Preface

For the description of modern technical and technological devices and systems, mathematical models are used that incorporate different types of differential equations, including ordinary differential equations, partial differential equations, operator equations, etc. The stability analysis of the solutions to such equations is successfully carried out by the classical methods developed in the stability theory of motion. These general methods are the direct Lyapunov method, the method of integral inequalities and the comparison technique.

One of the first models that described the complex structure of a real system was a hybrid Wittenenhausen system (see [27] Chapter 1). In this model, the system state is determined by two components: continuous and discrete time. The continuous system state is described by a system of ordinary differential equations the right-side part of which depends on the discrete state. The discrete state is changed when the continuous state gets into some region of the state space.

"Hybridity" of the mathematical model of a real system occurs when its behaviour is described by different types of equations. The examples of such physical systems are:

- continuous systems with phase changes (bouncing ball, walking robot, the growth of biological cells and their division);
- continuous systems controlled by discrete automation devices (thermostat, chemical production with discretely introduced catalysts, autopilot);
- coordinated processes (aircraft takeoff and landing in a large airport, control of car streams on autobahns).

Also, the term "hybrid system" is currently used to describe the dynamics of objects containing neural networks, various fuzzy logic devices, electrical and mechanical components in complex systems and many other cases. Another important example of hybrid systems is systems consisting of a digital control system and a continuous component describing the model of the process under consideration. Traditionally, in the study of such systems, the mathematical model of the continuous component is discretised and, as a result, a system of difference equations is obtained that is to be analysed. Such an approach may not

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be acceptable in the modern theories of robust control, where both continuous and discrete components play an important part in their natural representation.

Therefore, the construction of the stability theory of hybrid and shock systems is an important scientific challenge at the present stage of development of this academic direction.

Chapters 1 – 6 discuss stability problems of certain classes of hybrid and shock systems. Namely:

- the hybrid systems on a time scale, whose behaviour is described by dynamic equations (not to be confused with the dynamic systems in the Nemytsky–Stepanov sense[1*]);
- the hybrid systems with aftereffects under pulse perturbations;
- the hybrid weakly coupled systems, whose subsystems are defined in the Banach spaces;
- mechanical systems with impacts described by the Poincaré mapping;
- the bouncing ball model expressed by discrete mapping and difference equations;
- common recurrence equations and inequalities, linearization techniques and global estimates.

The above systems, under certain assumptions, fall into the category of hybrid dynamic systems, as they include at least two different types of equations that describe the dynamics of independent subsystems.

Chapter 1 provides some knowledge of mathematical analysis on the time scale applicable in the stability investigation of a hybrid system on the time scale. Here, the main approaches used in the analysis of motion stability on the time scale are discussed. These approaches are based on the dynamic integral inequalities, the generalised direct Lyapunov method and the comparison principle. The development of these approaches is based on the results of the classical stability theory of motion adapted to the problems under consideration regarding mathematical analysis on the times scale.

Chapter 2 addresses hybrid systems with aftereffects under impulse perturbations. An impulse system is considered to be hybrid if, in the formation of the dynamic behaviour of the entire system, its components (continuous and discrete) play an equal part. Besides, a number of the conditions of various types of stability are based on the matrix-valued functions defined on the product of spaces. This class of auxiliary functions allows one to simplify the application of

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[1*] V.V. Nemytsky, V.V. Stepanov. Qualitative theory of differential equations. – 1949 (Russia), Moscow, GITL. English Transl. Princeton, Princeton Univ. Press, 1960. – 534 p.

the direct Lyapunov method for the systems with after-effects and impulse perturbation.

Chapter 3 presents the results of the stability analysis of hybrid systems consisting of subsystems given in some Banach spaces. The analysis is done by the method of Lyapunov vector functions, whose components are associated with independent subsystems. In the case of weakly coupled subsystems, the algebraic conditions of various types of stability are established. The conditions are expressed in terms of the special matrices property of having a fixed sign under the constraints on the small parameter value at the subsystem coupling functions.

Chapter 4 is devoted to the research of hybrid systems with impacts in which moments of collisions are not predetermined. The Poincaré mapping is employed for illustrating and solving the problem of the designation of moments of the perturbations. Stability conditions for periodic points are formulated for a general form of the discrete periodic mapping; explanations of how to investigate the stability of periodic points for fixed and multi-cycle cases are demonstrated as well. Because of the analysis of hybrid systems with impacts in the extended phase space, it has been asserted that, if solutions of the equation of motion are known for every initial condition, then and only then a hypersurface of jumps of state variables is determined and can be computed.

Chapter 5 deals with the dynamic problems implied by mathematical models of vibro-impacting systems embodied by some hybrid systems with impacts. Particularly, the motion of a material point and its impacts with the moving limiter are researched. Special consideration is given to periodic events such as one impact every period, one impact every multiple period and multiple impacts every one period of motion of the limiter. In all of these occurrences, the dynamic behaviour of the system and the stability of fixed points is investigated. Furthermore, specific instances of the chaotic motion are examined and reported for specific values of control parameters. The numerical solutions reveal a broad variety of non-linear behaviours encompassing irregular non-periodic orbits, the subharmonic motion, chaotic episodes, grazing and chattering processes.

Chapter 6 provides a broad look at the recurrence equations and their properties; difference equations and inequalities. It includes the linearization technique for a general form of implicit difference equations and some global estimates of solutions of difference equations. Conditions of stability of periodic points of the discrete periodic mapping are established. In applications dealing with complex systems, the researchers are quite often more interested in describing and understanding the time evolution of the solutions of non-linear equations, rather than in finding their exact values. Thanks to the difference approach and its association with the Poincaré mapping, evaluation of the time evolution becomes not only possible but also easily accessible.

Some features of this book include the following. It is the first book in which:

(a) the stability of a two-component hybrid system is investigated on the time scale;

(b) the stability of a hybrid impulse system with aftereffects is studied by the direct Lyapunov method based on the matrix functions;

(c) for the stability analysis of a hybrid system in the Banach space a generalisation of the direct Lyapunov method is proposed;

(d) stability investigations of periodic points and k-cycles for systems with impacts are effectively carried out;

(e) the dynamics of a material point moving in a gravitational, viscous field and colliding with a moving limiter is modelled and investigated as a continuous-time and discrete-time dynamic problem;

(f) some novel difference inequalities and new qualitative properties of the difference system solutions are explored.

This book also shows that different models of hybrid and shock systems described by the systems of ordinary differential equations, systems of dynamic equations on time scales, systems of impulse equations with aftereffects and equations in the Banach space allow the generalised direct Lyapunov method to be applied for the analysis of motion stability.

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