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**G. Garbacz**[1*], **L. Kyzioł**[2]

[1] *Opole University of Technology, Faculty of Production Engineering and Logistics, Opole, Poland*
[2] *Gdynia Maritime University, Faculty of Mechanical Engineering, Gdynia, Poland*
[*] *g.garbacz@po.opole.pl*

# APPLICATION OF METRIC ENTROPY FOR RESULTS INTERPRETATION OF COMPOSITE MATERIALS MECHANICAL TESTS

## ABSTRACT

In this paper the results of mechanical studies of the Aropol 536 composite on the epoxy-resin base are described. The aim of the studies was to measure elastic-plastic changes in the composite during its deformation. The obtained results were analyzed using Kolmogorov-Sinai metric entropy. The entropy was computed applying phase portraits reconstructed from a phase plane using delayed coordinates. Resolution of the particular experimental setup limits the number of the acquired data points, i.e., from several to tens of thousands of points and it has significant influence on accuracy of the obtained results. In conclusion, in the tested composites elastic-plastic deformations are periodic and repeat in a distinctive way in a wide range of deformations of the sample. Deformation of the elastic-plastic composite are associated with its complex structure and studies of its mechanical properties require more advanced methods such as use of Kolmogorov-Sinai metric entropy.

***Keywords:*** *composite, elastic-plastic deformation, Kolmogorov-Sinai metric entropy, phase plane*

## INTRODUCTION

Composite materials based on glass fibre are a group of engineering materials that are widely applied in engineering constructions. The structure of composites, i.e. the combination number of joining different fibers and matrix types makes it possible to generate new engineering materials. The interpretation of mechanical tests results of the uniaxial tensile test of this group of materials is the subject of the work presented. An attempt to apply Kolmogorov – Sinai metric entropy was taken to show the tensile test as a process that has certain stages of elastic and permanent deformation of a composite. The application of such a method results from the fact that the characteristics of a composite material tension till its rupture does not have characteristic points such as structural steel has, for example. The proposed method requires conducting tensile tests together with the registration of a large number of measuring points, that is minimum a few hundred points registered during one test which is necessary for the calculation of data metric entropy [1]. On the basis of the calculated metric entropy values, their phase portraits were made which allowed to indicate potential characteristic points and the regularities of the tensile process [3]. The purpose of the paper is the presentation of a new method of testing construction materials and comparison between results obtained using this method and other methods for a chosen composite.

## EXPERIMENTAL

### *Material*

The material for the research was obtained in the shape of rectangular plates approximately 150 mm long and 20 mm wide. The polyester-glass composites are classified as orthrotropic materials. In case of this composite, it was difficult to present mathematical description of its mechanical properties due to the lack of data available. The material was received in the form of "composite pieces" and so the technology of its production, its chemical constitution, fibre layout were unknown. Regardless of many unknown aspects, it was decided to determine composite mechanical properties. The samples were composed of six different materials of various chemical composition, various content of polyester-glass material and glass mat. However the basic test material was a laminate as glass mat of 350 basis weight and Aropol 536 resin. The basic properties of glass fibre [5,6] are shown in Table.1.

***Table 1.*** *Basic properties of glass fibre*

| Density $kg/m^3$ | $R_m$ MPa | $\varepsilon$ % | E MPa | Poisson coefficient $\nu$ |
|---|---|---|---|---|
| 2450 | 200 | 3 | 1800 | 0.23 |

Composites for the research were marked from 1 to 4 and 6. These composites were made of glass mat and resin only. The thickness of particular materials was as follows: material „1"– 2,40 mm, „2" – 1,45 mm, „3" – 4,75 mm, „4" – 7,60 mm, „6" – 3,15 mm.

### *Preliminary tests*

Preliminary tests were conducted in order to determine the glaze content. The glaze content was measured by material roasting in a muffle furnace at the temperature of 1000° C, until the mass in the pot changed. Table 2 shows the measurements results [5].

***Table 2.*** *Mass content of glass fibre in composites*

| Material number | Material thickness mm | Glaze content % |
|---|---|---|
| 1. | 2.40 | 41.5 |
| 2. | 1.45 | 36 |
| 3. | 4.75 | 35 |
| 4. | 7.60 | 62 |
| 6. | 3.15 | 38 |

### *Mechanical tests*

Static tensile tests were performed. The material examined was a glass mat composite of basis weight 350 on the Aropol 536 resin matrix [5,6]. Rectangular plate samples of 150 mm in length were tested. 15 samples of different width and thickness, and various glass fibre content underwent testing. Obtained data is shown in Table 3 and Table 4 [5, 6].

