PRIORITY SCHEDULING IN THE PLANNING OF MULTIPLE-STRUCTURE CONSTRUCTION PROJECTS

E. RADZISZEWSKA-ZIELINA¹, B. SROKA²

The paper presents a method of priority scheduling that is useful during the planning of multiple-structure construction projects. This approach is an extension of the concept of interactive scheduling. In priority scheduling, it is the planner that can determine how important each of the technological and organisational constraints are to them. A planner's preferences can be defined through developing a ranking list that defines which constraints are the most important, and those whose completion can come second. The planner will be able to model the constraints that appear at a construction site more flexibly. The article presents a general linear programming model of the planning of multiple-structure construction projects, as well as various values of each of the parameters that allow us to obtain different planning effects. The proposed model has been implemented in a computer program and its effectiveness has been presented on a calculation example.

Keywords: time coupling method, linear programming, multiple-structure projects, scheduling, interactive scheduling, priority scheduling

1. INTRODUCTION

General methods of planning construction projects can be divided into two groups: those which assume a non-determined structure of the projects that are being planned, e.g. [2, 19, 20], and those which assume a determined one. Both groups of methods are still being developed. A determined

¹ Assoc. Prof. PhD. Eng., Cracow University of Technology, Faculty of Civil Engineering, ul. Warszawska 24, 31-155 Cracow, Poland, e-mail: eradzisz@izwbit.pk.edu.pl
² M.Sc. Eng., Cracow University of Technology, Faculty of Civil Engineering, ul. Warszawska 24, 31-155 Cracow, Poland, e-mail: bsroka@izwbit.pk.edu.pl
structure of multiple-structure construction projects was assumed in the article, with completion time being a deterministic variable.

The basic methods of planning multiple-structure construction projects are, for instance: Line of Balance (LOB)[1], Horizontal and Vertical Logic Scheduling for Multistory Projects (HVLS)[22], Repetitive Scheduling Method (RSM)[3] and others [5,7,8]. The methods of planning the carrying out of multiple-structure projects that have been presented do not, however, take into account technological and organisational constraints. The time coupling method [14,21] makes it possible to model both technological and organisational constraints that are encountered during the carrying out of multiple-structure construction projects [17].

This article presents an expansion of the concept of the interactive scheduling method presented in publication [11]. The concept of interactive scheduling assumes that the algorithm of the optimisation method will not be dependent on each of the stages of data calculations, which reflect the planning situation. The approach proposed in this article assumes that a planner (usually the construction site director) can define their preferences regarding time couplings. Time couplings will be a mapping of the technological and organisational constraints that are encountered during the carrying out of a multiple-structure construction project. A planner can sort time couplings, indicating which of them are a priority and need to be completed, and which are secondary and their completion is of lesser significance. In general, these couplings can, but do not need to be, dependent on the order in which structures are built. The appropriate determining and implementation of a planner's organisational preferences can have a positive impact on the relations between a general contractor and subcontractors [16, 18], reduce the probability of the occurrence of organisational problems that influence the proper course of a project's implementation [4], limit the amount of problematic decisions that need to be made during such an implementation [10], help in the selection of appropriate contractor or subcontractors during the stage of preparing to initiate the carrying out of a project [6,9] and serve as an application that can support software based both on BIM [12] and other types of technology[13].

The goal of the article is to create a model of linear programming that can be used to determine an optimal schedule of the carrying out of a multiple-structure project, taking into account the technological and organisational preferences of a planner. The possibility to change the order of the selection of the structures has been omitted in the proposed model.
2. PRIORITY SCHEDULING LINEAR MODEL

We are presented with a multiple-structure construction project that is being carried out using a pipeline work system. There are \( m \) tasks to be completed by \( m \) specialised brigades on \( n \) structures. The duration of each task is deterministic and known from the outset. Once a task has begun it cannot be stopped. The following sets of index pairs have been determined below:

\[ \hat{O} = \{(1,1),(2,1),(n-1,1),(1,2),(n,2),\ldots,(n-1,m)\} \] - helpful in determining time couplings between the tasks performed by a brigade on different structures;

\[ \hat{B} = \{(1,1),(2,1),(n,1),(1,2),\ldots,(m-1,m)\} \] - helpful in determining time couplings between the tasks performed by different brigades on a single structure;

\[ \hat{W} = \{(1,1),(2,1),\ldots,(n,1),(1,2),\ldots,(n,m)\} \] - the set of index pairs for all tasks.

