



## ANALYSIS OF THE MATHEMATICAL MODEL OF THE SPRAY DOSE APPLIED BY THE FIELD SPRAYER NOZZLES

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### ABSTRACT

The article presents analysis of the mathematical model for determination of a momentary dose of spray applied by the field sprayer nozzles which move on the curve with the forward speed the value of which may differ from the speed accepted for regulation. Regulation speed and regulation dose, real forward speed of a sprayer, angular velocity during the curve movement, and the coefficient of the nozzle location towards the axis of the sprayer turn are independent variables in the suggested model. Based on the mathematical model, plots were drawn and analyses of relation of the spray dose to particular variables were carried out including inter alia, a repeated field spray, application of a dose which considerably differs from the regulation dose and diversity of the dose on the working width of the sprayer.

## Introduction

A Pesticide spraying is the most often used and the most effective method of crop protection. Requirements of environment protection enforce application of plant protection substances in the dose compliant to the producer's recommendations (Langman and Pedryc, 2008). Proper use of sprayers is most often evaluated based on the level and regularity of application of working spray. One of the most significant factors which influence the regularity of the working spray distribution is the position of the field boom in comparison to the sprayed surface (Langenakens et al., 2000; Szewczyk, 2000). During working crossings of the field sprayer, a boom on which sprayers are placed performs unfavourable horizontal movements (Lardoux et al., 2007a; Lardoux et al., 2007b). Non-uniform boom speed and sometimes its several movements over the same surface of the field cause momentary changes of spray doses (Ramon et al., 1998; Antonis et al., 2000). It may lead to accumulation of the crop protection substance and local contamination, which is the main source of threat for environment (Doruchowski and Hołownicki, 2003). Due to the above reasons, optimization of solutions applied with regard to the reduction of horizontal fluctuations of the boom, which have been unfairly neglected so far is necessary (Lipiński et al., 2011).

During chemical protection treatment a sprayer should be operated linearly and with the constant speed. Based on the data obtained during control of the sprayer's operation which does not have the system of automatic pressure control it was found out that due to the

errors in operating the aggregate on the field with irregular shape, only 25% of the surface was covered with the assumed amount of spray (Garbiak and Jurga, 2015). According to DAMMAN company, in case the field sprayer with the working width of 36 m is used, the spray dose during chemical protection treatments is within 40% to 160% in comparison to the set value. It is caused mainly by irregular shapes of fields or avoiding various types of impediments (Curves Control Application).

According to many specialists, investigations on determination of the impact of the field boom location and aggregate movement on the distribution of spray and covering of the sprayed objects are recommended. Development of a mathematical model of spray distribution in relation to the assumed parameters in the variable conditions of sprayer work would be favourable (Szewczyk, 2009).

### The objective of the research

The objective of the research was the analysis of the methodological model of the spray dose applied by field sprayers which includes the regulation speed and dose, real forward speed of the sprayer, angular velocity during curve movement and coordinate of the sprayer location towards the axis of sprayer turn on the dose of spray applied by this sprayer. In order to realize the objective the following research question was formulated:

1. Does the nozzle speed towards the sprayer depend on the sprayer speed?
2. What impact on the spray dose applied by the sprayer nozzles have angle speed and speed of the sprayer during the curve if this speed depends on the speed assumed in order to regulate the dose?
3. Does the increase of doses of spray applied by nozzles which are located on one side of the symmetrical sprayer boom, which is induced by the curve movement, is balanced with an identical reduction of doses on the opposite side?
4. What impact on the spray dose applied by the sprayer nozzles have angular velocity and speed of the sprayer during the curve crossing if the sprayer speed depends on the speed accepted in order to regulate the sprayer dose?
5. At which values of independent variables the maximum values of doses occur?
6. At which values of independent variables a repeated field spray takes place?

