force.

The solution of the auxiliary problem for the case of arbitrary mechanical loading and a perfect contact is presented by Panasyuk et al. (1976), Pasternak et al. (2010), Piskozub and Sulim (2008), Sulym (2007). To illustration the considered approach, consider its elementary applications, assuming that half-spaces are compressed by a uniform pressure

$$\sigma_{yyk} = -P \;\; (k = 1,2; \;\; x \in L). \tag{5}$$

For the primary problem with the account of Hook's law, Eq (4) writes as:

$$\begin{gathered}
\left[\sigma_{yz}\right]_{L'_n} \equiv \sigma_{yz}^- - \sigma_{yz}^+ = f_3^n(x,t), \\
\left[\frac{\partial w}{\partial x}\right]_{L'_n} \equiv \frac{\partial w^-}{\partial x} - \frac{\partial w^+}{\partial x} = \left[\frac{\sigma_{xz}}{G}\right]_{L'_n} \equiv \\
\equiv \frac{\sigma_{xz}^-}{G_1} - \frac{\sigma_{xz}^+}{G_2} = f_6^n(x,t), \;\; x \in L'_n; \\
f_3^n(x,t) = f_6^n(x,t) = 0, \text{If} x \notin \gamma_n.
\end{gathered} \tag{6}$$

Assuming that the magnitude and a direction of action of the mechanical loads, which perform longitudinal shear, change quasistatically (so slowly that there is no necessity to consider inertial terms) and change under the certain law, which can be arbitrary. Thus, external loading of the primary problem is defined by the stress $\sigma_{yz}^\infty = \tau(t), \sigma_{xz}^\infty = \tau_k(t)$ uniformly distributed at the infinity, the concentrated forces of intensity $Q_k(t)$ and screw dislocations with Burgers vectors $b_k(t)$ applied at the points $z_{*k} \in S_k (k = 1, 2)$. According to Eq (20.5) of (Sulym, 2007) stresses at the infinity at arbitrary time should satisfy the condition

$$\tau_2(t) G_1 = \tau_1(t) G_2, \tag{7}$$

which provides the straightness of the material interface at the infinity.

After the antiplane problem is solved accounting for friction and for each contact lines γ_n the displacement discontinuity is determined, it is possible to calculate the work of friction forces. This work, and hence, the energy dissipation at γ_n for a single step of change in external loading from zero to the maximum value, is calculated by means of the integrals

$$W_n^d = -\int_{\gamma_n} \tau^{\max} |(w^- - w^+)| dx. \tag{8}$$

3. THE PROBLEM SOLUTION

Applying the results of sec. 20.2 (Sulym, 2007) to the solution of the primary problem, one can obtain the following relations, which state that components of stress tensor and derivatives of displacement at the line L of infinite plane S, and inside the latter are equal to

$$\begin{gathered}
\begin{cases}
\sigma_{yz}^{\pm}(x,t) = \mp p_k f_3^n(x,t) - C g_6^n(x,t) + \sigma_{yz}^{0\pm}(x,t), \\
\sigma_{xz}^{\pm}(x,t) = \mp C f_6^n(x,t) + p_k g_3^n(x,t) + \sigma_{xz}^{0\pm}(x,t),
\end{cases} \\
(z \in S_k; \; r = 3,6; \; k = 1,2; \; j = 3-k); \\
g_r^n(z,t) \equiv \frac{1}{\pi} \int_{L'_n} \frac{f_r^n(x,t)dx}{x-z}, \\
p = \frac{1}{G_1 + G_2}, \; p_k = G_k p, \; C = G_1 G_2 p, \\
\sigma_{yz}(z,t) + i\sigma_{xz}(z,t) = \sigma_{yz}^0(z,t) + i\sigma_{xz}^0(z,t) + \\
+ i p_k g_3^n(z,t) - C g_6^n(z,t).
\end{gathered} \tag{9}$$