A model of linear programming, which will be called model A, has been developed for the conditions and sets defined as above.

Data: \( t_{i,j} \)

Parameters: \( t_{c_{ij}} \), \( t_{c_{ij}}^{eg} \), \( t_{c_{ij}}^{bd} \), \( t_{c_{ij}} \), \( t_{c_{ij}}^{bg} \), \( c_{w_{ij}}^{eg} \), \( c_{w_{ij}}^{bd} \), \( c_{w_{ij}} \), \( c_{w_{ij}}^{bg} \)

Decision variables: \( W_{R_{i,j}}, W_{Z_{i,j}}, W_{P_{i,j}}, Z_{C_{i,j}}, c_{k_{ij}}^{ad}, c_{k_{ij}}^{eg}, c_{k_{ij}}^{bd}, c_{k_{ij}}^{bg} \)

Goal function:

\[ (2.1) \quad \min FC = KC_{O} + KC_{B} + KT + NWZ_{n,m} \]
\[ (2.2) \quad KC_{O} = \sum_{(i,j) \in \hat{O}} (c_{k_{ij}}^{ad} \cdot c_{w_{ij}}^{eg} + c_{k_{ij}}^{eg} \cdot c_{w_{ij}}^{bd}) \]
\[ (2.3) \quad KC_{B} = \sum_{(i,j) \in \hat{B}} (c_{k_{ij}}^{bd} \cdot c_{w_{ij}}^{eg} + c_{k_{ij}}^{bg} \cdot c_{w_{ij}}^{bg}) \]
\[ (2.4) \quad KT = \rho \cdot \sum_{(i,j) \in \hat{W}} (NWZ_{i,j} - NPZ_{i,j}), \text{ where } \rho << 1 \]

Constraints:

\[ (2.5) \quad NWZ_{i,j} = W_{R_{i,j}} + t_{i,j}, \text{ where } (i,j) \in \hat{W} \]
\[ (2.6) \quad NPZ_{i,j} = W_{P_{i,j}} + t_{i,j}, \text{ where } (i,j) \in \hat{W} \]
\[ (2.7) \quad ZC_{i,j} = NPZ_{i,j} - NWZ_{i,j}, \text{ where } (i,j) \in \hat{W} \]
\[ (2.8) \quad W_{R_{i+1,j}} \geq NWZ_{i,j} + t_{c_{ij}}^{ad} - c_{k_{ij}}^{ad}, \text{ where } (i,j) \in \hat{O} \]
\[ (2.9) \quad W_{R_{i+1,j}} \leq NWZ_{i,j} + t_{c_{ij}}^{eg} + c_{k_{ij}}^{eg}, \text{ where } (i,j) \in \hat{O} \]
\[ (2.10) \quad W_{P_{i+1,j}} \geq NPZ_{i,j} + t_{c_{ij}}^{ad} - c_{k_{ij}}^{ad}, \text{ where } (i,j) \in \hat{O} \]
The model A that has been presented requires the introduction of the task performance duration for tasks performed on structure $i$ by brigade $j$ ($t_{i,j}$).

The parameter models are: $t_{i,j}^{od}, t_{i,j}^{og}$ - the value of the limiting lower and upper time couplings between work on successive structures, as well as their respective unit weights $c_{i,j}^{od}, c_{i,j}^{og}$ for failing to ensure a coupling; $t_{i,j}^{bd}, t_{i,j}^{bg}$ - the value of limiting upper and lower time couplings between the work of successive brigades and their respective unit weights $c_{i,j}^{bd}, c_{i,j}^{bg}$ for failing to ensure a coupling.

