Ulrich Gerhafer, Alfred Strauss, Konrad Bergmeister

Institute for Structural Engineering, BOKU University, Peter-Jordan-Strasse 82, 1190 Vienna, Austria

ulrich.gerhafer@boku.ac.at

NUMERICAL MODELING OF ELASTOMERIC BEARINGS IN STRUCTURAL ENGINEERING

ABSTRACT

The intention of this paper is the discussion of different possibilities for the numerical modeling of elastomeric bearings. It is focused on issues of the complex material behavior and the specific mechanical conditions of the bearings. The relevant considerations are based on the experiences from extensive material tests on elastomer and reinforcement as well as from loading and deformation tests on full scale bearings. It shows the possibilities and the limitations of the numerical bearing models as a tool for research and bearing design.

INTRODUCTION

Elastomeric bearings are frequently used in structural engineering for over 50 years [1]. Nevertheless the current European structural codes [2] [3] still provide rough simplifications for their design. In the codes the complex material behavior is reduced to a few parameters and a strictly linear force displacement behavior is assumed. These deficits are generally known [4] [5] but only few research projects are concerned with the development of advanced design guidelines to be applied in practice.

The nonlinear material behavior and geometrical behavior are too complex to be described in strictly mathematical approaches. To analyze all relevant influences and the mechanical coherences parametric studies based on numerical models or loading tests are needed [6].

The bearing geometry for the numerical models is fairly simple. Nevertheless there are several material and geometrical associated determinations and assumptions that have an essential impact on the accuracy, the computational costs and on the possible range of studies on the bearing behavior. In particular essential determinations are associated with the appropriate selection of

- a) the material models capturing the mechanical characteristics of the elastomer,
- b) the material models for the reinforcement,
- c) the element types according to the occurring strain states and
- d) the numerical calculation algorithm.

The target of this paper is the discussion of the most important aspects of the numerical modeling of elastomeric bearings. It shall provide practical advices for an efficient development of bearing models with consideration of the required accuracy and complexity as well as the

computational effort. Special attention is paid to the elastomeric material behavior and the embedded fiber reinforcement.

NUMERICAL MATERIAL MODELS

Reinforced elastomeric bearings are composed of quadratic or rectangular sheets of elastomer and reinforcement sheets of steel or fiber which are assembled in alternating order to form a bearing cube.

The horizontal oriented reinforcement in the block bearing restrict the strain field in horizontal direction due to vertical loading and in consequences increases the vertical stiffness and load capacity of the bearing.

This arrangement of the reinforcement causes shear strain in the elastomeric sheets as well as in the interface between the elastomer and the reinforcement sheets and a tensile loading of the reinforcement sheets.

The previously mentioned increase of the vertical stiffness results from the restricted vertical compression deformation due to the horizontal displacement restriction and the compressive modulus properties of elastomer.

The additional compressive deformation of the elastomer is small due to its large compression modulus. Beyond the geometrical parameters the vertical stiffness of an elastomeric bearing is therefore determined by the shear modulus and compressive modulus of the elastomer as well as by the elastic modulus of the reinforcement.

The horizontal deformation (parallel to the reinforcement layers) of the bearing causes additional shear strain in the elastomer with a constant value in all parts of the elastomer. As both – the vertical loading and the horizontal deformation of the bearing – result in a shear deformation of the elastomer the dominant material parameter determining the behavior of elastomeric bearings is the shear modulus of the elastomer.

Hyperelastic material models for elastomer

The maximum shear strain for elastomer is limited according to [2] to $\varepsilon_{u,k} = 500\%$. These large strain ranges of the elastomer require the usage of hyperelastic material models based on strain energy potentials for the capturing of the stress strain performance.

There are two associated approaches a) the Neo Hooke Model, which uses model parameters that are explicit related to the mechanical properties and b) phenomenological models like the Ogden- or Yeoh-Model. These models require the adjustments of their input parameters as they are not explicit related to material properties from material tests.

The parameters (e.g. shear modulus or bulk modulus) of the mechanical based models can directly be measured in material tests .