***Table 3.*** *Characteristics of samples tested by extensometer of the 50 mm base*

| Sample number | Glass fibre content % | Sample dimensions mm | Sample cross section $mm^2$ | Measuring length $L_0$ mm | Length after rupture $L_1$ mm |
|---|---|---|---|---|---|
| 11 | 41.5 | 2.40x10.0 | 24.0 | 50 | 51.90 |
| 12 | | 2.40x10.0 | | | 52.00 |
| 13 | | 2.40x10.0 | | | 51.80 |
| 21 | 36.0 | 1.45x10.0 | 14.5 | 50 | 52.00 |
| 22 | | 1.45x10.0 | | | 52.00 |
| 23 | | 1.45x10.0 | | | 51.90 |
| 31 | 35.0 | 4.75x10.0 | 47.5 | 50 | 51.60 |
| 32 | | 4.75x10.0 | | | 51.55 |
| 33 | | 4.75x10.0 | | | 51.50 |
| 41 | 62.0 | 7.60x10.0 | 76.0 | 50 | 51.55 |
| 42 | | 7.60x10.0 | | | 51.70 |
| 43 | | 7.60x10.0 | | | 51.50 |
| 61 | 38.0 | 3.15x10.0 | 31.5 | 50 | 52.05 |
| 62 | | 3.15x10.0 | | | 51.95 |
| 63 | | 3.15x10.0 | | | 52.00 |

***Table 4. T****est results*

| Sample number | Sample final deformation ε % | | Destructive force N | Rupture strength $R_m$, MPa | |
|---|---|---|---|---|---|
| | single | mean | | single | mean |
| 11 | 3.8 | 3.8 | 2352 | 98 | 97 |
| 12 | 4.0 | | 2270 | 95 | |
| 13 | 3.6 | | 2400 | 100 | |
| 21 | 4.0 | 3.9 | 942 | 65 | 60 |
| 22 | 4.0 | | 800 | 55 | |
| 23 | 3.8 | | 870 | 60 | |
| 31 | 3.2 | 3.1 | 4560 | 96 | 94 |
| 32 | 3.1 | | 4465 | 94 | |
| 33 | 3.0 | | 4300 | 91 | |
| 41 | 3.1 | 3.2 | 8968 | 118 | 118 |
| 42 | 3.4 | | 9120 | 120 | |
| 43 | 3.0 | | 8710 | 115 | |
| 61 | 4.1 | 4.0 | 3717 | 118 | 120 |
| 62 | 3.9 | | 3780 | 120 | |
| 63 | 4.0 | | 3835 | 122 | |

Tensile strength tests were carried out at Gdańsk University of Technology, the Department of Material Engineering and Welding on a universal testing machine INSTRON, at the strain rate of $\dot{\varepsilon} = 10^{-3} \div 10^{-4}$ s-1. Strain was registered by extensometer with the base of $l = 50\ \text{mm}$ .

**Fig. 1.** Measurement of strain performed by extensometer on universal testing machine INSTRON

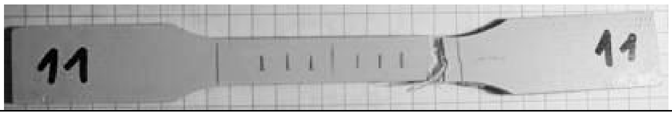
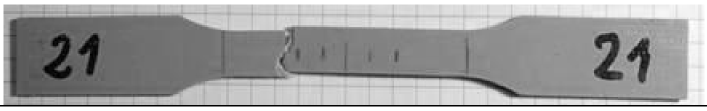
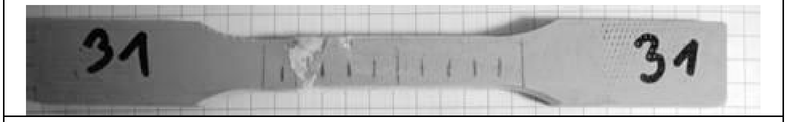
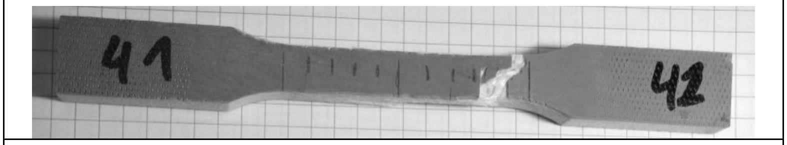
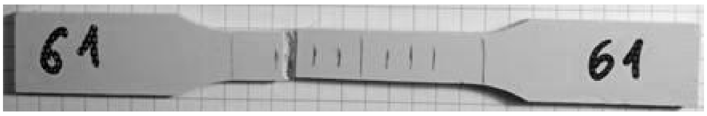
| Sample number |
|---|
| 11<br> |
| 21<br> |
| 31<br> |
| 41<br> |
| 61<br> <br>62 |