The model's decision variables are: $NWR_{i,j}, NWZ_{i,j}, NPR_{i,j}, NPZ_{i,j}, ZC_{i,j}, c_{i,j}^{od}, c_{i,j}^{og}, c_{i,j}^{bd}, c_{i,j}^{bg}$ - auxiliary variables, that allow us to determine by how much did the lower and upper time coupling for performing tasks on structures, as well as the lower and upper time coupling for the work of brigades miss their marks.

The $cw$ parameters and $ck$ variables require additional elaboration. The $cw$ parameters mark the unit increment of the goal function, in the event that the respective $tc$ time coupling fails to be ensured.

For instance: if the value of the lower limit is $t_{i,j}^{od} = 3$ while $c_{i,j}^{od} = 100$, then if the calculated coupling between structure O1 and O2 is 1 (which means that $NWR_{1,2} - NWZ_{1,2} = 1$) then the value of the goal function will reach 200 (because the value of the ancillary variable is $c_{k_{1,2}} = 2$ – according to condition 2.8). If, however, the real coupling reaches a value equal to or higher than 3, then it will not have an impact on the value of the goal function ($c_{k_{1,2}} = 0$ - according to condition 2.8).
The goal function (2.1) is the sum of a number of elements. Based on formulas (2.2) and (2.3) we can calculate the value of the failure to ensure lower and upper time coupling between structures and brigades. Formula (2.4) will cause the determining of the shortest and longest completion times for each task. The parameter $\rho$ takes on a very low value (a parameter value lower by at least two orders of magnitude than the minimum task duration time was experimentally assigned) This will lead to a situation in which the value of variables responsible for the shortest and longest completion times will be correctly assigned, while the impact on the goal function will be negligible. In addition, the goal function includes the completion time of the entire project. The goal function will be minimised.

The following constraints are featured in the model. Formula (2.5) makes it possible to link the shortest initiation and completion times of all tasks performed during the carrying out of a project. Formula (2.6) makes it possible to link the longest times of the initiation and completion of all tasks performed during the carrying out of a project. Formula (2.7) makes it possible to determine the total amount of spare time. Formulas (2.8-2.15) preserve the dependencies of CPM networks, taking into account lower and upper time couplings both for structures and brigades (for the shortest and longest completion times), in addition to making it possible to determine by how much time did the time couplings miss their mark ($ck$ variables). Formula (2.16) implements the assumption that the earliest completion time of the project is equal to the longest project completion time. All the variables take on non-negative values (2.17).

Various types of models, whose use can produce different planning effects have been presented in table 1. In table 1, $A$ means a sufficiently large number. It is difficult to determine the precise value of parameter $A$. An experimental analysis showed that a parameter value that is higher by at least two orders of magnitude than the maximum of completion times needs to be assumed. In addition: $A << B << C ... << X << Z$ ($A << B$ means that parameter B is much higher than A - it was experimentally determined that it is higher by two orders of magnitude).

Model A.1 meets the constraints of the CPM method (in time coupling theory it is a TCM III model). Each task can begin only when all the preceding ones have been completed. Model A.2 serves to model the continuity of the work of brigades (TCM I). Model A.3 models the continuity of work on structures (TCM II). In Model A.4, the decision-maker determines the structures and brigades for which work continuity is to be ensured. The structures for which work continuity is to be ensured are marked by the set $\bar{K}$. The set $\bar{L}$ defines which brigades are to have their work continuity ensured. In model A.5, the performance of work by a brigade on different structures has been allowed, with an
overlap of a maximum of $s$ days. In model A.6, the possibility of performing work on a single structure by multiple brigades has been allowed, with the overlap of the work performed by each brigade having a maximum of $s$ days. Model A.7 simultaneously implements the constraints for models A.5 and A.6.