The superscript "+" corresponds to $k = 2$ and "- " corresponds to $k = 1$. The values denoted with superscript "0" characterize corresponding quantities in a continuous medium without cracks under the corresponding external loading (the homogeneous solution):

$$\begin{gathered}
\sigma_{yz}^0(z,t) + i\sigma_{xz}^0(z,t) = \tau(t) + i\{\tau_k(t) + \\
+ D_k(z,t) + (p_k - p_j)\overline{D}_k(z,t) + 2p_k D_j(z,t)\},
\end{gathered} \tag{10}$$

$$D_k(z,t) = -\frac{Q_k(t) + iG_k b_k(t)}{2\pi(z - z_{*k})} \quad \begin{pmatrix} z \in S_k, \\ k = 1,2; \;\; j = 3-k \end{pmatrix}.$$

Using Eqs (9), (10) and the condition (1) of presence of limiting friction force at the crack slippage lines γ_n under the mutual displacement of faces in the out-of-plane direction z (at other crack zones friction forces corresponds to the value defined by a condition of a perfect mechanical contact) one obtains the system of $2N$ singular integral-differential equations

$$\begin{cases}
f_3^n(x,t) = 0, \;\; (x \in L') \\
g_6^n(x,t) = \frac{1}{2C}\left(\langle\sigma_{yz}^0(x,t)\rangle + 2\operatorname{sgn}[w]\tau_{yz}^{\max}\right),
\end{cases} \tag{11}$$

which solution is known (Sulym, 2007).

For the detailed analysis of the solution of the problem consider a special case of presence of a single $(N = 1)$ crack (contact defect) with $L'_1 = [-b; \;\; b]$ and the slippage line arising at $\gamma_1 = [-a; \;\; a] \;\; (a \le b)$ under symmetric $(z_{*k} = \pm id)$ loading. The solution of the integral equation (11) after calculation of corresponding integrals is as follows

$$\begin{gathered}
f_6(x,t) = \frac{1}{\pi C\sqrt{a^2 - x^2}}\Big\{\pi\big(\tau(t) + \operatorname{sgn}[w]\tau_{yz}^{\max}\big)x + \\
+ \sum_{k=1}^{2} p_{3-k}\left(Q_k(t)\operatorname{Im}\frac{\sqrt{z_{*k}{}^2 - a^2}}{x - z_{*k}} + \right. \\
\left. + \left(G_k b_k(t)\operatorname{Re}\left(\frac{\sqrt{z_{*k}{}^2 - a^2}}{x - z_{*k}} + 1\right)\right)\right\} \;\; (x \in [-a; \;\; a]).
\end{gathered} \tag{12}$$

The function $X(z) = \sqrt{z^2 - a^2}$ is understand as a branch, satisfying the condition $\sqrt{z^2 - a^2}/z \to 1$ as $z \to \infty$. Similar reasoning is used for a choice of branches of functions $\sqrt{z_{*k}^2 - a^2}$ and $\sqrt{\bar{z}_{*k}^2 - a^2}$, $k = 1, \; 2$.

Expression for displacement discontinuity $[w]$ is received with

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integration of Eq (12):

$$[w](x,t)=\int_{-a}^{x} f_6(x,t)dx=-\frac{1}{C}(\tau(t)+ +\mathrm{sgn}[w]\tau_{yz}^{\max})\sqrt{a^2-x^2}+ +\frac{1}{\pi C}\{\sum_{k=1}^{2} p_{2-k}\,(Q_k(t)\mathrm{Im}I(x,z_{*k})+ +G_k b_k(t)\left(\pi+2\arcsin\frac{x}{a}+\mathrm{Re}I(x,a,z_{*k})\right))\} \quad (13)$$

where

$$I(x,a,z)\equiv\sqrt{z^2-a^2}\int_{-a}^{x}\frac{dx}{\sqrt{a^2-t^2}(x-z)}= =i\ln\frac{a(z-x)}{a^2-xz-i\sqrt{a^2-x^2}\sqrt{z^2-a^2}}. \quad (14)$$

Introducing the stress intensity factor (SIF) with the relation

$$K_3=\lim_{r\to o\,(\theta=0)}\sqrt{\pi r}\sigma_{yz}, \quad (15)$$

it is simple to obtain analytical expression for SIF in the case of a crack with a slippage line $\gamma_1=[-a;\ a]\ \ (a\le b)$:

$$K_3^{\pm}(t)=\frac{1}{2\sqrt{\pi a}}\int_{-a}^{a}\sqrt{\frac{a\pm x}{a\mp x}}\left(\langle\sigma_{yz}^{0}(x,t)\rangle+ +2\mathrm{sgn}[w]\tau_{yz}^{\max}\right)dx=\frac{1}{\sqrt{\pi a}}\{\pi a(\tau(t)+ +\mathrm{sgn}[w]\tau_{yz}^{\max})-\sum_{k=1}^{2}p_{2-k}\left(Q_k(t)\mathrm{Im}\frac{a\pm z_{*k}}{\sqrt{z_{*k}^2-a^2}}+ +b_k(t)G_k\mathrm{Re}\left(\frac{a\pm z_{*k}}{\sqrt{z_{*k}^2-a^2}}\mp 1\right)\right)\}. \quad (16)$$

Let's address the question on the size a of a slippage zone. In the course of increase in loading it is possible to allocate three phases essentially different from the point of view of the development of crack face slippage under the longitudinal shear:

1. The applied loading is still so small, that the condition (2) holds everywhere along L, i.e. slippage does not arise in general.
2. The applied loading is already sufficient for conditions (1) to hold at least at some small line $\gamma_1=[-a;\ a]\ \ (a\le b)$, but its size is less than established size $L'_1=[-b;\ \ b]$ of a crack. Loading at which the slippage first occurs is further named the first critical loading. While loading at a stage of the first phase of its change has not reached the first critical size, existence of a crack will not have any influence on the stress strain state of a solid. Everything occur the same as if two half-spaces pressed to each other are the object of research. In case of transition to the second loading phase, for determination of the size a of a slippage zone it is possible to use a condition of equality to zero of stress intensity factor given by Eq (22) in (Cherepanov, 1966).
3. If the loading increases such that the size of a crack (natural cohesive or adhesive forces of half-spaces outside the cracks) limits the slippage zone (in the absence of restrictions, a tends to exceed b), then at the crack tips singular stresses are present, and hence, SIF is nonzero. In this case the slippage zone γ_1 coincides with a crack L'_1. The minimal loading, for which the slippage line equals to the crack length thus initiating stress singularity at its tips, is further called the second critical loading.

Presence of the analytical solution for all parameters of stress strain state, and in particular, for SIF allows to calculate analytically the work of friction forces at the line L'_1 of slippage for any considered kind of loading. This work, and hence, energy dissipated on L'_1 due to change in external loading at certain time t is determined through the integral

$$W_1^d(t)=-\int_{-a}^{a}\tau_{yz}^{\max}|[w](x,t)|\,dx= =-\frac{\tau_{yz}^{\max}}{C}\left|\frac{\pi a^2}{2}\left(\tau(t)+\mathrm{sgn}[w]\tau_{yz}^{\max}\right)+ +\sum_{k=1}^{2}p_{2-k}\left(Q_k(t)\mathrm{Im}\left(\sqrt{z_{*k}^2-a^2}-z_{*k}\right)+ +G_k b_k(t)\mathrm{Re}\left(\sqrt{z_{*k}^2-a^2}-z_{*k}\right)\right)\right|. \quad (17)$$

Let's analyze expressions (12- 13), (16- 17) for the most indicative variant of loading by concentrated force $Q_2(t)$, which changes monotonously from zero to its maximum value $Q_{\max}$. The force is applied at the point $z_{*2}=id$ of an upper half-space. Then from expression

$$K_3(t)=-\sqrt{\pi a}\tau_{yz}^{\max}+\sqrt{\frac{a}{\pi}}\frac{p_1Q_2(t)}{\sqrt{a^2+d^2}} \quad (18)$$

accounting for the fact that $\mathrm{sgn}[w]=-1$ under the monotonous increase in loading, one obtains the slippage condition

$$Q_2(t)\ge Q_2^{*}=\frac{\pi d\tau_{yz}^{\max}}{p_1} \quad (19)$$

and the size of a slippage zone

$$\boldsymbol{a(t)}=\sqrt{\frac{\boldsymbol{p_1^2Q_2(t)^2}}{\boldsymbol{\pi^2\tau_{yz}^{\max 2}}}-\boldsymbol{d^2}}=\boldsymbol{d}\sqrt{\frac{\boldsymbol{Q_2(t)^2}}{\boldsymbol{Q_2^{*2}}}-\boldsymbol{1}}. \quad (20)$$