**Fig. 2.** Selected samples after rupture

For samples presented in Picture 2, five data sets of absolute strain signal and force corresponding to strain were registered during the performance of the tests. Appropriate characteristics in the measurements set of measuring points and tensile strength are presented in the graphs 1,3,5,7. On the basis of the photographs of these samples it is possible to conclude that rupture was of brittle type.

With regard to composite plastic characteristics it is difficult to comment on the tensile process phases as it was in case of metal alloys that showed characteristic points such as plasticity limit. Hence the necessity to apply other analytical tools for the examination of processes occurring in the composite during tensile test. The paper proposes the usage of Kolmogorov-Sinai metric entropy as well as the analysis of metric entropy phase portraits.

## CONCEPT OF KOLMOGOROV–SINAI METRIC ENTROPY

The Kolmogorov-Sinai metric entropy [4,7] for the probability discrete distribution is defined by the formula:

$$S = -\sum_{i=1}^{N} p_i \, lnp_i \qquad (1)$$

where:
N – is the number of partitions into which a set of all possible results was divided,
pi - is the probability of results occurrence in i-th partition, (while in the definition if $p \ln p \equiv 0$, if p = 0)

If the partitions are equiprobable, that is $p_i = 1/N$ for all, the entropy is defined by formula $S = \ln N$ assuming the maximum value. However, if the results are known to be within certain partitions, then entropy reaches the minimum value S = 0, because $p_i = 1$.

The Kolmogorov-Sinai entropy is calculated many times. Every time it is calculated for a certain number of a constant quantity k of the selected sub sets, that is parts of a set n being a set of probable results recorded during tensile test of a composite. The selected k – elements sub sets are each time divided into N sub partitions in order to make calculations of Kolmogorov-Sinai entropy. On the basis of the calculated values, a Kolgomorov-Sinai metric entropy graph can be created in the function of consecutive measuring points, taking into account the quantity of k elements set.

The Kolmogorov-Sinai metric entropy has a dynamic character and therefore it is suitable to describe and examine the phenomena of such nature as for example the deformation or damage of composites. The fluctuation of metric entropy is inseparably connected to the dynamics changes of physical processes as well as the energy dissipation that accompanies these changes.

### *How does Kolmogorov-Sinai formula work?*

Let us exemplify the procedure of computing the Kolmogorov-Sinai entropy using a set of $k = 20$ randomly chosen numbers from the interval from 0 to 15. In the first step this set is divided into $N = 5$ subsets. Then, the probability that an element of the set belongs to a half-bounded interval is estimated (Table 5). Finally, using these probabilities for each of intervals one finds that the Kolmogorov-Sinai metric entropy is:

$$S = -\sum_{i=1}^{5} p_i \, ln p_i \quad = - \; (- 0{,}3466 - 0{,}2846 - 0{,}3612 - 0{,}3219 - 0{,}2303 ) \approx 1{,}5445$$

***Table 5.*** *Procedure of computing the K-S metric entropy from simulated data*

| Interval | ≥ 0 k < 3 | ≥3 k <6 | ≥6 k <9 | ≥9 k <12 | ≥12 k <15 |
|---|---|---|---|---|---|
| Data | 2.6<br>2.7<br>1.1<br>0<br>0.5 | 3.4<br>4.5<br>3.6 | 6.6<br>7.5<br>8.9<br>6.1<br>7.8<br>7.9 | 9.2<br>10.3.<br>11.5<br>11.1 | 13.5<br>12.8 |
| Probability $p_i$ that an element is in the i-th interval | 5/20 | 3/20 | 6/20 | 4/20 | 2/20 |
| $p_i \ln p_i$ | -0.3466 | -0.2846 | -0.3612 | -0.3219 | -0.2303 |

Figures 1 and 2 show graphically a calculation of K - S entropy for consecutive location of n- numerous measurement set, with reference to experimental data.