Models A.8a and A.8b implement the concept of priority scheduling. Through priority scheduling we should understand scheduling in which the decision-maker (planner) determines the priority (importance) of each technological and organisational constraint. Model A.8a allows the possibility for the overlap of work on structures or the work of an individual brigade on multiple structures. Model A.8b does not allow such a possibility. The planner’s preference regarding the priority of couplings have been determined using the sets: $\mathbf{B}, \mathbf{C}, ..., \mathbf{X}$. Set $\mathbf{B}$ defines the lowest priority, while set $\mathbf{X}$ the highest. Every set is determined as follows: $\mathbf{B} = \{ (y_1, i, j_1), (y_2, i, j_2), ..., (y_k, i, j_k) \}$, where $k$ defines the amount of couplings of the same priority, $y_l$ determines the type of coupling $l$: $y_l \in \{ o, b \}$ ($o$ - couplings between structures, $b$ – couplings between brigades), the parameters $i, j$ define between which tasks should coupling be implemented. In the software dedicated to planners, users will not introduce the values of individual parameters independently. This requires the development of more accessible options for the software, such as a multiple choice list, setting up a ranking in the form of a list, the graphical introduction of couplings and determining their priorities in the form of a numerical order. Then, the algorithm implemented in the program would select the appropriate parameter values based on the options chosen by the user.

In the case of model A.1, A.2, A.3, A.5, A.6, A.7 the value of the goal function will be approximately equal to the completion time of the entire project. This is a result of the fact that in the goal function formula, one of the elements (formula 2.4) is responsible for determining the shortest and longest times. The influence of this factor is negligible in comparison to the completion time of a multiple-structure construction project, because the value of the parameter $\rho << 1$.