In general, the calibration of phenomenological models is carried out with a fitting algorithm comparing measured test data and numerical model response. There is no direct correlation between the stiffness values of types of strain in contrast to conventional linear elastic models. Therefore test data from different types of material tests are required to carry out a reasonable calibration. Even though the shear strain stress behavior is significant for the bearing behavior a calibration based exclusively on shear test data can lead to instability when the model subjected to compression or tension.

Leading finite element programs provide several standard hyperelastic models. Some programs as for instance ABAQUS [7] provide the application of additional mechanical material

degradation (Mullins Effekt [7]) or hysteretic behavior in a restricted manner (e.g. only strain-rate dependent hysteretic behavior and not hysteresis as inner friction). A more satisfying model is the parallel network model [8] which can be implemented as a user subroutine. It is based on the definition of several parallel components of conventional hyperelastic models with additional degradation or damping elements. It was used partly for bearing models presented here to study the damping behavior of elastomeric bearings.

Calibration of the elastomer material models

A conventional hyperelastic material model without mechanical degradation or hysteretic behavior does not completely cover the existing elastomeric performance or behavior. In general the calibration of material models relies on stress strain distribution curves as for example shown in Fig. 1. Depending on the specific model it may be necessary to use test data from qualitatively different material tests (e.g. uniaxial tensile test and simple shear test) to achieve an accurate performance of the material in the specific field of application. Furthermore the limitations of conventional hyperelastic material models without mechanical degradation or hysteretic behavior may require a selection of part of the test data to be used in the calibration process as demonstrated in Fig. 1 (test data from $n_{\text{total}} = 12$ deformation cycles).

Fig. 1. Strain-Stress-Diagram from cyclic uniaxial loading tests used for the calibration of material model parameters; n = number of the specific deformation cycle used for calibration

The following statements have to be taken into account for the selection of calibration test data:

- When a state of strain is applied to an elastomer specimen for the first time parts of the sulphur bridges connecting the long-chained molecules get broken. Due to this mechanical material degradation (Mullins effect) the material behaves less stiff in the following strain cycles. Therefore the shear modulus of the elastomer shows different values at different load levels of the bearing depending on the maximum strain ϵ_{max} ever applied.
- Due to the hysteretic behavior the branch of ascending and descending strain are not equal. When applying a vertical load on a bearing the elastomer undergoes shear strain in positive and negative direction. A subsequent horizontal shear deformation results in an addi-

tional shear strain with equal value for all parts of the elastomer. The combination of vertical loading and horizontal deformation means an increase or decrease of the shear strain of the elastomer, depending on the location in the bearing.

The material degradation and hysteresis characteristics are also valid for shear deformation of the elastomer, as shown in Fig.2. In the figure the comparison of shear test data and results from a numerical parallel network model [8] shows good compliance for the previously mentioned material degradation process and the nonlinear stiffness.

Fig. 2. Left: Comparison of data from a parallel network model and data from shear tests used for calibration; Right: simplified illustration of a parallel network model with springs, friction elements and viscous dampers

A generic algorithm can be used for the calibration of the parallel network model. The error value has been computed by summation of the stress difference between numerical one-element models and test data from shear and uniaxial tests. As the material behavior under shear strain is decisive for bearing design the error associated error value has been weighted stronger. A more accurate fitting of the damping value could be obtained by the improvement of the calibration algorithm (e.g. by additional comparison of the loop area of numerical and test data) or by enhancement of the model with additional parallel components.

The secant shear modulus of the elastomer is the most significant material parameter for bearing design. This modulus is responsible for the horizontal stiffness of the bearing, the restoring force and the dynamic isolation properties. In the current design codes[2] and [3] the complex material behavior related to the shear modulus G is represented by the assumption a constant value of $G = 0.9 \text{ N/mm}^2$. For an exact calculation of the bearing behavior it is therefore recommended to use the shear modulus $G(\epsilon_{\max}, \epsilon)$ as a function of the prestrain $\epsilon_{\max}$ (maximum shear strain ever applied to the material) and the actual strain for the design of elastomeric bearings [6].

Fig. 3 presents the test results with respect to the parallel network model of the tested specimen with minor derivations of between both data. An analog comparison of the values of equivalent viscous damping $\xi(\epsilon_{\max}, \epsilon)$ of numerical and experimental data shows that a satisfying agreement of the data was only achieved for large strain amplitudes ϵ , as mentioned previously. The influence of prestrain showed a negligible change in the damping values [6].

Fig. 3. Secant shear modulus of the elastomer dependent on the prestrain $\varepsilon_{\max}$ and the actual strain amplitude ε compared to the results of the parallel network model [6]

Material models for modeling of elastomer

The appropriate material model strongly depends on the issue of interest. The Neo-Hooke model with constant shear modulus allows (a) an isolated view on the mechanical bearing properties and (b) the simplified investigation of geometrical influences on the bearing stiffness [6]. The appropriate description of the damping value requires the selection of a hysteresis incorporated material model as documented in [6].

Material model for the modeling of carbon and steel sheets

For conventional bearings reinforced with steel plates the choice of the linear elastic isotropic model is obvious as the tensile stress has to be below the yield stress to avoid fatigue. Fiber reinforcement required more detailed considerations [9].

Tensile tests on glass and carbon fiber fabric vulcanized in elastomer sheets showed very low stiffness at the beginning of the first loading cycle. Up to a strain value of $\varepsilon = 0.5$ to 1.5 % the elastic modulus is by a factor of 10 to 50 smaller than the maximum measured value [6]. Subsequent loading cycles (maximum tensile forces between 10 and 40 N/mm) with almost complete unloading were carried out to measure the secant modulus without the low stiffness of the initial fiber orientation process.

The elastic modulus of glass fiber fabric (style 03798 from Interglas, raw fiber weight 275 g/m²) reached values of $E_{g,\text{warp}} = 2.2$ to 2.7 kN/mm or $E_{g,\text{weft}} = 3.5$ to 4.0 kN/mm in the warp or weft direction. The respective maximum value of the tensile force of a loading cycle had only minor influence on the elastic modulus. Analogous tests on carbon fiber fabric (style 05208 from Interglas, raw fiber weight 305 g/m²) showed values of $E_c = 7.1$ to 11.3 kN/mm without significant effect of the thread orientation but strong influence of the load level.

Assuming a minimum thickness of reinforcement steel plates of $t_s = 2$ mm according to [2] the stiffness of the fabric is 50 to 150 times lower and therefore has to be considered in the design process. For steel reinforced bearings the reinforcement can be considered as rigid; the vertical bearing stiffness only depends on the shear and compression moduli of the elastomer as well as on the geometry. The estimation of the total vertical deformation of a fiber rein-

forced bearing is difficult due to the large distribution of the initial reinforcement stiffness depending on the assembly process.

More important for the design process and therefore for the development of numerical models is the vertical bearing stiffness under load variations. This partial secant stiffness is the relevant value to calculate differential deformations and is therefore important for the dimensioning of joints and the calculation of the vertical vibration behavior. For dynamic applications the determination of the corresponding damping value is required.

For this purpose tensile tests with partial unloading were carried out on carbon fabric specimens vulcanized on elastomer sheets [6]. The specimens with a length of 120 mm and a width of 30 to 45 mm were loaded with various levels of uniaxial tension $n_{\max} = 10$ to 50 N/mm. The measurements were carried out for repeated cycles of partial unloading of $\Delta n = 5$ to 80 %.

The partial secant moduli $E_f(\Delta n, n_{\max})$ of the carbon fabric specimens (geometry see before) have been between 8.7 and 16.6 kN/mm, as shown in Fig. 4. The corresponding equivalent damping values range from $\xi_f = 2,4$ to 12,8 %.

Fig. 4. Partial secant modulus $E_f(n_{\max}; \Delta n)$ [kN/mm] of carbon fabric specimens at different levels of maximal force $n_{\max}$ (see above) and various unloading amplitudes Δn measured at the third repetition cycle

It is not possible to assign a specific stiffness value to the reinforcement material model as the distribution of the tensile stress in the reinforcement is non uniform: the stress due to vertical load has a maximum value in the center of the bearing and declines to zero at the side faces. Furthermore Fig. 4 shows that neither the choice of a nonlinear model with a strain dependent stiffness is a satisfying solution as there is a strong influence of the load amplitude.

As the stiffness of the reinforcement has only minor influence on the horizontal bearing behavior the choice of the linear elastic model with an average stiffness seems a suitable approximation for investigations on the horizontal bearing stiffness. It can also be used for a limit valuation of the vertical behavior. To avoid in plane shear stiffness when modeling fiber reinforcement the choice of an orthotropic linear elastic material model with shear modulus close to zero is recommended.

The share of the internal work performed in the reinforcement and its influence on the bearing damping value can be estimated in a separated consideration based on the results of the material test data [6]. These investigations show that the additional damping caused by the variation of the tensile force in the fabric can be significant especially for the vertical direction.

FINITE ELEMENTS FOR MODELING OF ELASTOMER AND REINFORCEMENT

The choice of an appropriate type of finite elements strongly depends on the geometry of the structure and on the occurring material distortion. The decision is crucial to the quality of the numerical results and to the computational costs; a suitable compromise has to be arrived.

Element type of the elastomer

The numerical simulation of bearings with horizontal or rotational deformations requires three dimensional models. The reduction to a computationally cheap model of rotationally symmetric solid elements is only possible for circular bearings with vertical deformations. For rectangular bearings the choice of hexahedral (brick-shaped) elements is obvious. Common finite element programs provide elements with first or second order (linear or quadratic) approach for the strain throughout the element. These elements can be used with full or reduced integration, see Fig. 5, and diverse additional formulations.

First order brick elements use eight nodes and therefore are cheaper in calculation than second order elements with 20 nodes. Due to the linear deformation between two nodes of a first order element the parabolic lateral bulging of the elastomer in between two reinforcement layers (as a result of vertical deformation) is approximated by a polygonal shape. This may lead to an overestimation of the vertical stiffness of the bearing. The horizontal stiffness of the bearing model is barely influenced by the choice of first or second order formulation as horizontal deformations particularly lead to constant shear deformations of the elastomer elements.

In addition, the effects of shear locking and hourglassing by the modeling of Finite Elements depend strongly on the element type [7]: The bending deformation of the full integrated elements (see Fig. 5) due to the subjected moment result in a change in length of the horizontal lines that pass through the integration points. The length of the vertical lines does not change. In addition the angle between horizontal and vertical lines remains rectangular in the second order elements but changes in the first order elements. The material of the first order element suffers a shear deformation due to bending. This unrealistic shear strain entails an additional stiffness called shear locking.

Fig. 5. Illustration of first and second order elements with full or reduced integration; occurrence of shear locking and hourglassing effect

To eliminate this effect ABAQUS provides first order elements that are enhanced by incompatible modes. In addition to the standard displacement degrees of freedom additional deformation modes are defined to prevent shear locking [7]. The costs in calculation are still smaller than those of second order elements.

The computational effort can be additionally restricted by the use of reduced integration. First order elements with reduced integration use only one integration point on each element surface as shown in Fig. 5. It has to be noticed that a bending deformation of these elements does not result in a change in length of the lines passing through the integration points and therefore does not result in a reaction force [7]. Therefore the elements may tend to instability which can be observed as hourglassing effect. The effect causes distorted elements as shown in Fig. 6. This effect may occur especially at models with high vertical compression, large horizontal deformations or with complex material deformations (e.g. parallel network model). Leading finite element programs provide several numerical tools to control hourglassing to a certain degree by adding artificial correction forces [7]. Comparative analyses showed that a singular occurrence of the effect has no significant effect on the horizontal stiffness but reduces the vertical stiffness. The hourglass controlling also increases the internal energy resulting in an incorrect high damping value [6].

Fig. 6. Corner section of a bearing model with hourglassing effect with typical rhomboid element distortion

In summary it can be stated that:

- First order elements with reduced integration (Fig. 5, b) lead to the lowest computational costs but may lead to a significant hourglassing effect
- First order elements with full integration (Fig. 5, a) prevent hourglassing but may suffer shear locking if bending deformations occur. For bearings that are fixed to the loading planes (no rolling off) these elements should lead to good results as the elastomer is mainly subjected to shear deformation.
- Second order elements (Fig. 5, c and d) lead to the best results but also cause the largest computational costs.

In addition the large compression modulus of elastomer can lead to major changes in compression even for small deformations. This may lead to convergence problems or to rounding differences of the numerical solution. ABAQUS provides hybrid formulations of the elements which calculate the compression using independent system variables to avoid this effect [7] but only for implicit time integration analysis and not for explicit central-difference time integration analysis.

Due to all these considerations most numerical studies in [6] are based on bearing models of first order elements with reduced integration. Only in some cases elements with full integration have been used for damping calculations.

Finite elements for the modeling of the reinforcement

The thickness of the reinforcement layers is usually small compared to the thickness of the elastomer layers. Therefore the additional height of the reinforcement has only minor influence on the bearing behavior. The shear deformation of steel reinforcement plates is negligible compared to the shear deformation of the elastomer. When fiber reinforcement layers are used there might be a small share of additional shear deformation in the fabric that has to be analyzed. In general the use of shell or membrane elements without geometrical thickness can be assumed to be suitable.

The tensile stress and strain is domination in the reinforcement. The plane shear deformation has only minor influence on the bearing behavior. Therefore four node linear elements can be considered as appropriate.

The horizontal deformation of elastomeric bearings leads to a bending of the reinforcement layers. The amount of out of plane bending is significant when unfixed bearings (bearings that are not fixed to the horizontal loading planes as seen in

Fig. 8) start to roll off from the horizontal loading planes [10]. Therefore the fiber reinforcement was modeled using (flexible) membrane elements instead of shell elements with a bending stiffness related to the calculated thickness.

Modeling of the whole elastomeric bearings

According to the current European standard [2] elastomeric bearings can be designed with or without covering steel plates at the bottom or top of the bearing. The steel plates allow the fixing of the bearings to the structure to prevent rolling off from the loading planes or horizontal slipping due to insufficient friction. The boundary conditions of fixed bearings can be realized by coupling the nodes of each surface to a reference node which is either fixed or is used for the application of loads or deformations.

The loading and deformation of unfixed bearings requires the definition of two rigid planes with related reference points as shown in Fig. 7. In addition an appropriate contact definition allows the lifting off of the elastomer body in cause of large deformations while the horizontal reaction force is transferred by remaining contact areas as shown in

Fig. 8. A number of twenty elements in horizontal direction proofed to lead to an acceptable smoothness of the rolling movement (see Fig. 11) with small computational effort.

Fig. 7. Finite element model of a square bearing of side length a (with $n = 4$ layers of reinforcement) with rigid loading planes; only half of the bearing was modeled (x - y plane = symmetry plane)

Each elastomeric layer between the reinforcement was modeled with three elements in the height. A comparison of models with a number of three, four and five elements in height showed that the resulting variation of the vertical stiffness is less than 5 % for common bearing dimensions [6]. The influence on the horizontal stiffness is even less. The top and bottom cover of elastomer of unfixed bearings usually have a reduced thickness and were therefore modeled with only two elements in height.

Bearing models that are used for simulations of vertical compression and horizontal deformation tests (parallel to one of the vertical symmetry planes) can be reduced to one half due to the symmetry as shown in Fig. 7. For symmetry reason a fixed boundary condition in the y-direction is applied to all nodes of the symmetry plane x-z have.

Fig. 8. Rolling off of the unfixed top and bottom plane of the bearing model (left) with contact of the lateral faces to the loading planes for large deformations (right)

The contact definitions of unfixed bearings as well as the nonlinear material behavior of hyperelastic material models result in a nonlinear behavior of the bearing models. Most commercial finite element programs use Newton's method to solve nonlinear problems, which proved to be usable for all models with limited element distortion. When large horizontal deformations are applied strong element distortions can be observed at the edges orthogonal to the deformation direction, see

Fig. 8, detail b. In addition parts of the lateral faces get in contact with the horizontal loading planes constituting a numerical contact problem, see detail a. Even for small iteration steps the program may not find a suitable state of equilibrium.

In this case the use of a quasi static analysis based on the explicit central-difference time integration rule (explicit dynamic analysis) [7] is recommended. This procedure does not require the formation of the target stiffness matrices for each step of iteration to find an equilibrium state for the whole model. The calculation of the state of each element at the end of a time increment is based on the displacement, the velocity and the acceleration resulting from the nodal forces at the beginning of the time increment. The calculation of the model state at each time increment is numerically inexpensive. In order to obtain accurate results, the time increments must be extremely small (e.g. $\Delta t = 1e-6$ sec in a load step of horizontal deformation from 0 to 100 % in $t_{\text{total}} = 2$ sec). Comparisons with variations of the time increments are necessary to judge the quality of the results.

Experience has shown that the explicit dynamic analysis leads to satisfying results even for extreme states of deformation of the bearing models. In addition the process can be parallelized very effectively.

VALIDATION, SCOPE AND LIMITATIONS

The results of the simulations have been validated using data from loading and deformation tests: The comparisons of data from deformation tests and numerical results in Fig. 9 and Fig. 10 demonstrate the compliance of the nonlinear geometrical behavior of the test specimens with respect to the numerical results. The loss of horizontal stiffness due to rolling off and vertical load as well as the rise of stiffness due to lateral contact can be observed.

Fig. 9. Test data versus numerical model with Neo-Hooke material model of an unfixed bearing with low damping ratio [11]; horizontal stiffness $K_h \approx 0$ at $v_x = 100$ % due to rolling off and vertical load [10]

Fig. 10. Data versus numerical model with Neo-Hooke material model of an unfixed bearing with low damping ratio [11]; rising horizontal stiffness K_h for deformations $v_x > 125$ % due to lateral contact [10]

For these models the simple Neo-Hooke material model was used to allow an isolated view on the geometrical bearing behavior. Complex material effects as the hysteretic behavior or

the material degradation are not taken into account. These models can be used as basis for parametric studies on the geometrical influences on the bearing stiffness.

Fig. 11 shows a comparison of data from deformation tests on unfixed bearings with high damping elastomer and numerical results of bearing models with the parallel network material model. The numerical model a) shows a realistic stiffness behavior including material degradation and gives a qualitative impression of the bearing damping due to hysteresis of the elastomer. Evaluations of extensive deformation test data and numerical calculations in [6] show that the quantitative damping results do only agree for very limited conditions. This is because the parallel network material model can not describe the additional inner friction in the elastomeric material dependent on the vertical compression.

Fig. 11. Comparison 3 of test data from unfixed bearings with high damping ratio [6] and numerical simulations with a) parallel network material model and b) Neo-Hooke material model

Nevertheless these hysteretic bearing models can be used to study diverse influences on the damping behavior and can therefore be regarded as a tool of research. To develop models that describe the damping behavior of bearings considering all relevant influences it would be necessary to extend the material model. In addition adequate test data for the calibration process including the influences of material compression should be used.

SUMMARY

The deliberations of this paper provide an overview on the modeling of elastomeric bearings. The influence of the complex material behavior of elastomer and its implementation in the numerical model are discussed from a practical point of view. The differences of various material models are discussed in respect of the calibration, the computational effort and the possible range of studies that can be carried out with the bearing models. A detailed explanation of the pros and cons of different finite element types and of possible compositions of the bearing model is given. Practical experiences supply support for the most important issues of the modeling of elastomeric bearings.

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