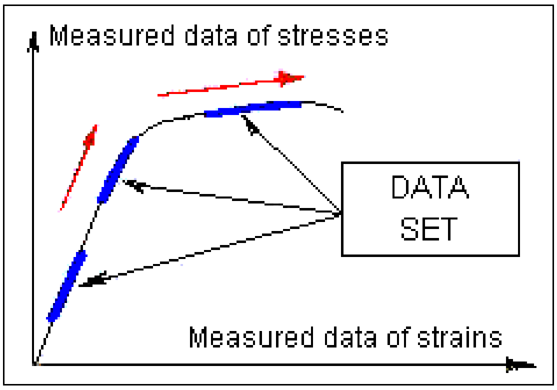


**Fig. 3.** Demonstration of data set's dislocation in reference to stresses or strains

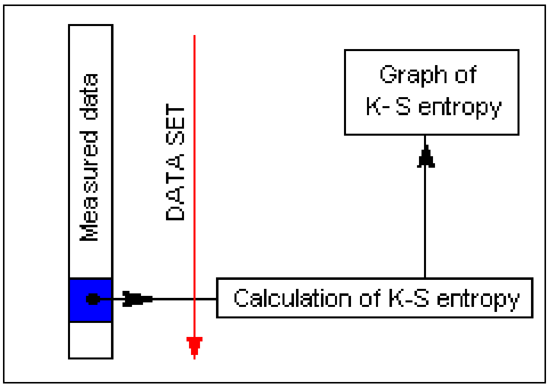


**Fig. 4.** Block diagram for the K-S metric entropy calculation

## DETECTING REGULARITIES OF INTERNAL NONLINEAR DYNAMICS OF PHYSICAL PROCESSES BY THE RECONSTRUCTION OF PHASE SPACE

The method of examining time series proposed by F. Takens [8], makes it possible to reveal internal regularities or otherwise, to classify the nature of the phenomenon as purely stochastic. If the results of the experiment, as the value x(t), can be obtained in the form of time series when sampling it e.g. every $\tau$ seconds, the following sequence of numbers can be obtained:

$$x0 = x(0), \quad x1 = x(\tau), \quad x2 = x(2\tau), \quad x3 = x(3\tau), \ldots\ldots\ldots\ldots\ldots, xn = x(n\tau)$$

A graph is made, a graph of values x(t) shifted in time by a definite value T which is a multiple of $\tau$, in the so called delayed coordinates. Thus for two dimensions the following sequence of vectors is obtained:

$$
\begin{aligned}
&( x(0), x(T) ),\\
&( x(\tau), x(T + \tau) ),\\
&( x(2\tau), x(T + 2\tau) ),\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&( x(n\tau), x(T + n\tau) )
\end{aligned}
$$

For two dimensions, the phase space reconstruction comes down to the phase plane reconstruction and is said to constitute a phase portrait. The procedure of creating phase portraits out of one time set of mechanical tests data is also presented in paper [3].