For model A.4, A.8a and A.8b, the value of the goal function can reach a value close to the project completion time when all of a planner’s preferences have been met. In the case when meeting a planner’s preference is impossible, the value of the goal function informs to what degree the required conditions had not been met.
<table>
<thead>
<tr>
<th>Type of model</th>
<th>Value of parameters</th>
<th>Planning effect</th>
<th>Goal function value</th>
</tr>
</thead>
</table>
| A.1 | \( t_{ci,j}^{og}, t_{ci,j}^{bg}, cw_{ci,j}^{od}, cw_{ci,j}^{og}, cw_{ci,j}^{bd}, cw_{ci,j}^{bg} = A \)  
\( t_{ci,j}^{ad}, t_{ci,j}^{bd} = 0 \) | Constraints like in the CPM method. | \( FC = NWZ_{n,m} \) |
| A.2 | \( t_{ci,j}^{bg}, cw_{ci,j}^{od}, cw_{ci,j}^{og}, cw_{ci,j}^{bd}, cw_{ci,j}^{bg} = A \)  
\( t_{ci,j}^{ad}, t_{ci,j}^{cd}, t_{ci,j}^{bd} = 0 \) | Continuity in the work of brigades. | \( FC = NWZ_{n,m} \) |
| A.3 | \( t_{ci,j}^{og}, cw_{ci,j}^{ad}, cw_{ci,j}^{og}, cw_{ci,j}^{bd}, cw_{ci,j}^{bg} = A \)  
\( t_{ci,j}^{ad}, t_{ci,j}^{cd}, t_{ci,j}^{bd} = 0 \) | Continuity of work on structures | \( FC = NWZ_{n,m} \) |
| A.4 | \( cw_{ci,j}^{og}, cw_{ci,j}^{bg} = A \), if \( j \notin K \)  
\( cw_{ci,j}^{bd}, cw_{ci,j}^{bg} = A \), if \( i \notin L \)  
\( cw_{i,j}^{ad}, cw_{i,j}^{og} = B \), if \( j \in K \)  
\( cw_{i,j}^{bd}, cw_{i,j}^{og} = B \), if \( i \in L \)  
\( t_{ci,j}^{og} = A \), if \( j \notin K \)  
\( t_{ci,j}^{bg} = A \), if \( i \notin L \)  
\( t_{ci,j}^{ad}, t_{ci,j}^{bd} = 0 \)  
\( t_{ci,j}^{og} = 0 \), if \( j \in K \)  
\( t_{ci,j}^{bg} = 0 \), if \( i \in L \) | Continuity of work on structure \( k \in K \) and continuity in the work of brigades \( l \in L \). | \( FC = NWZ_{n,m} \)  
If:  
\( ck_{ij}^{ad}, ck_{ij}^{og}, ck_{ij}^{bd}, ck_{ij}^{bg} = 0 \)  
then: \( FC = NWZ_{n,m} \)  
Otherwise:  
\( FC \) determines the degree of meeting a planner's preferences |
| A.5 | \( t_{ci,j}^{og}, t_{ci,j}^{bg}, cw_{ci,j}^{ad}, cw_{ci,j}^{od}, cw_{ci,j}^{og}, cw_{ci,j}^{bd}, cw_{ci,j}^{bg} = A \)  
\( t_{ci,j}^{ad} = -s \)  
\( t_{ci,j}^{bd} = 0 \) | Possibility of performing tasks on different structures by an individual brigade, with an \( s \) amount of days of work overlap | \( FC = NWZ_{n,m} \) |
| A.6 | \( t_{ci,j}^{og}, t_{ci,j}^{bg}, cw_{ci,j}^{ad}, cw_{ci,j}^{od}, cw_{ci,j}^{og}, cw_{ci,j}^{bd}, cw_{ci,j}^{bg} = A \)  
\( t_{ci,j}^{og} = 0 \)  
\( t_{ci,j}^{bg} = -s \) | Possibility of performing tasks on one structure by a group of brigades, with an \( s \) amount of days of work overlap | \( FC = NWZ_{n,m} \) |
| A.7 | \( t_{ci,j}^{og}, t_{ci,j}^{bg}, cw_{ci,j}^{ad}, cw_{ci,j}^{od}, cw_{ci,j}^{og}, cw_{ci,j}^{bd}, cw_{ci,j}^{bg} = A \)  
\( t_{ci,j}^{ad}, t_{ci,j}^{bd} = -s \) | Combination of models 5 and 6 | \( FC = NWZ_{n,m} \) |
Model A has been implemented in the Python programming language. The PyMathProg [15] environment has been used to solve the model. PyMathProg is an environment which is used to model, solve and analyse linear programming problems. The environment uses the GLPK solver (GNU Linear Programming Kit). GLPK uses the Simplex method to solve linear programming problems. The script, along with solved calculation examples, had been written in the Jupyter application and posted to GitHub3. The program’s code is also available at the author’s discretion.

### 3. Calculation example

In order to present the models that have been shown in the article, a project composed of 3 structures has been used. Each of the structures are to have 4 types of tasks performed on them. The completion

---

3 Source code: https://github.com/bsrokapk/TCM/blob/master/TCM.ipynb
time of each task has been shown in table 2. Every model (A.1-A.8) has been calculated for this data, with the results shown in table 3.

Table 2. Task completion times for tasks performed by brigade B on structure O.

<table>
<thead>
<tr>
<th></th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>O2</td>
<td>9</td>
<td>4</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>O3</td>
<td>10</td>
<td>7</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3. Results obtained for the calculation example, for models from A.1 to A.8

