Fuzzy adaptive Kalman filter for the drive system with an elastic coupling

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Abstract: In the paper issues related to the design of a robust adaptive fuzzy estimator for a drive system with a flexible joint is presented. The proposed estimator ensures variable Kalman gain (based on the Mahalanobis distance) as well as the estimation of the system parameters (based on the fuzzy system). The obtained value of the time constant of the load machine is used to change the values in the system state matrix and to retune the parameters of the state controller. The proposed control structure (fuzzy Kalman filter and adaptive state controller) is investigated in simulation and experimental tests.

Key words: adaptive KF, two-mass drive, fuzzy logic, robust estimation

1. Introduction

Torsional oscillations in mechanical couplings are one of the most significant issues which restrict the performance of speed control loops in industrial drives and servo systems. This is especially evident in multi-mass systems, in which the load (machine) is connected to a driving motor through one or multiple flexible shafts. During transient operation (e.g. start-up), large inertias of the motor-load system and shaft elasticity greatly influence the torque transmission properties of the drive system and result in increased angular vibrations of the shaft. Excessive shaft twists and poorly damped torsional vibrations are detrimental to the drive’s performance greatly compromising product quality and system reliability, and in some cases leading to instability and failure of the entire drive system. This problem commonly occurs in rolling-mill drives, belt-conveyors, paper machines, robotic-arm drives including space manipulators, servo-drives and throttle systems [1]-[12].

Many control structures have been proposed in order to improve the torsional characteristics of two-mass drives, for instance sliding mode control, PI(D) with additional feedback loops, resonance-ratio control, digital filters, robust and adaptive control [4-18]. The control problem becomes especially difficult when the parameters of the plant change with time or
with the change of the operation point. The mentioned situation occurs, for instance, in robot-arm drives, where the moment of inertia changes according to the movement of the arm.

The most advanced control structures, ensuring very good performance of the system, are based on control structures with additional feedbacks from all system state variables: torsional torque, load speed and disturbance torque simultaneously. Still, the direct feedbacks from these signals are very often impossible, because additional measurements of these mechanical variables are difficult, costly and, what is more important, they deteriorate the reliability of the system. Therefore, special systems for estimation of these variables are necessary [19].

In many papers the Luenberger observers have been applied to the non-measurable state variables reconstruction. In linear systems with non-changeable parameters and small values of measurement noises, this estimator can ensure good accuracy of the estimated states. Nevertheless, changes or improper identification of the plant parameters require a bigger value of the observer gains, which accelerates the convergence between the real and estimated states. However, contrary to the case above, in the system with measurement noises there is a need for reducing the observer gains to avoid oscillations of the estimated state variables [19].

For systems with high values of parameter and measurement noises the Kalman filter (KF) can ensure good estimation accuracy, as opposed to the Luenberger Observer. It results from the fact that KF algorithm relies directly on the parameters and measurement noises. One of the drawbacks of KF is its complicated algorithm which result in quite a long computational time. It means that practical implementation of a KF algorithm requires a faster processor than for a Luenberger observer. The classical KF is said to be robust to parameter noises, yet a change of measurement or system noises affects its performance. Therefore, advanced solutions based on the adaptation of the KF parameters are sought after [20-23].

In recent years, interest in estimation methods based on artificial intelligence has increased [24, 25]. The most common approach is using different structures of neural networks and their combination with fuzzy logic. Estimators of this type do not require knowledge of the mathematical model of the object, only the data vectors relevant for the training process. However, these estimators are not popular in industrial applications.

In this paper a fuzzy KF for the plant with parameters and measurements noises is proposed. The paper is divided into six sections. After a short introduction the mathematical model of the drive and the control structure are described briefly. Then the proposed adaptive fuzzy KF is introduced. Next the information related to the simulation and experimental studies are presented. A short conclusion is submitted at the end of the paper.

2. Mathematical model of the drive and the control structure

In this work, a commonly used mathematical model of the drive system with an elastic joint is used (in p.u. system):

$$T_i \frac{d\omega_i}{dt} = m_v - m_s,$$  \hspace{1cm} (1)
\[ T_2 \frac{d\omega_2}{dt} = m_s - m_L, \]  
\[ T_c \frac{dm_s}{dt} = \omega_1 - \omega_2, \]

where: \( \omega_1, \omega_2 \) – motor and load speeds, \( m_s, m_s, m_L \) – electromagnetic, shaft and load torques, \( T_1, T_2 \) – mechanical time constant of the motor and the load machine, \( T_c \) – stiffness time constant.

In this paper a structure with a state controller which allows free location of the closed-loop poles is considered. The control law is defined as follows:

\[ m_{\text{ref}}^e = k_1 \int e dt + k_1 \omega_1 + k_2 m_s + k_3 \omega_2, \]

where: \( m_{\text{ref}}^e \) – reference value of the electromagnetic torque (control signal), \( e \) – is a regulation error and \( k_1, k_2, k_3 \) – are state controller coefficients in the feedbacks from motor speed, shaft torque and load speed respectively.

The parameters of the control structure are set with the use of the pole-placement method, with the general methodology presented in [19], according to the following equations

\[ k_1 = T_1 T_2 T_c \omega_1^4, \]
\[ k_1 = 4T_1 \omega_1 \omega_c, \]
\[ k_2 = T_1 T_2 \left( 2\omega_1^2 + 4\omega_1^2 \omega_c^2 - \frac{1}{T_2 T_c} - \frac{1}{T_1 T_c} \right), \]
\[ k_3 = k_1 \left( \omega_1^2 T_1 T_2 - 1 \right). \]

All coefficients in (5)-(8) depend on the parameters of plant. Changing the parameters results in deterioration of the dynamic properties of the plant and hence transients of the system state variables are different from desired. The proposed control structure requires information of all system states because the direct feedbacks from shaft torque and load speed are very often impossible to obtain in industrial drives.

In order to ensure the robustness of the system state transients an adaptive control structure working with the fuzzy KF is proposed. The block diagram of the considered system is shown in Figure 1. The proposed control structure consists of two main loops, namely the inner (electromagnetic torque control) and outer (speed) loop.

The electromagnetic torque control loop consists of a torque controller (usually PI), power converter, electromagnetic part of the motor and a current sensor. The parameters of the torque controller are selected in such a way that they ensure an adequately quick torque regulation, usually with the help of the modulus criteria. The transfer function of the torque controller compensates the delay caused by the electromagnetic part of the motor. The delay caused by the power converter and current sensor, on the other hand, are uncompensated. The
latter is very small and is usually neglected, so the main uncompensated element is the power converter (based on the IGBT transistors). The delay of these switches can be calculated as the inverse of the switching frequency which in the present study is set to 10 kHz. It should be noted that in many cases the delay caused by the hardware of the system can be smaller than the sampling time. In the presented study the hardware delay has the value of 100 μs, while the sampling time is equal to 500 μs.

The speed control loop comprises a state controller, optimized electromagnetic torque control loop, mechanical part of the drive, speed sensor and fuzzy adaptive KF. The coefficients of the state controller (5)-(8) are retuned according to the value of the time constant of the load machine estimated by KF. The exact description of the fuzzy adaptive KF is given in the next section.

Classical KF

According to the KF theory, the system is disturbed by Gaussian white noises, which represent process and measurement errors \( w(t), v(t) \). The system is described by Equation (9):

\[
\begin{align*}
\dot{x}(t) & = Ax(t) + Bu(t) + w(t) \\
y(t) & = Cx(t) + v(t),
\end{align*}
\tag{9}
\]

where \( x(t), y(t) \) are the input and the output vectors, \( A, B, C \) are the state, input (control) and output matrices of the system and \( w(t), v(t) \) – represent process and measurement errors.

3. Adaptive Fuzzy Kalman filter

The main part of the adaptive control structure is a KF, which estimates the system state variables and the values of the time constant of the load machine. A block diagram of the adaptive fuzzy KF is presented in Figure 2.

In the estimation system presented in Figure 2 three main parts can be distinguished, as presented below.

Classical KF

According to the KF theory, the system is disturbed by Gaussian white noises, which represent process and measurement errors \( w(t), v(t) \). The system is described by Equation (9):
After discretization of Equation (8) with $T_s$ sampling step, the state estimation using a KF algorithm is calculated by equation:

$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k) + K(k+1)[y(k+1) - C(k+1)\hat{x}(k+1/k)],$$

where the gain matrix is obtained by the following numerical procedure:

$$P(k+1/k) = A(k)P(k/k)A(k)^T + Q(k)$$

$$K(k+1) = P(k+1/k)C(k+1)^T[C(k+1)P(k+1/k)C(k+1)^T + R]^{-1}$$

$$P(k+1/k+1) = [P(k+1/k) - K(k+1)C(k+1)]P(k+1/k),$$

where $K$ is KF gain, $P$ is a estimate covariance, $I$ is a identity matrix, $Q$, $R$ are covariance matrices. The parameters $Q$, $R$ and $A$ are retuned by the fuzzy adaptation mechanism.

**Fuzzy system used for estimation of the time constant of the load machine**

The input signals of the fuzzy system are the load torque and the speed error. During the start-up and reversal the mechanical time constant of the load machine is estimated on the basis of the above-mentioned signals. The estimation error of the load torque is negative when the real time constant $T_2$ is bigger than set in the state matrix. The error is positive for a smal-
ler $T_2$ evident in the estimator. In order to avoid the activation of the system for small fluctuation of the speed caused, for example by rapid changing of the load torque, the dead-zones are used.

The input value of the fuzzy system is described as follows:

$$er = \frac{1}{t_2} \left[ D_2 \left( m^e_{T} \right) \left( \omega^{ref} - \omega_1 \right) k_T \cdot \tilde{k}_T \right],$$

where: $D_1()$ – indicates the dead zone, $m^e_{T}$ – error of the estimated load torque, $\omega^{ref}$ – reference speed, $\omega_1$ – motor speed, $k_T$ – fuzzy gain, $t_1$ and $t_2$ – starting and stopping time of the algorithm.

The values calculated according to Equation (11) and the reference speed are the input signals of the fuzzy system. Each input has three triangular membership functions (Fig. 4a, b) and the fuzzy system has four outputs with membership functions presented in Figure 3c, d, e, f. Parameters of the fuzzy system are obtained by trial and error methodology.

![Fig. 3. The input (a, b) and output (a, d, e, f) membership functions of the fuzzy system](image)

The fuzzy system is based on the rules with the following form:

$$R_1: \text{IF } er \text{ is } A^e_{1} \text{ AND } \omega^{ref} \text{ is } A^i_{2} \text{ THEN }$$

$$T^e_{2} \text{ is } B^h_{1} \text{ AND } k_T \text{ is } B^i_{2} \text{ AND } Q(T_2) \text{ is } B^e_{3} \text{ AND } R(T_2) \text{ is } B^i_{4},$$

where: $A_{1, 2} –$ input sets, $A_{1, 2, 3, 4} –$ output sets, $T_2 –$ estimated mechanical time constant, $k_T –$ gain, $Q(T_2), R(T_2) –$ the value of the covariance matrices.

The fuzzy system has 18 rules and the relationship between the inputs and four system outputs is presented in Figure 4.
Fuzzy adaptation of the KF gain

The third part of the estimator is responsible for adaptation of the KF gain (the system is shown in red in Figure 2). In order to calculate the level of the estimation error the Mahalanobis distance is used [22]. Mahalanobis distance is based on correlations between variables by which different patterns can be identified. It takes into account the variability of the components during determination of its distance to the corresponding center. For two vectors \( \mathbf{X} = (x_1, x_2, x_3, ..., x_n) \) and \( \mathbf{Y} = (y_1, y_2, y_3, ..., y_n) \) the Mahalanobis distance is defined as:

\[
D^2(x, y) = (x_i - y_i)^T \mathbf{P}^{-1} (x_i - y_i),
\]

where: \( x, y \) – two input vectors, \( \mathbf{P} \) covariance matrix.

The affinity between the real and estimated vectors is represented by the following equation:

\[
D^2 = \begin{pmatrix}
\omega_1^T & \omega_2^T \\
\omega_1 & \omega_2 \\
m_1 & m_2 \\
m_L & m_T
\end{pmatrix} \begin{pmatrix}
p_{11} & 0 & 0 & 0 \\
0 & p_{22} & 0 & 0 \\
0 & 0 & p_{33} & 0 \\
0 & 0 & 0 & p_{44}
\end{pmatrix} \begin{pmatrix}
\omega_1 \\
\omega_2 \\
m_1 \\
m_T
\end{pmatrix} = \begin{pmatrix}
\omega_1^T \\
\omega_2^T \\
m_1^T \\
m_T^T
\end{pmatrix}
\]

\[
(14)
\]
Because only the motor speed is known it can be written as follows:

\[ D^2 = p_{11}(\omega - \omega_i)^2. \]  \hfill (15)

The factor (14) depends on the level of measurement noises. For the operation-point dependent disturbances, it should be calculated in the shifted window. Additionally, its value is scaled with relation to the KF gain:

\[ \bar{D}^2 = \frac{1}{K_{\text{gain}}} \left\{ \frac{1}{N} \sum_{n=1}^{N} [D^2(k) + D^2(k - n)] \right\}. \]  \hfill (16)

The value of (16) depends on the relationship between the measured and predicted samples. For the steady-stay condition it results only from the level of measurement noises. In order to ensure the optimal estimation of states the following rules should be fulfilled. Firstly, the bigger level of noises the smaller gain of the KF. Secondly, a smaller value of noises means a bigger gain of the KF. In order to obtain the optimal value of the KF gain a fuzzy system which transforms the above-mentioned rules into non-linear relationship is utilized. The calculated value (16) comes to the fuzzy block with four input membership functions (Fig. 5a). The output value of the system is presented in Figure 5b.

![Fig. 5. Fuzzy input membership functions (a) and resulting characteristic of the fuzzy system (b)](image)

Parameters of the fuzzy system are obtained by the trial and error methodology.

### 4. Simulation study

In this section, the proposed fuzzy KF strategy for the drive system with an elastic coupling will be evaluated through simulations tests. It should be emphasized that contrary to the classical approach (linearization around the operation point) the value of the time constant of the load machine is updated in the estimator by the fuzzy system. Also the gain of the KF depends on the Mahalanobis distance.

The nominal parameters of the tested drive are as follows: the mechanical time constant of the load machine equals to \( T_2 = 0.203 \text{ s} \), mechanical time constant of the driving motor \( T_1 = 0.203 \text{ s} \) and stiffness time constant \(-T_c = 1.2 \text{ ms}\).
At the beginning the classical KF with constant values in the covariance matrices $Q$ and $R$ is investigated. In Figure 6 the transients of the selected state values of the system are presented. The system speed and electromagnetic torque are disturbed by additional white noises with the following magnitude: $\pm 0.003$ for $\omega_1$ and $\pm 0.001$ for $m_e$ which emulates noises in the experimental set-up. In order to illustrate the drawbacks of the classical solution the values of the covariance matrices are not set properly. The tested estimators work in a closed-loop control structure (Fig. 1).

![Fig. 6. Transients of the selected system values: motor, load and reference speeds (a), electromagnetic, shaft, and load torques (b), estimation errors for the motor speed (c) and load torque (d) for the control system working with the classical KF](image)

As can be concluded from the transients presented in Figure 6, the control structure works in a stable way. However, in the electromagnetic torque transients (Fig. 6b) very big high-frequency oscillations which cause vibrations in the motor speed are visible (Fig. 6a). The estimation errors in motor speed are small, yet very large in the load speed transients.

Then the control structure with adaptation of the Kalman gain is investigated. In order to show the effectiveness of the proposed solution the noises with variable magnitude are added to the signals. In the first 1.5 s the magnitude of the noises is the biggest, then for the next 1.5 s it decreases by half and, finally, from the third second it has a smaller magnitude. The system transients are shown in Figure 7.
The application of the KF with the variable gain improves the performance of the control structure significantly. The noises evident in the system state variables are almost unnoticeable in the presented scales (Fig. 7a, b). Only during transients the load speed error is big, then it drops to a very small value, though (Fig. 7c). The transients of the Kalman gain are presented in Figure 7d.

The system with simultaneous adaptation of the Kalman gain and estimation of the mechanical time constant is considered next. Selected transients of the system are presented in Figure 8.

The initial value of the mechanical time constant is set to 0.2. Then at the time $t = 5$ s it increases to 0.4. Next at the time $t = 10$ s and $15$ s $T_2$ changes to 0.1 and 0.2, respectively. The motor speed transients are presented in Figure 8b. The visible fluctuations in the speed result from the changeable load torque (Fig. 8b). The estimate of the time constant of the load machines follows its real value with sufficient accuracy. The Kalman gain transient is presented in Figure 8d.

5. Experimental results

A pilot-scale laboratory set-up composed of two 500 W DC-motors connected by a long steel shaft (length 600 mm, diameter 7 mm) is used in the tests. The driving motor and load machine are controlled by a dSpace 1103 control platform via two separate power converters.
The inner torque control loop consists of a PI torque controller, power converter (transistor H-bridge), armature winding and armature current sensor. The parameters of the torque controller are tuned to ensure fast aperiodic step response of the torque control loop. In the experimental set-up the motor speed measurement is realized with an incremental encoder (36000 pulses per rotation).

The proposed fuzzy adaptive Kalman filter is investigated through a series of experimental tests. First, the drive working in the low speed region is examined. The initial mechanical time constant of the load machine is set to 0.203 s when the real value is $T_2 = 0.406$ ms (the time constant of the load machine is changed by adding another flywheel). The selected system transients are shown in Figure 9.

The system speed transients have a desired shape (Fig. 9a). Also there are not any visible oscillations in the shaft and electromagnetic torques transients (Fig. 9b). The estimate of the time constant of the load machines reaches its real value very fast. Some quickly eliminated disturbances are visible during the reversal. The KF gain varies in relation to the present system speed in that it reaches the smallest value for the zero speed, which results from the fact that for a smaller value of the motor speed the measurement noises (encoder quantization errors) increase. The biggest estimation errors into the motor and load speed exist during the reversal (Fig. 9e, f).

Then the drive system working for the nominal value of the speed is investigated. The selected transients of the system are shown in Figure 10.
The system speed transients have a desired shape (Fig. 10a). There appear no visible oscillations in the shaft and electromagnetic torques transients (Fig. 10b). The estimate of the $T_2$ follows the real value (form 408 to 203 ms) relatively quickly. The KF gain varies due to the value of the speed. The speed estimation errors are presented in Figure 10e, f. They have rather small values, which suggests that the adaptive control structure works in a proper way.

6. Summary

In the paper an application of the adaptive control structure to a drive system with a flexible coupling is presented. The adaptive state controller is supplied with the system state
variables and parameter $T_2$ by the fuzzy KF. The estimated value of $T_2$ updates the matrices of the KF in every calculation step. The KF gain is modified with the use of the value of the measurements noises. The proposed KF works properly for a wide range of changes of the mechanical time constant of the load machine and different levels of the measurements noises. Its effectiveness has been confirmed through the simulation and experimental studies.

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