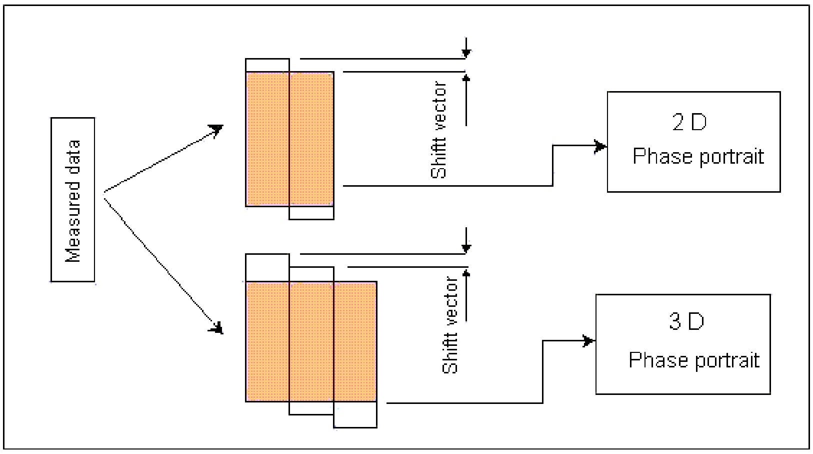


**Fig. 5.** **S**cheme of the phase portrait (A) and the phase space (B) reconstruction

## DISCUSSION

Metric entropy was calculated for the acquired records of data, whose graphs compared to the characteristic of tension in the function of measuring points were depicted in figures 4, 6, 8, 10 and 12. The areas of a significant drop of metric entropy are crucial and constitute the premise to indicate the change of material structure. Figures 5, 7, 9, 11 and 13 depict metric entropy phase portraits that correspond to tested samples. These portraits correspond to the oval spaces that are marked in comparison with tension characteristics.

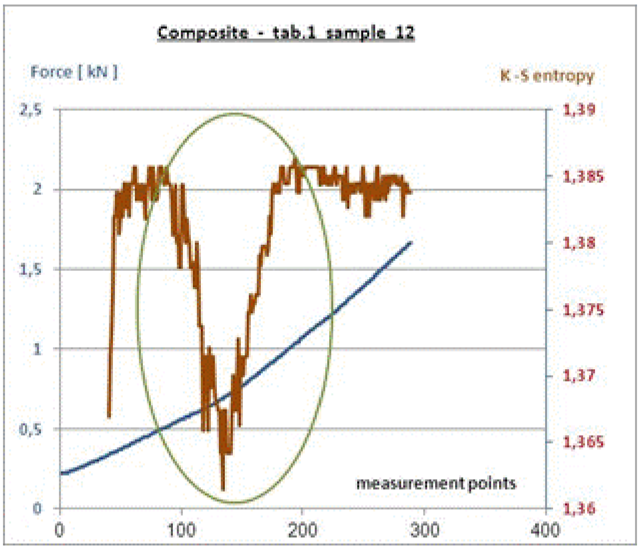


**Fig. 6.** Tension graph of composite sample 12 in comparison with K-S entropy graph

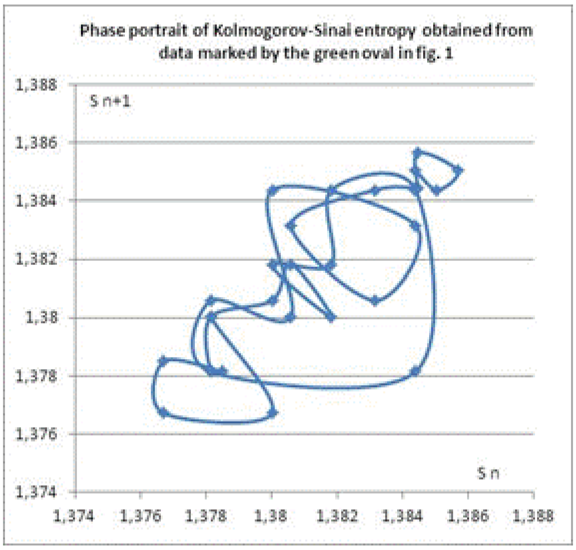


**Fig. 7.** Phase portrait of metric entropy, whose graph is shown in Figure 4 for the measuring set of n = 80 points, divided into N = 4 sets of entropy calculations. Shiftt vector is = 1

Figure 4 presents the sample tension graph of composite 12 together with Kolmogorov-Sinai entropy graph. The tension graph does not show any characteristic points, whereas the

K-S entropy changes significantly in the area marked by the oval. It can be concluded that considerably intensive structural changes occur inside the composite in the oval area. Figure 5 depicts entropy phase portrait of Figure 4. It is oval in shape and the phase trajectory closes. It can be concluded that material structural changes are of reproducible type.

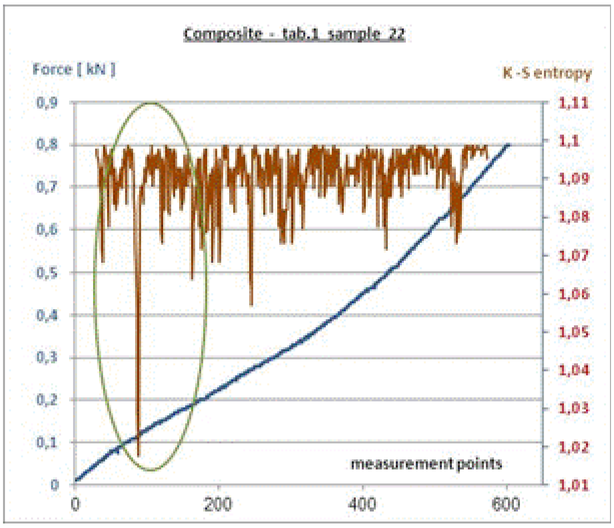


**Fig. 8.** Tension graph of composite sample 22 in comparison with K-S entropy graph

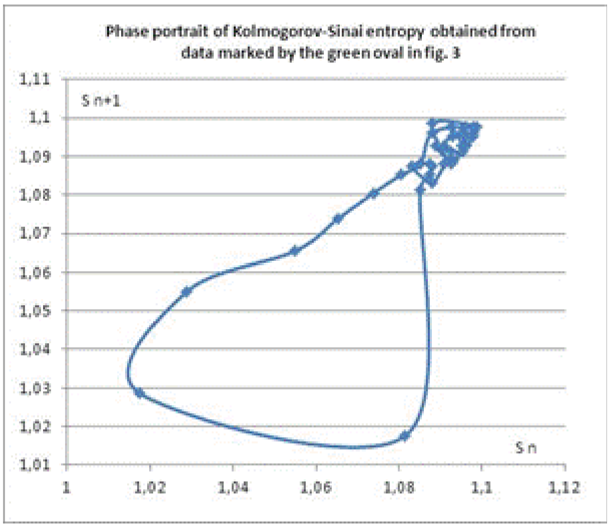


**Fig. 9.** Phase portrait of metric entropy, whose graph is shown in Figure 7 for the measuring set of n = 60 points, divided into N = 3 sets of entropy calculations. Shift vector is = 1

In Fig. 6, the K-S minimum entropy falls into the initial phase of tensile process, which proves the most advanced structural changes in this phase of the test. The corresponding area of K-S entropy phase portrait is shown in Fig. 7. On the basis of this portrait it is possible to recognize reproducible structural changes inside the composite during tensile test. The results presented in Fig. 8 and 9 can be interpreted similarly in Fig. 7 and 8.

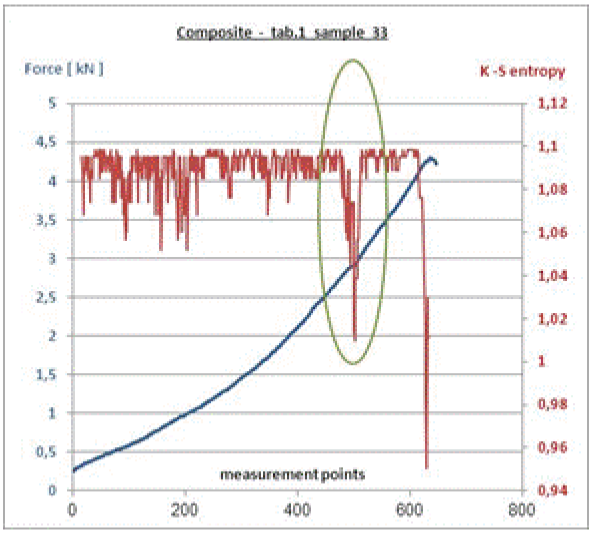


**Fig. 10.** Tension graph of composite sample 33 in comparison with K-S entropy graph

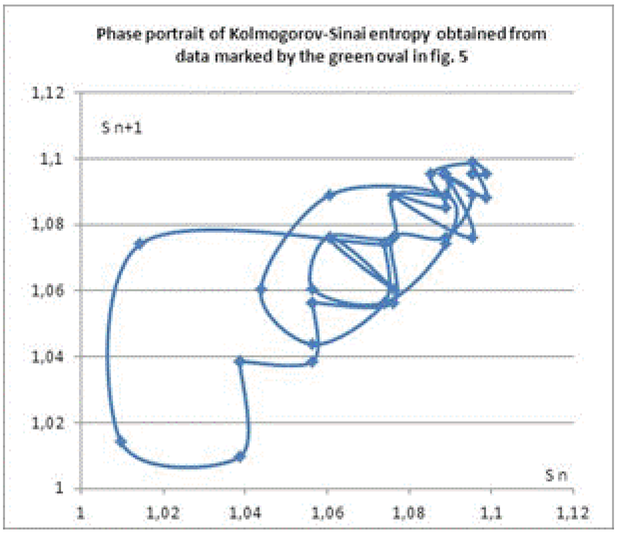


**Fig. 11.** Phase portrait of metric entropy, whose graph is shown in Figure 8 for the measuring set of n = 30 points, divided into N = 3 sets of entropy calculations. Shift vector is = 1

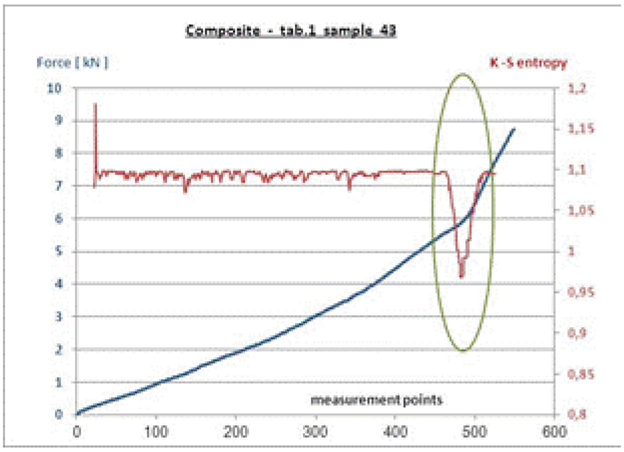


**Fig. 12.** Tension graph of composite sample 43 in comparison with K-S entropy graph

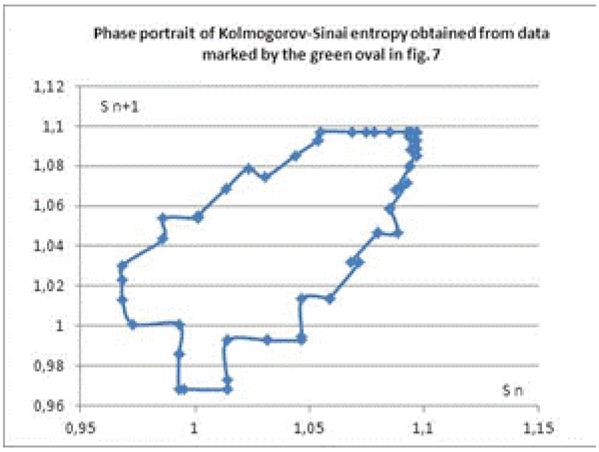


**Fig. 13.** Phase portrait of metric entropy, whose graph is shown in Figure 10 for the measuring set of n = 60 points, divided into N = 2 sets of entropy calculations. Shift vector = 5

Figures 10 and 11 depict the entropy cyclic nature and its corresponding structural changes which are clearly seen.

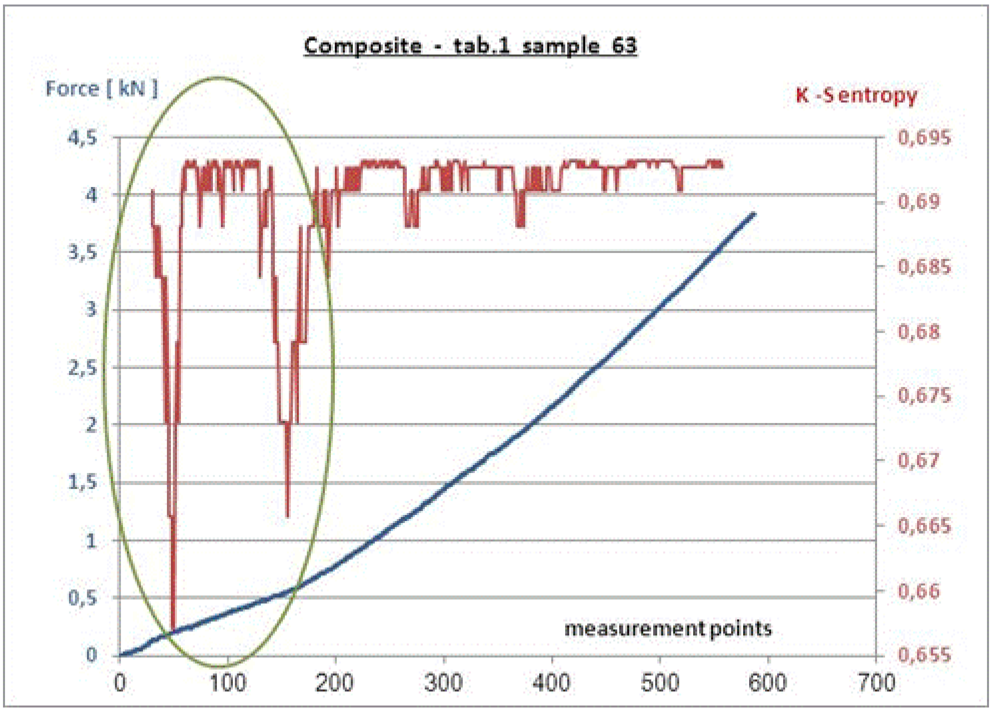


**Fig. 14.** Tension graph of composite sample 63 in comparison with K-S entropy graph

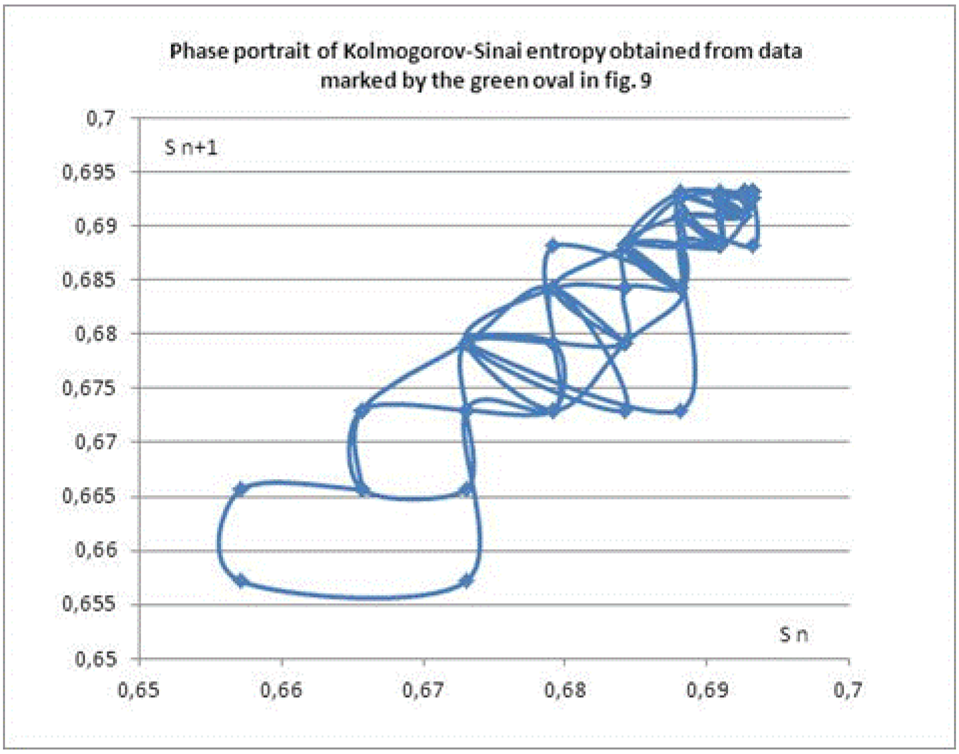


**Fig. 13.** Phase portrait of metric entropy, whose graph is shown in Fig. 12 for the measuring set of n = 60 points, divided into N = 2 sets of entropy calculations. Shift vector is = 5

Figures 12 and 13 for sample 63 confirm the regularity of internal deformation process of the tested materials.

## CONCLUSIONS

Summing up, during mechanical testing of composites the phenomena of elastic - plastic deformation that are periodic, recur in a characteristic way in a wide range of strains during sample loading. This reproducible nature is seen in the shapes of phase portraits. Elastic – plastic deformations of the composite are related to its complex structure, i.e. various mechanical properties of the matrix and filler. Moreover, connection between the matrix and the strengthening phase also plays an important role. The calculation of Kolmogorov-Sinai metric entropy of measuring data sets allows studies of the inner structure of tensile process. The graphs of metric entropy and their phase portraits enable a holistic evaluation of material deformation without the analysis of the tensile curve.

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