<table>
<thead>
<tr>
<th>Model</th>
<th>Constraints</th>
<th>Goal function value</th>
<th>Completion time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>Constraints like in the CPM model</td>
<td>= 44</td>
<td>44</td>
</tr>
<tr>
<td>A.2</td>
<td>Continuity of the work of brigades</td>
<td>= 48</td>
<td>48</td>
</tr>
<tr>
<td>A.3</td>
<td>Continuity of work on structures</td>
<td>= 45</td>
<td>45</td>
</tr>
<tr>
<td>A.4</td>
<td>Continuity of work for structure 3 and brigade 3</td>
<td>= 244</td>
<td>44</td>
</tr>
<tr>
<td>A.5</td>
<td>One brigade can work on several structures (1 day of overlap)</td>
<td>= 42</td>
<td>42</td>
</tr>
<tr>
<td>A.6</td>
<td>More than one brigade can work on one structure (a day of overlap)</td>
<td>= 41</td>
<td>41</td>
</tr>
<tr>
<td>A.7</td>
<td>A combination of A.5 and A.6</td>
<td>= 39</td>
<td>39</td>
</tr>
<tr>
<td>A.8a</td>
<td>Priority scheduling: 1) Continuity of work of brigade 3 2) Continuity of work on structure 2 3) Continuity of work of brigade 2</td>
<td>= 840</td>
<td>40</td>
</tr>
<tr>
<td>A.8b</td>
<td>Priority scheduling: 1) Continuity of work of brigade 3 2) Continuity of work on structure 2 3) Continuity of work of brigade 2</td>
<td>= 6020046</td>
<td>46</td>
</tr>
</tbody>
</table>

Detailed solutions have been provided for types A.8a and A.8b. The solution shown on figure 1 has been achieved for model A.8a. All of the continuity conditions specified by the planner have been met. This, however, required an overlap of the work of brigade 1 on structure 1 and 2 for 6 days, as well as an overlap of the work performed by brigades 2 and 3 on structure 1 for 2 days.
It was not possible to meet all of the planner's expectations for model A.8b. The solution has been presented on figure 2. There is no possibility of work overlap in this model. The priority preference, and thus ensuring the continuity of the work of brigade 3, has been met. It was not possible to ensure the continuity of work on structure 2 (lack of continuity over a total of 6 days), in addition to ensuring the continuity of the work of brigade 2 (lack of continuity over a period of 2 days).

4. CONCLUSIONS

The model that has been presented is useful in the planning of multiple-structure construction projects. Priority scheduling is an innovative approach to the planning of multiple-structure projects that makes it possible to easily take into account a planner's preferences regarding technological and organisation constraints at a construction site. The greatest flaw of the proposed model is that a planner needs to know the completion times of all the tasks on all structures during a schedule's development. In addition, in the current version of the software, it is the planner that must assign
appropriate weights to each parameter in order to obtain appropriate planning effects. The implementation of the proposed method in a digital system that will possess a graphical user interface will eliminate the problem of setting weights by the planner by hand. In the future, the model will be expanded to include the possibility of altering the order of the construction of structures.

REFERENCES


**LIST OF FIGURES AND TABLES:**

Fig. 1. Detailed solution obtained for model A.8a
Rys. 1. Rozwiązanie szczegółowe uzyskane dla modelu A.8a
Fig. 2. Detailed solution obtained for model A.8b
Rys. 2. Rozwiązanie szczegółowe uzyskane dla modelu A.8b
Tab. 1. Parameter values depending on the desired planning effect
Tab. 1. Wartość parametrów w zależności od oczekiwanej efekt planistycznego
Tab. 2. Task completion times for tasks performed by brigade B on structure O.
Tab. 2. Czas trwania pracy wykonywanej przez brygadę B na obiekcie O
Tab. 3. Results obtained for the calculation example, for models from A.1 to A.8
Tab. 3. Otrzymane wyniki dla przykładu obliczeniowego dla modeli od A.1-A.8

Received 31.05.2017
Revised 29.11.2017
HARMONOGRAMOWANIE PRIORYTETOWE W PLANOWANIU PRZEDSIĘWZIĘĆ WIELOOBIEKTOWYCH

Słowa kluczowe: metoda sprzężeń czasowych, programowanie liniowego, przedsięwzięcia wieloobiektowe, harmonogramowanie, harmonogramowanie interaktywne, harmonogramowanie priorytetowe

STRESZCZENIE: