DOI: 10.2478/v10077-009-0011-3

Eugeniusz Ranatowski
University of Technology and Life Science, Mechanical Department, Bydgoszcz, Poland

MATHEMATICAL MODELLING OF THE THERMAL CYCLE IN HEAT AFFECTED ZONE WITH THREE DIMENSIONAL HEAT SOURCE MODELS AND PULSED POWER WELDING

PART I

ABSTRACT

The non-linear analytic-numerically computed calculations with use of two different heat source models are adopted for Pulsed Power Welding (PPW). The following heat source models are used in this study: cylindrical-involution-normal (C-I-N) and double-ellipsoidal (D-E). At first the temperature fields generated by C-I-N and D-E heat sources with pulsed power in both stationary and moving co-ordinates systems (Fourier transformation form) are established.

Key words: *temperature, power input, heat source.*

INTRODUCTION

Pulsed power welding (PPW) is a useful welding method and possesses several benefits over the conventional welding techniques - flexible and accurate heat input control and high effectiveness. Under PPW the basic problem with an appropriate assessment of weldability is about defining the relation between material structure and the function of mechanical properties. The assessment of the structure change is joined with a suitable analysis of the thermal cycle under welding.

As a rule, recently, the evaluation of thermal cycles has been performed using either pure numerical or analytic methods. As counterpoise, an analytic-numerical method [2] was presented in order to make possible the effective elastic cycle calculation for a plate with optional thickness and radiate heat transfer on both surfaces.

Furthermore, the temperature dependent material properties are also considered, which makes this method very competitive with other non-linear ones.

For finding the solution for PPW at first it is assumed that moving heat source (HS) during a very short period of time, generates an impulse of energy which induces an instantaneous thermal field in the plate area. Total temperature distribution from moving HS can be achieved by summing HS impulse results on its movement path.

Finally, the obtained temperature field solution has algebraic form and must be discretised in order to make computer calculations possible. For this purpose we will

use calculations in Mathcad programme, which enable calculations with temperature dependent physical parameters: λ(T), cp(T), ρ(T), a(T) and pulsed heat input q(t).
The following three dimensional heat source models are used in this study:

- cylindrical-involution-normal (CIN) [2],
- double ellipsoidal (DE) [3].

The model that previously and mainly was used is Gaussian surface flux distribution model.
Unfortunately this one does not reflect heat power input change for "z" variable. For example Wei and Shian [4] constituted that under high-intensity laser beam welding the incident energy rate distribution is assumed to be Gaussian distribution and HS model is idealised by a paraboloid of revolution. This is one of main reasons why two more accurate models are implemented.

BASIC FORMULATION FOR TRANSIENT HEAT FLOW

Energy heat transport in heat effected zone (HAZ) is mainly progressed by thermal conduction and can be described by Fourier – Kirchhoff (F-K) parabolic differential equation. Then the temperature $T(x_0, y_0, z_0, t)$ as a function of spatial stationary coordinates x_0, y_0, z_0 - Fig. 1 and time, t, satisfies the following F-K equation at every point in the domain, Ω:

$$\mathrm{div}(\lambda\,\mathrm{grad}\,T) - c_p\rho\frac{\partial T}{\partial t} = -q_v(x_0, y_0, z_0, t) \tag{1}$$

where:

- T - temperature, °C or K,
- λ - thermal conductivity, W $cm^{-1}K^{-1}$,
- c_p - specific heat, J $kg^{-1}K^{-1}$,
- ρ - mass density, kg cm^{-3},
- t - time, sec,
- q_v - power input in volume, W cm^{-3}.

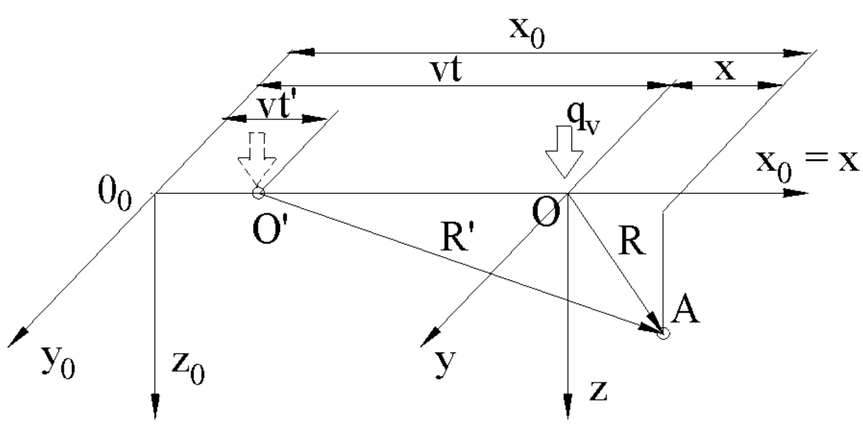


Fig. 1. Scheme of the co-ordinate system under welding

In order to solve equation (1) the following assumptions must be satisfied:

- welded sheet has infinite width and length but finite thickness g,
- the structure of welded parts is homogeneous with isotropic physical properties,
- physical parameters (λ, c_p , ρ , a) are temperature invariant,
- at sheet surfaces z = 0 and z = g, heat transfer occurs with enviroment at surface coefficients of conductance α_0 , α_1 ,
- the following boundary conditions are assumed:

$$\frac{\partial T}{\partial x_0} = 0\,;\; x_0 \to \infty\,, x_0 \to -\infty \tag{2a}$$

$$\frac{\partial T}{\partial y_0} = 0\,;\; y_0 \to \infty\,, y_0 \to -\infty \tag{2b}$$

$$\lambda \frac{\partial T}{\partial z_0} = \alpha_0 T\,;\; z_0 = 0 \tag{2c}$$

$$\lambda \frac{\partial T}{\partial z_0} = \alpha_1 T\,;\; z_0 = g \tag{2d}$$

where:

α_0	-	the surface coefficient of conductance at z = 0, W cm^{-2}K^{-1},
α_1	-	the surface coefficient of conductance at z = g, W cm^{-2}K^{-1},
g	-	thickness plate, cm.

Graphic interpretation of above conditions is shown in Fig. 2.

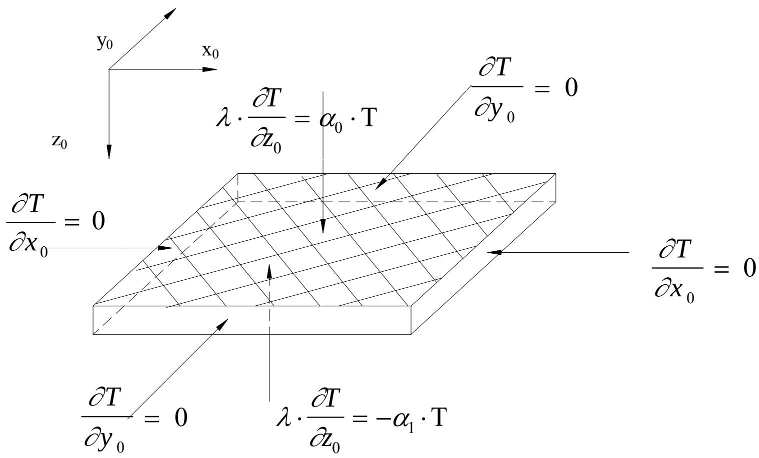


Fig. 2: Graphic interpretation of boundary conditions

In addition, the initial condition must be specified for $x_0 , y_0 , z \in \Omega_0$:

$$T(x_0 , y_0 , z_0 , t = 0) = T_0 = 0 \tag{2e}$$

If the partial differential eq. (1) is linear, the boundary conditions (2a)÷(2d) and initial condition (2e) are consistent, the problem is well posed and a unique solution exists. If parameters λ, c_p , ρ , a are functions of temperature T, eq. (1) is nonlinear and it makes pure analytical calculation impossible. So, there is a specific and logic assumption performed in order to numerically approximate well the solution that in fact does not exists.