Hereinafter Q_2^{*} is the first critical value of the force, at which slippage starts at certain time t^{*}. Assuming that $a=b$ in (20) one can obtain the second critical value for the concentrated force loading, which induces nonzero SIF and stress singularity at crack tips,

$$Q_2^{**}=\frac{\pi\tau_{yz}^{\max}}{p_1}\sqrt{d^2+b^2}=Q_2^{*}\frac{\sqrt{d^2+b^2}}{d}. \quad (21)$$

Hence, accepting for a generality, that $Q_{\max}\ge Q_2^{**}$, the following values are obtained for the stress strain state parameters

for $Q_2(t)\le Q_2^{*}$:

$$f_6(x,t)=[w](x,t)=g_6(z,t)\equiv 0; \quad (22)$$

for $Q_2^{*}\le Q_2(t)\le Q_2^{**}$:

$$f_6(x,t)=\frac{x}{C\sqrt{a^2-x^2}}\left(-\tau_{yz}^{\max}+ +\frac{p_1Q_2(t)\sqrt{a^2+d^2}}{\pi(x^2+d^2)}\right), \quad [w](x,t)=\frac{p_1Q_2(t)}{2\pi C}\ln\frac{\sqrt{a^2+d^2}-\sqrt{a^2-x^2}}{\sqrt{a^2+d^2}+\sqrt{a^2-x^2}}+ +\frac{1}{C}\tau_{yz}^{\max}\sqrt{a^2-x^2}\ \ (|x|\le a); \quad (23)$$

$$W_1^d(t)=\frac{\pi a^2\tau_{yz}^{\max 2}}{2C}- -\frac{\tau_{yz}^{\max}}{C}p_1Q_2(t)\left(\sqrt{a^2+d^2}-d\right) \quad (24)$$

or accounting for Eq (19)

$$W_1^d(t)=-\frac{{p_1}^2}{2\pi C}(Q_2(t)-Q_2^{*})^2; \quad (25)$$

for $Q_2^{**}\le Q_2(t)\le Q_{\max}$ one should replace a with b in Eqs (18), (23) and (24).

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In case of identical materials of half-spaces ($G_1 = G_2 = G$) one should use in the abovementioned equations the following parameters: $C = G/2,\ p_1 = p_2 = 1/2$.

At a smooth contact between crack edges (friction coefficient is zero) one should assume that $\tau_{yz}^{\max} = 0$ in the abovementioned equations. In this case at arbitrary magnitude of the considered variants of loading the slippage zone instantly grows to the size of a crack.

For solution of the problem for various combinations of loading it is necessary to consider that superposition of solution for variants of loading can be not always used due to the nonlinearity of the problem considered.

4. THE NUMERICAL ANALYSIS

On an example of the aforementioned variant of loading let us illustrate the application of the proposed approach to determination of the size of a slippage zone, the displacement discontinuity at L'_1, and energy dissipation depending on the basic parameters of stress strain state (the magnitude and remoteness of the applied force, friction coefficient, material properties). Based on Eqs (20)–(25) the following dimensionless values are further considered: $\tilde{a} = a/b,\ \tilde{x} = x/b,\ \tilde{d} = d/b$, which are normalized length of slippage line, x coordinate and remoteness of a force application point, respectively; $\tilde{Q}_2(t) = Q_2(t)/\pi bP,\ \tilde{Q}_2^* = Q_2^*/\pi bP = \tilde{d}\alpha/p_1$, which are normalized magnitudes of acting force and the first critical force; and $\tilde{w}(x,t) = [w]C/bP,\ \tilde{W}_1^d = W_1^d C/\pi b^2 P^2, \tilde{K}_3 = K_3/\sqrt{\pi b}P$, which are normalized displacement discontinuity, dissipation energy and SIF, respectively.

Fig. 2 shows the dependence of the dimensionless length $\tilde{a}$ of a slippage zone on the ratio $\tilde{Q}_2(t)/\tilde{Q}_2^*$ in the range from zero to $\tilde{Q}_{\max}/\tilde{Q}_2^* \geq \tilde{Q}_2^{**}/\tilde{Q}_2^* = \sqrt{1+\tilde{d}^2}/\tilde{d}$ for various values of the remoteness parameter $\tilde{d}$. It is well noticed that with remoteness of a force application point together with natural increase in the first critical force the relative slippage zone growth rate also essentially increases.

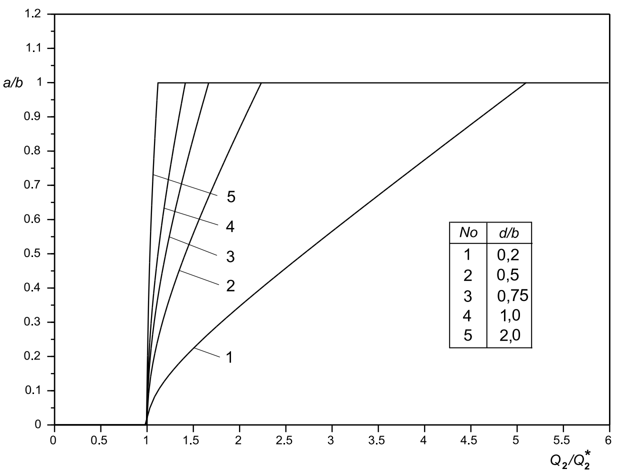

Fig. 2. Dependence of the size of a slippage zone on the loading parameter

Figs 3 and 4 depict the change of the displacement discontinuity function $\tilde{w}(x,t)$ depending on the x/b for various cases of change in the magnitude and remoteness of the applied force $\tilde{Q}_2(t)$, and friction coefficient α. Predictably, the increase in force magnitude leads to growth of displacements, and increase in α contrarily decrease them. Presence of the loading concentrated force renders considerably greater influence on the displacement discontinuity, if it is applied in the less rigid medium (Fig. 5).

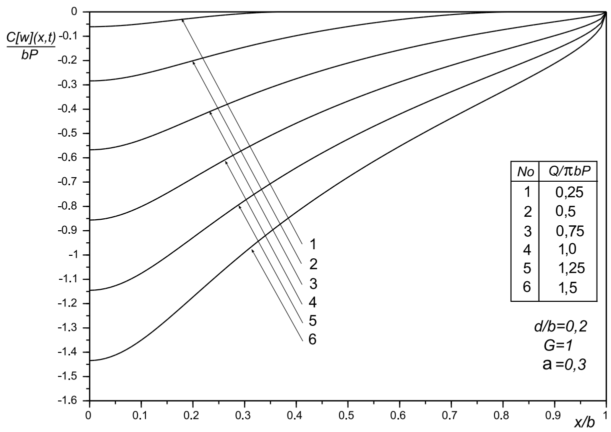

Fig. 3. Displacement discontinuity dependence on the magnitude of the applied force

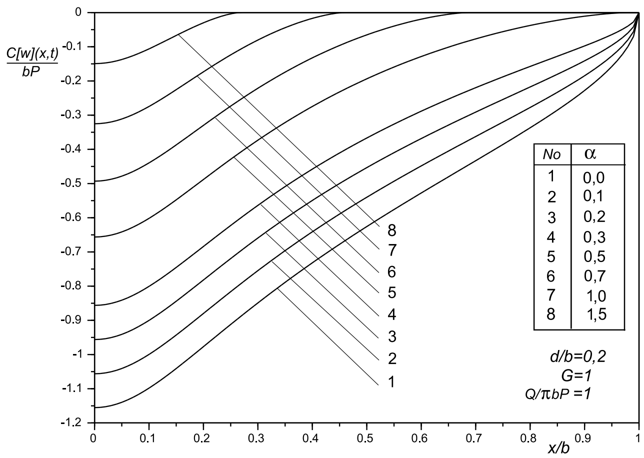

Fig. 4. Influence of friction coefficient on the displacement discontinuity

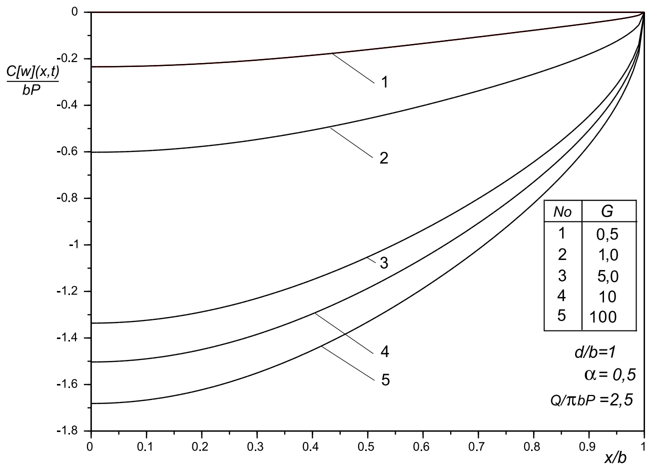

Fig. 5. Influence of shear modulus ratio of the materials on the displacement discontinuity

Figs 6 and 7 plot the dependence of the dissipated energy $W_1^d C/\pi b^2 P^2$ on the magnitude and remoteness of the applied force $\tilde{Q}_2(t)$ at various combinations of parameters G_1/G_2,d/b,

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and friction coefficient α. In the plots the continuous line corresponds to energy dissipation at the second stage of loading, and das-dot one corresponds to the third stage of loading. The general tendency of the energy dissipation change can be formulated as follows: the less is relative distance d/b of the force application point and the greater is the friction coefficient, the bigger is the energy dissipation.

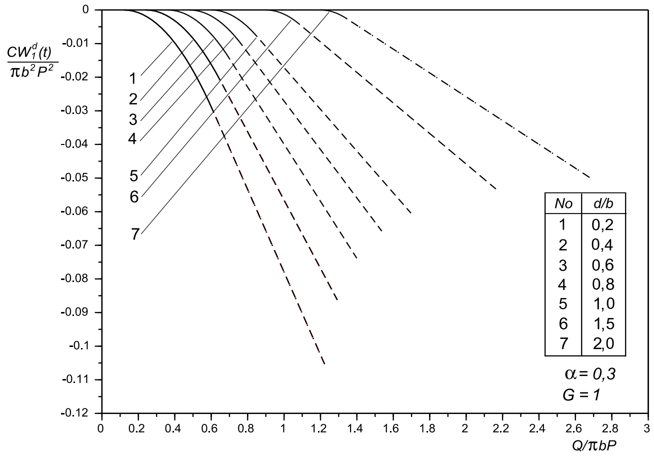

Fig. 6. Influence of the remoteness of the force application point on the energy dissipation

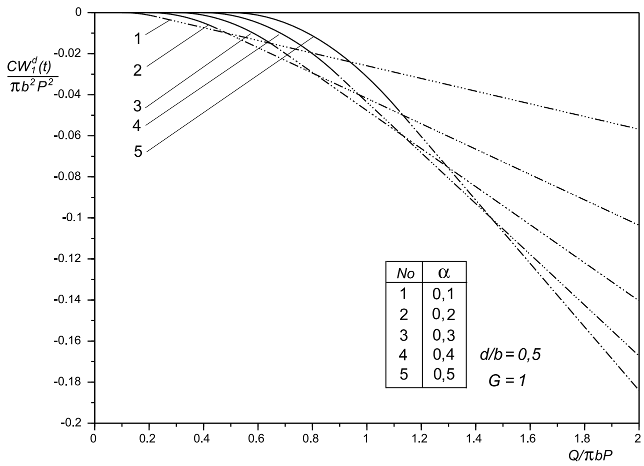

Fig. 7. Influence of the friction coefficient on the energy dissipation

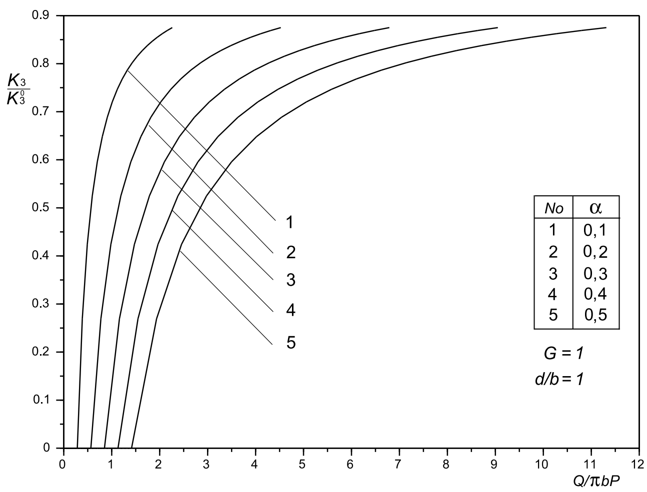

Fig. 8. Friction influence on SIF for a crack (here K_3^0 is SIF in the absence of friction)

Figs. 8-10 illustrate the influence of friction on the reduction of SIF K_3 arising at the third phase of loading, comparing to classic SIF K_3^0 calculated in the absence of friction. Dependence on the magnitude and remoteness of the applied force $\tilde{Q}_2(t)$ is studied for various combinations of parameters G_1/G_2 and d/b. The general qualitative tendency of influence of these parameters on SIF is the same as those for energy dissipation.

Doubtless interest of the further research is in the study of influence of friction slippage under the multiple, in particular cyclic, loading of a medium containing crack-like defects of contact at the interface.

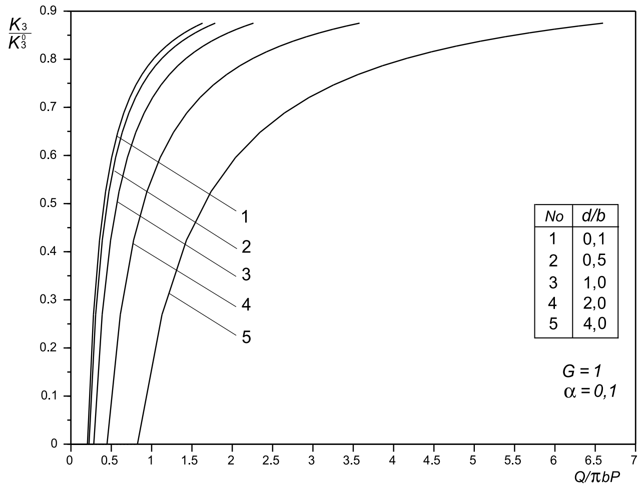

Fig. 9. Influence of remoteness of the force application point on SIF for a crack (here K_3^0 is SIF in the absence of friction)

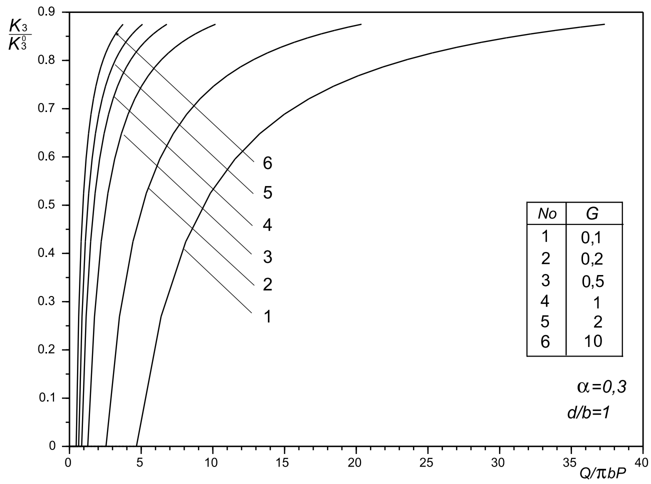

Fig. 10. Influence of shear modulus ratio of the materials on SIF for a crack (here K_3^0 is SIF in the absence of friction)

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