### Methodology

Majority of field sprayers used in the country is not equipped with the system of automatic pressure control respectively to the present forward speed of the sprayer. The following mathematical model of the dose applied by nozzles in the function of particular variables was suggested for sprayers:

$$q = \frac{Q_{reg} \cdot v_{reg}}{v_m + \frac{\alpha r}{t}}, \quad (1)$$

where:

- $q$  – dose of spray applied by the sprayer sprinkler, ( $\text{dm}^3 \cdot \text{ha}^{-1}$ )
- $Q_{reg}$  – regulation dose, ( $\text{dm}^3 \cdot \text{ha}^{-1}$ )
- $v_{reg}$  - working speed of a sprayer assumed for regulation, ( $\text{m} \cdot \text{s}^{-1}$ )

- $v_m$  – real forward speed of a sprayer, ( $\text{m}\cdot\text{s}^{-1}$ )  
 $\alpha$  – turn angle of a sprayer, (rad)  
 $r$  – coordinate of the nozzle towards the turn axis of the sprayer, (m)  
 $t$  – turn time by angle  $\alpha$ , (s).

Since  $\alpha\cdot t^{-1}=\omega$ , where  $\omega$  is angular velocity, the model takes the following form:

$$q = \frac{Q_{reg}\cdot v_{reg}}{v_m + \omega\cdot r}. \quad (2)$$

Values of variables which are in the model meter (2) are determined by the user and do not change during the spraying process. The value of variables which are in the denominator may be subject to dynamic changes, the reason for which is not only the need to change the speed and the route of movement caused mainly by the field shape and the presence of field obstacles such as e.g. masts, trees or garden pounds, but resulting sometimes from a low level of training of the machine operator.

For the needs of this study, a working width of the sprayer of 20 m and the angular velocity were assumed during the change of the movement direction which do not exceed the value of  $0.2 \text{ rad}\cdot\text{s}^{-1}$ . When making plots indispensable for the analysis of the model, it was assumed that the turn angle of the sprayer to the left has a positive value and to the right a negative value. Looking in the direction of the sprayer movement, coordinate of the nozzle located on the longitudinal axis of the turn of the sprayer has a zero value, sprayers which are on the right from the axis have increasing coordinates with positive values expressed in metres. Coordinates of sprayers from the left side of the turn axis have negative values. In calculations a relatively low regulation dose of  $Q_{reg} = 100 \text{ dm}^3\cdot\text{ha}^{-1}$  was assumed which allowed a simple identification of the value of the dose applied by nozzles, not only in  $\text{dm}^3\cdot\text{ha}^{-1}$  but also in percentages of the regulation dose ( $Q_{reg}$ ).

The presented model (2) may be applied for determination of the dose of spray applied by the nozzles of the sprayer equipped with the pressure regulation system in relation to the real speed of the sprayer ( $v_m$ ). However, for such sprayers, the model assumes a slightly different form, i.e. in the numerator of the expression a real speed of the sprayer ( $v_m$ ) should be placed instead of the speed for which regulation was carried out ( $v_{reg}$ ).

## Results of analysis

Figure 1 presents plots of relation of the nozzle speed ( $v_d = \omega\cdot r$ ) to the coordinate ( $r$ ) of its location on the sprayer's boom. Plots were made for angular velocities within  $-0.15\div 0.15 \text{ rad}\cdot\text{s}^{-1}$  in the step whose value is  $0.05 \text{ rad}\cdot\text{s}^{-1}$ . It may be observed that in case when the forward movement speed of the sprayer ( $v_m$ ), as well as its angular velocity ( $\omega$ ) during the turn do not change, the course of the changes of the sprayer speed in relation to the coordinate ( $r$ ) its location on the sprayer beam is linear. In case of all curves, one may notice that the higher is the value of the angular velocity the higher is also the value of the slope of the straight line which defines the nozzle velocity ( $v_d$ ). For the pencil of curves which characterize the nozzle velocity ( $v_d$ ) at variable values of angular velocity the point whose abscissa is zero is the only common point and the ordinate has a value equal to the forward speed of the machine ( $v_m$ ).

Comparing curves presented in figure 1a and 1b concerning a uniform angular velocity, one may notice that for any identical coordinates ( $r$ ), changes in the nozzle speed towards the sprayer are the same although forward speeds of the sprayer ( $v_m$ ) are varied. At the same time, one may state that the relative speed of the sprayer and nozzle with the constant coordinate  $r \neq 0$  is higher if the absolute value of angular velocity of the sprayer is higher. At the constant angular velocity of  $\omega \neq 0$ , the mentioned absolute speed increases along with the increase of the absolute value of the location coordinate of a nozzle.

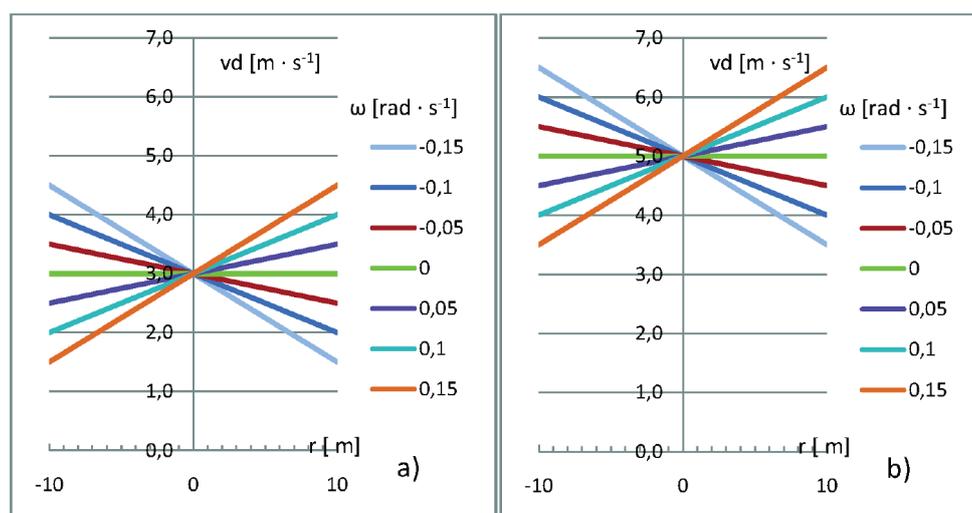


Figure.1. Relation of the nozzle speed ( $v_d$ ) towards the surface area from the location of the coordinate ( $r$ ) with selected angular velocities ( $\omega = -0.15; -0.1; -0.05; 0; 0.05; 0.1; 0.15 \text{ rad}\cdot\text{s}^{-1}$ ) and forward speeds of the sprayer a)  $v_m = 3 \text{ m}\cdot\text{s}^{-1}$ , b)  $v_m = 5 \text{ m}\cdot\text{s}^{-1}$

Figure 2 presents the relation of the dose ( $q$ ) to the coordinate ( $r$ ) of the nozzle location at selected angular velocities of the sprayer in a situation when the sprayer speed ( $v_m$ ) complies with the speed ( $v_{reg}$ ) accepted for the regulation of expense ( $Q_{reg}$ ). The presented plots may be also used for description of the dose ( $q$ ) for sprayers equipped with the system of automatic pressure control in the function of the sprayer speed ( $v_m$ ), within the range of the speeds which guarantee efficient regulation but in relation to the applied nozzles this scope may be varied. If the angular velocity is  $\omega \neq 0$ , the course of changes of the dose ( $q$ ) in relation to the coordinate of the nozzle location ( $r$ ) is similar to the course of the exponential function. The point whose abscissa is zero is a common point of crossing of the pencil of all curves which define the relation of the dose to the coordinate of the nozzle location at variable values of the angular velocity and the coordinate has a value compliant to the regulation dose value ( $Q_{reg}$ ). For nozzles with a uniform coordinate the change of the spray dose is higher if the absolute value of the angular velocity is higher. In case of each curve which characterizes the dose ( $q$ ) at the angular velocity  $\omega \neq 0$ , the increase of this dose above the regulation dose ( $Q_{reg}$ ) for the nozzle with any coordinate  $r \neq 0$  is higher than the decrease for

the nozzle with the coordinate ( $-r$ ). When comparing the plots for two varied regulation speeds ( $v_{reg}$ ) of the sprayer one may state that the doses ( $q$ ) applied by nozzles with the identical coordinate ( $r$ ) and the same value of angular velocity ( $\omega$ ), the higher is the regulation speed ( $v_{reg}$ ) the less they differ from the regulation dose ( $Q_{reg}$ ).

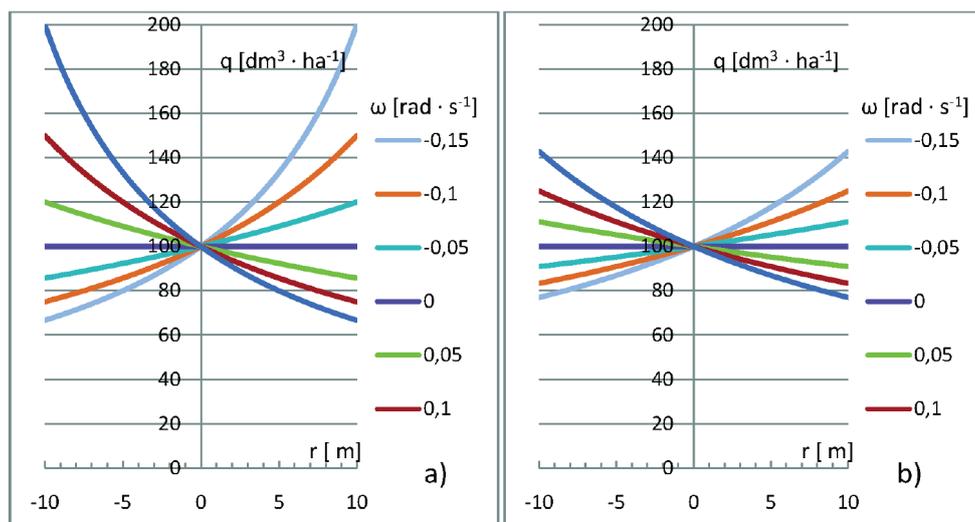


Figure. 2. Relation of the dose ( $q$ ) to the coordinate ( $r$ ) of location of a nozzle with the selected angular velocities of a sprayer ( $\omega$ ); regulation dose  $Q_{reg}=100 \text{ dm}^3 \cdot \text{ha}^{-1}$ ; a)  $v_{reg}=v_m=3 \text{ m} \cdot \text{s}^{-1}$ ; b)  $v_{reg}=v_m=5 \text{ m} \cdot \text{s}^{-1}$

The model behaves slightly different if  $v_{reg} \neq v_m$ . Figure 3 presents the relation of the dose ( $q$ ) to the coordinate ( $r$ ) of the nozzle location at selected angular velocities of the sprayer in a situation when the sprayer speed ( $v_m$ ) does not comply with the speed ( $v_{reg}$ ) accepted for the regulation of expense ( $Q_{reg}$ ). It was found out that the course of dose changes ( $q$ ) in relation to the coordinate of the nozzle location ( $r$ ) is similar to the course of the exponential function. The point whose abscissa is zero is a common point of crossing of the pencil of all curves which define the relation of the dose to the coordinate of the nozzle location at variable values of the angular velocity and the coordinate has a value different than the regulation dose value ( $Q_{reg}$ ) which is  $q=Q_{reg} \cdot v_{reg} \cdot v_m^{-1}$ . If the speed of the sprayer is lower than the speed accepted for the regulation dose, then the mentioned value ( $q$ ) is higher than ( $Q_{reg}$ ) (fig.3a), and if the machine speed is higher, the value  $q$  is lower than  $Q_{reg}$  (fig.3b). At the constant speed of the sprayer ( $v_m$ ), for the nozzle with the coordinate ( $r$ ) the spray dose ( $q$ ) is higher if the absolute value of angular velocity is higher. When comparing the plots for two different speeds of the movement ( $v_m$ ) of a sprayer (fig. 3a and fig. 3b) one may state that the lower is this value the higher are doses ( $q$ ) of spray applied by nozzles with an identical coordinate ( $r$ ) and uniform value of the angular velocity ( $\omega$ ). Moreover, the difference between extreme values of doses of spray applied by nozzles with coordinates ( $r$ ) and ( $-r$ ) is higher. Furthermore, for the curve which defines the dose ( $q$ ) at the angular velocity

$\omega \neq 0$ , the increase of the spray dose, caused by the curve movement, exceeding the regulation dose value ( $Q_{reg}$ ) for the sprayer with the coordinate ( $r \neq 0$ ) is higher than the reduction of the spray dose which corresponds to the nozzle located on the opposite side of the field boom i.e. with the opposite coordinate.

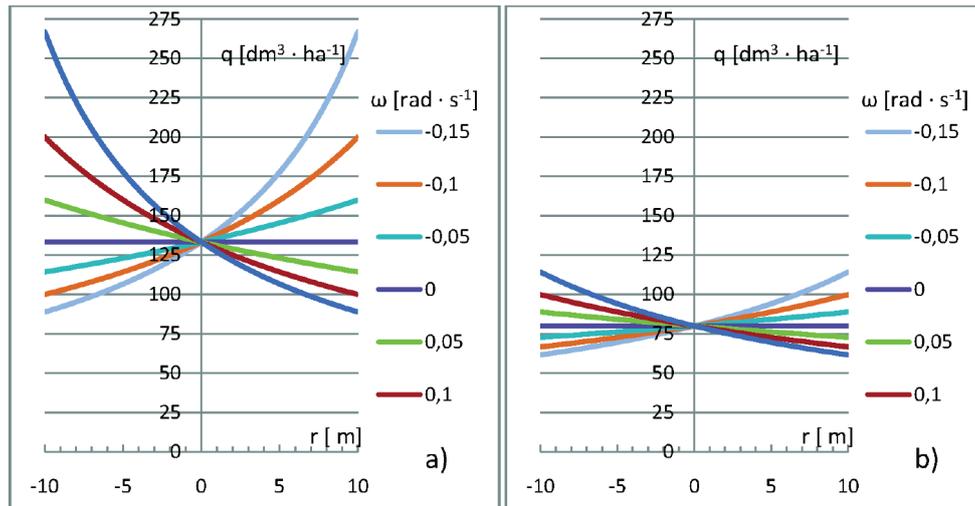


Figure 3. Relation of the dose ( $q$ ) to the coordinate ( $r$ ) of the nozzle location at the selected angular speeds of the sprayer ( $\omega$ ); regulation dose  $Q_{reg}=100 \text{ dm}^3 \cdot \text{ha}^{-1}$ ,  $v_{reg}=4 \text{ m} \cdot \text{s}^{-1}$ : a)  $v_m=3 \text{ m} \cdot \text{s}^{-1}$  b)  $v_m=5 \text{ m} \cdot \text{s}^{-1}$

Figure 4 presents the relation of the dose ( $q$ ) to all variables which would be changed during the spray process. The presented plots concern two variable speeds ( $v_{reg}$ ) accepted for the regulation of the dose ( $Q_{reg}$ ). There is a product ( $\omega \cdot r$ ) on the horizontal axis which characterizes the nozzle speed towards the sprayer and on the vertical axis of the sprayer speed ( $v_m$ ). The variability of the dose ( $q$ ) was presented on the surface plot which covers 9 ranges from 0 to 450  $\text{dm}^3 \cdot \text{ha}^{-1}$  with the step of 50  $\text{dm}^3 \cdot \text{ha}^{-1}$ . Moreover, one of the ranges includes the higher dose from 50  $\text{dm}^3 \cdot \text{ha}^{-1}$  and the range marked with 2X symbol illustrates on the plot the repeated spray of the field surface. Omitting the area of the plot which concerns the repeated spray of the field, the dose decreases when the values ( $\omega \cdot r$ ) and ( $v_m$ ) are higher. When comparing the plots made for two different speeds ( $v_{reg}$ ) accepted for regulation of the spray dose ( $Q_{reg}$ ) one may state that the width of zones which include ranges of particular doses is higher on the plot on the right namely concerning the higher speed ( $v_{reg}$ ). It means that assuming higher speed ( $v_{reg}$ ) for regulation of the dose ( $Q_{reg}$ ) causes lower dynamics of the dose changes ( $q$ ) as a result of speed changes of the sprayer and nozzle speed. A repeated spray of the field on both plots covers the same scope of values. A rectilinear border of this area runs from the point with coordinates (0.0) to the point (-2.2) namely the border is marked by all points where the absolute nozzle speed ( $\omega \cdot r$ ) with the turn contrary to the direction of the sprayer movement balances the sprayer movement speed ( $v_m$ ).

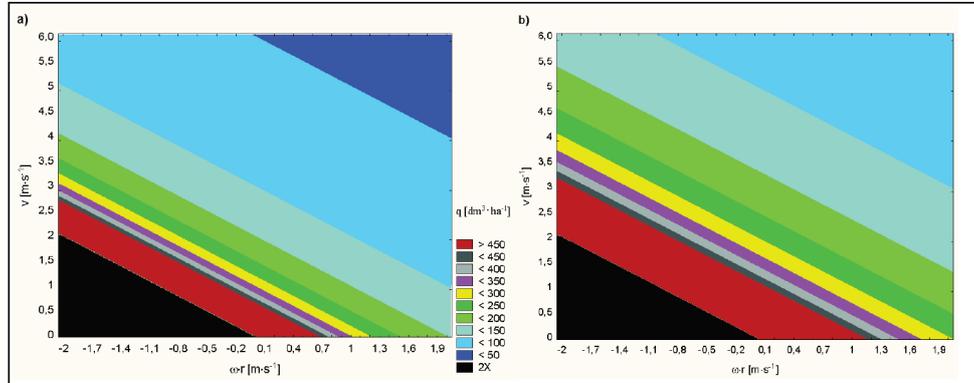


Figure 4. Relation of the dose ( $q$ ) to the forward speed of the sprayer ( $v_m$ ) and the quotient of the coordinate ( $r$ ) of the nozzle location and the angular velocity of the sprayer ( $\omega$ ); the regulation dose  $Q=100 \text{ dm}^3 \cdot \text{ha}^{-1}$ , a)  $v_{reg}=3 \text{ m} \cdot \text{s}^{-1}$ , b)  $v_{reg}=5 \text{ m} \cdot \text{s}^{-1}$

Since the doses presented in figure 4 include mainly ranges of the step which is  $50 \text{ dm}^3 \cdot \text{ha}^{-1}$ , it is purposeful to present a precise course of the dose variability ( $q$ ) in relation to the nozzle speed ( $\omega \cdot r$ ) towards the sprayer at the constant forward speed ( $v_m$ ). In order to illustrate the occurrence and the course of doses ( $q$ ) concerning the repeated spray of the field, the forward speed of a machine which is  $1.5 \text{ m} \cdot \text{s}^{-1}$  was accepted, i.e. two times lower than the regulation speed ( $v_{reg}$ ). On the presented plot (fig. 5) the lowest value of dose ( $q$ ) occurs at the highest absolute speed of the nozzle with the turn compliant to the turn of the sprayer speed vector, when the nozzle speed towards both the field and the sprayer is the highest. When the positive value of the product ( $\omega \cdot r$ ) decreases, namely when the nozzle speed towards the sprayer decreases, the value of the dose increases. At the nozzle speed towards the sprayer, which is zero, the dose according to the model has the value of ( $q=Q_{reg} v_{reg} v_m^{-1}$ ). Negative values of the product ( $\omega \cdot r$ ) mean the nozzle movement towards the field with the speed lower than the machine speed. When the positive value of expression ( $v_m + \omega \cdot r$ ) drops and gets closer to zero, the spray doze reaches the maximum value. In such case, the real value of the dose will at the constant expense of spray depend on the time the nozzle stays over the same fragment of a field and on the size of the area, on which spray is applied. In case of nozzles which have a flat, fan stream, the dose applied by the nozzle with the slight speed towards the surface of the field may many times exceed the regulation dose ( $Q_{reg}$ ). Further decrease of the nozzle speed ( $\omega \cdot r$ ) causes the change of the expression sign ( $v_m + \omega \cdot r$ ) into a negative one, which means that the nozzle comes back to the previously sprayed surface. Negative expression sign ( $v_m + \omega \cdot r$ ) causes the change of the dose sign ( $q$ ) defined by the model (2) which means the repeated field spray. The higher is the speed of reverse of the nozzle towards the field surface, namely the lower is the value of expression ( $v_m + \omega \cdot r$ ), the repeated dose becomes lower. It should be also noticed that the decrease of the dose at the repeated spray occurs only after the abscissa value ( $v_m + \omega \cdot r$ ) goes through the asymptote zone (vertical line on figure 5) in the neighbourhood of which the highest values of the dose of spray applied by the nozzle occur.

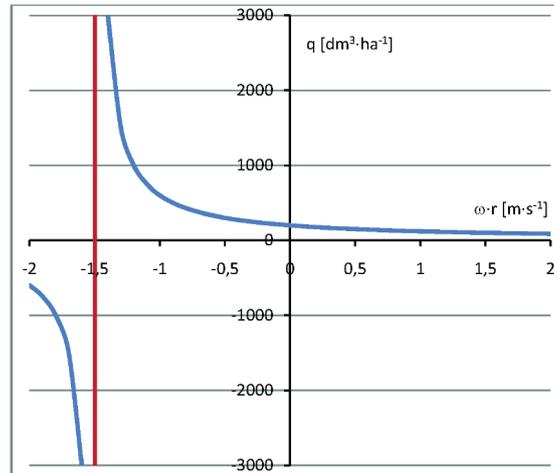


Figure. 5. Relation of the dose ( $q$ ) to the quotient coordinate ( $r$ ) of the nozzle location and the angular speed of the sprayer ( $\omega$ ); regulation dose  $Q_{reg}=100 \text{ dm}^3\cdot\text{ha}^{-1}$ ,  $v_{reg}=3 \text{ m}\cdot\text{s}^{-1}$ ,  $v_m=1,5 \text{ m}\cdot\text{s}^{-1}$

## Conclusions

Based on the analyses which were carried out, the following conclusions were formulated:

1. Relative nozzle and sprayer speed does not depend on the sprayer speed but on the nozzle coordinate and the angular speed.
2. If the sprayer movement speed complies with the regulation speed, this dose in the turn axis during the movement on the curve equals to the regulation dose but differentiation of the dose on the entire working width of the sprayer is higher if the speed of the sprayer movement is lower and the angular speed is higher.
3. During the curve movement of the sprayer, the increase of the dose of spray applied by nozzles whose speed towards the surface is lower than the forward speed of the sprayer, is higher than the reduction of doses of spray applied by nozzles with the speed higher than the sprayer speed.
4. If the sprayer movement speed does not comply with the speed accepted for regulation of the sprayer dose, this dose in the turn axis during the curve movement is different than the regulation dose but the range of the dose value on the entire working width of the sprayer is higher if the speed of the sprayer movement is lower and the angular speed is higher.
5. The highest dose of spray occurs when the nozzle speed towards the sprayer is close to the forward speed of the sprayer and senses of both speeds are opposite.
6. The repeated spray of the field surface occurs when the nozzle speed towards the sprayer is higher than the forward speed of the sprayer and senses of both speeds are different.

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## ANALIZA MODELU MATEMATYCZNEGO DAWKI OPRYSKU APLIKOWANEGO PRZEZ ROZPYLACZE OPRYSKIWACZA POLOWEGO

**Streszczenie.** W artykule przedstawiono analizę modelu matematycznego do wyznaczania chwilowej dawki oprysku aplikowanego przez rozpylacze opryskiwacza polowego poruszającego się po łuku z prędkością postępową, której wartość może odbiegać od prędkości przyjętej do regulacji. W proponowanym modelu zmiennymi niezależnymi są prędkość regulacyjna i dawka regulacyjna, rzeczywista prędkość postępową opryskiwacza, prędkość kątowna podczas ruchu po łuku i współrzędna położenia rozpylacza względem osi skrętu opryskiwacza. Na podstawie modelu matematycznego sporządzono wykresy i przeprowadzono analizy zależności dawki oprysku od poszczególnych zmiennych, z uwzględnieniem m.in. powtórnego oprysku pola, aplikowania dawki znacznie odbiegającej od dawki regulacyjnej oraz zróżnicowania dawki na szerokości roboczej opryskiwacza.

**Słowa kluczowe:** model, opryskiwacz, dawka, prędkość postępową, prędkość kątowną