The mathematical modelling of property – determining processes presents a modern and powerful tool to improve engineering materials and their processing such as welding process. Outgoing from the fundamental mechanisms and their physical representation in the form of equation systems, the effect of influencing factors on weldability can be simulated by numerical models. The application of this method results in a considerable reduction of the total development time and costs of experimental investigation. Beside, the mathematical modelling allows an optimisation of the numerous influencing parameters with the aim to increase the process reliability and to improve the welding construction properties. That means that the modelling of welding processes requires to take into account physical phenomena (thermic, metallurgy and mechanics) and their interactions. The analysis of the welding process from above point of view [1], enable to execute the algorithm which is presented in Fig. 1. The first step of our calculation (module I) effects on the character of heat flow in welding process and determines the nature of the weld thermal cycle and hence, in transformable alloys the metallurgical process and the microstructure of weld metal and heat affected zones (HAZ) (module II) and change of the mechanical specificity (module III). Besides, in agreement with Fig. 1 the estimate of the weldability consists with two stages:

- recurring projection process of the structure feature of weld metal and HAZ in comparison with base metal-submodules 1, 2, 3, 4,
- estimate of the result of this process through analysis of the feature of mechanical properties-submodules 5, 6, 7, 8.

Assessment of the step susceptibility of the base material on welding process is finally lean upon the fracture toughness parameter K_{mat} in terms of stress intensity factor K or his normalised value or others fracture parameters such as J, CTOD, G.

LINEAR ANALYSIS FOR PULSED POWER WELDING

We will follow PPW analytical scheme for time dependence of heat input q(t) -proposed by V.A. Karkhin, V.G Michailov and V.D. Akatsevich [5] – Fig. 3.

In the case when pulses have idealised trapezium waveform, the function q(t) can be described by 5 parameters: high pulse (peak) power q_p, high pulse time (peak duration) t_p, low pulse (background) power q_b, low pulse time (background duration) time t_b, and slope-up and slope-down pulse time t_s.

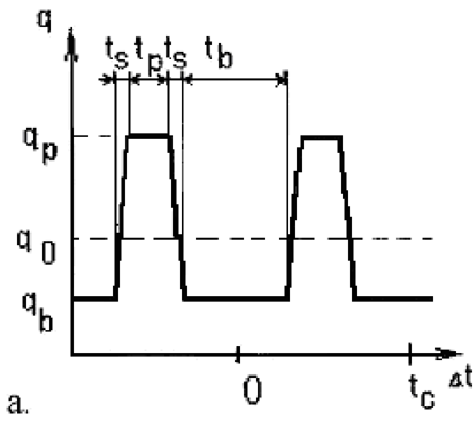
Heat input is a function of time q(t) and this will be included in Mathcad procedures which are very useful for modelling and simulation of welding thermal process [6].

This is an extra advantage to standard methods previously implemented by writers [2,7] of this article. Heat input is an additional parameter changing with time.

The following other assumptions are done:

- heat source energy is being input to the metal during time Δt,
- HS inputs are being summed up in points in distance $\Delta x = v\ \Delta t$. Considering this $t' = (j-1)\ \Delta t$, (j=1, 2, 3 …n).

- integrals were replaced by finished sums assuring their sufficient exactness.



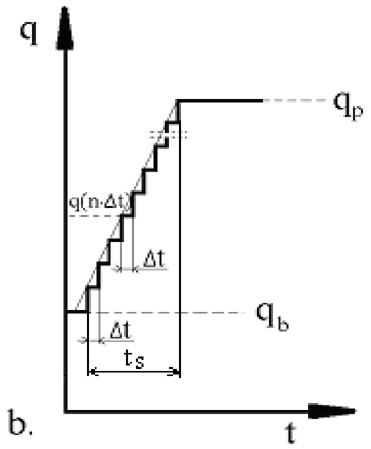


Fig. 3. Schematic diagram of pulsed power: a. course of function q(t) and her characteristic dimensions, b. details of course of function q(t) for t_s

Finally, the following computing expressions for linear heat flow solutions are obtained:

A. from Cylindrical-Involution-Normal heat source model:

Stationary co-ordinates system

$$T(x_0,y_0,z_0,t)=\sum_{j=1}^{n} \text{if}\left\{ t<(j-1)\cdot\Delta t, 0, \frac{q(t)\cdot k\cdot K_z}{\pi\cdot c_\gamma\cdot(1-\exp(-K\cdot s))}\cdot \right.$$
$$\frac{1}{4\cdot a\cdot k\cdot(t-(j-1)+1)}\cdot\exp\left[\frac{-k\left((x_0-(j-1)v\cdot\Delta t)^2+y_0^2\right)}{4\cdot a\cdot k\cdot(t-(j-1)\Delta t)+1}\right]\cdot \tag{3}$$
$$\left.\cdot\sum_{j=1}^{last} B_i\cdot C_i\cdot D_i\cdot\exp\left[-a\cdot r_i^2\cdot(t-(j-1)\Delta t)\right]\right\}$$

B. from Double Ellipsoidal configuration of source

Stationary co-ordinates system

$$T(x_0,y_0,z_0,t)=\sum_{j=1}^{n}\text{if}\{t<(j-1)\cdot$$
$$\cdot\Delta t,0,\frac{q_v(t)\cdot f_f\cdot 3\cdot\sqrt{3}\cdot\Delta t}{\pi\cdot\sqrt{\pi}\cdot\frac{\lambda}{a}\cdot c_f\cdot\sqrt{\left((12\cdot a\cdot(t-(j-1)\Delta t)+a_f{}^2)\cdot(12\cdot a\cdot(t-(j-1)\Delta t)+b_f{}^2)\right)}}\cdot$$
$$\cdot\exp\left(-\left(\frac{(x_0-v\cdot(j-1)\Delta t)^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\frac{1}{3}\cdot a_f^2}+\frac{y_0^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\frac{1}{3}\cdot b_f^2}\right)\right)+$$

$$+\frac{q_v(t)\cdot f_r\cdot 3\cdot\sqrt{3}\cdot\Delta t}{\pi\cdot\sqrt{\pi}\cdot\frac{\lambda}{a}\cdot c_r\cdot\sqrt{\left(\left(12\cdot a\cdot\left(t-(j-1)\Delta t\right)+a_r^{\,2}\right)\cdot\left(12\cdot a\cdot\left(t-(j-1)\Delta t\right)+b_r^{\,2}\right)\right)}}\cdot$$

$$\cdot\exp\left(-\left(\frac{\left(x_0-\nu\cdot\left(j-1\right)\Delta t\right)^2}{4\cdot a\cdot\left(t-\left(j-1\right)\Delta t\right)+\frac{1}{3}\cdot a_r^2}+\frac{y_0^2}{4\cdot a\cdot\left(t-\left(j-1\right)\Delta t\right)+\frac{1}{3}\cdot b_r^2}\right)\right)\cdot \tag{4}$$

$$\left.\cdot\sum_{i=1}^{last}B_i\cdot C_i\cdot E_i\cdot\exp\left(-a\cdot r_i^2\left(t-\left(j-1\right)\Delta t\right)\right)\right\}$$

where:

$$B_i=\cos\left(r_i\cdot z_0\right)+\frac{\alpha_0}{\lambda\cdot r_i}\cdot\sin\left(r_i\cdot z_0\right) \tag{5}$$

$$C_i=\frac{2\cdot r_i^{\,2}}{\left(\frac{\alpha_0^{\,2}}{\lambda^2}+r_i^{\,2}\right)\cdot\left(g+\frac{\alpha_1\cdot\lambda}{\alpha_1^{\,2}+r_i^{\,2}\cdot\lambda^2}\right)+\frac{\alpha_0}{\lambda}} \tag{6}$$

$$D_i=\exp\left(-K_z\cdot s\right)\cdot$$
$$\cdot\frac{\left(-K_z\cdot\cos\left(r_i\cdot s\right)\cdot\lambda\cdot r_i+r_i^{\,2}\cdot\sin\left(r_i\cdot s\right)\cdot\lambda-\alpha_0\cdot r_i\cdot\cos\left(r_i\cdot s\right)-\alpha_0\cdot K_z\cdot\sin\left(r_i\cdot s\right)\right)}{\left(K_z^{\,2}+r_i^{\,2}\right)\cdot\lambda\cdot r_i}+ \tag{7}$$
$$+\frac{K_z\cdot\lambda+\alpha_0}{\left(K_z^{\,2}+r_i^{\,2}\right)\cdot\lambda}$$

$$E_i=\int_0^g\left(\cos\left(r_i\cdot z\right)+\frac{\alpha_0}{\lambda\cdot r}\cdot\sin\left(r_i\cdot z\right)\right)\cdot\mathrm{approx}\left(z,c,nlast\right)dz \tag{8}$$

$$\mathrm{approx}(z,c,nlast)=1+\sum_{n=1}^{nlast}\frac{\left(-\frac{3}{c^2}\cdot z^2\right)}{\prod_{i=1}^{m}n} \tag{9}$$

where:

K_z	-	involution factor of HS, cm^{-1},
k	-	factor designating the HS concentration, cm^{-2},
$\prod_{i=1}^{m}n$	-	performs iterated multiplication of n over i=1, .., m-1, m,
s	-	HS penetration depth, cm,

$a_f, b_f, c_f, a_r, b_r, c_r$	-	ellipsoids semi axes, cm,
$a = a_f + a_r$;		
$b = b_f = b_r$;	-	dimension of D-E configuration of source, cm,
$c = c_f + c_r$		
f_r , f_f	-	fraction of heat deposits in front and rear quadrants,
q_{vf} , q_{vr}	-	power density distribution inside the front and rear quadrants respectively, W cm^{-3}.
$c_\gamma = c_p\,\rho$	-	volumetric specific heat, J K^{-1} cm^{-3}.

$r_1, r_2, r_3 \ldots r_i$ are roots of:

$$\mathrm{ctg}(r_i \cdot g) = \frac{\lambda^2 \cdot r_i^{\,2} - \alpha_0 \cdot \alpha_1}{\lambda \cdot r_i \cdot (\alpha_0 + \alpha_1)} \tag{10}$$

There are analogous expression for calculating welding cycles at moving co-ordinates. The difference is only in co-ordinates writing [2, 8]:

$$x = x_o - vt\ ,\ y = y_o\ ,\ z = z_o \tag{11}$$

CONCLUSION

Universalises analytical heat-flow solutions, which are capable of analysing thermal behaviour of the weldment under time-dependent power change during welding, are presenting. Pulsed power welding offers several advantages over the more conventional welding techniques making possible more flexible and accurate heat input control.
The following three dimensional heat source models are used in this study: CIN and DE.

Finally, the following computing expressions for linear heat flow solutions are obtained:

- for stationary co-ordinates system from CIN heat source model,
- for stationary co-ordinates system from DE heat source model.

REFERENCES

1. Mathcad-User's Guide. MathSoft, inc. Cambridge, MA02142. USA. 2001.
2. Ranatowski E, Poćwiardowski A.: An-analytic-numerical estimation of the thermal cycle during welding with various heat source model application. Mathematical Modelling of Weld Phenomena 5, H. Cerjak ed., The Institute of Materials, London, 2001.
3. Goldak J. et al.: A new finite element model for welding heat sources. Metal. Trans. B, 299–305. 1984.

4. Wei P.S., Shian M.D.: Three-dimensional analytical temperature field around the welding cavity produced by a moving distributed high-intensity beam. Journal of Heat Transfer, 115, 848 – 856. 1993.
5. Karkhin V.A. and Michailov V.G., Akatsevich V.D.: Modelling The Thermal Behaviour of Weld and Heat Affected Zone During Pulsed Power Welding. Mathematical Modelling of Weld Phenomena 4, H. Cerjak ed., The Institute of Materials, London, 1998.
6. Benker H.: Practical use of Mathcad : Solving of Mathematical Problems, Springer-Verlag, Berlin-Heidelberg, 1999.
7. Ranatowski E., Poćwiardowski A.: An-analytic-numerical evaluation of the thermal cycle in the HAZ during welding. Mathematical Modelling of Weld Phenomena 4, H. Cerjak ed., The Institute of Materials, London, 1998.