

## Economic Growth and Inequality with Tourism in an Integrated Walrasian-General Equilibrium and Neoclassical-Growth Theory\*

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**Abstract:** *This paper studies dynamic interdependence between economic growth, tourism, and inequalities in income and wealth in a small open economy. We build the dynamic model in an integrated Walrasian-general equilibrium and neoclassical-growth theory for a small open economy with multiple sectors and heterogeneous households in a perfectly competitive economy. The economy consists of one service sector which supplies non-traded services and one industrial sector which produces traded goods. We treat wealth accumulation and land distribution between housing and supply of services as endogenous variables. We show that the motion of the economy with  $J$  types of households is given by  $J$  nonlinear differential equations. We simulate the motion of the system with three groups of households. We also conduct comparative dynamic analysis with regards to the rate of interest, the price elasticity of tourism, the global economic condition, and the rich class' human capital, and the rich class' propensity to consume housing.*

**Keywords:** tourism; price elasticity of tourism; heterogeneous households; capital accumulation; land rent; integrated Walrasian and neoclassical-growth theory

**JEL Classification:** O, O4, O41

### Introduction

Tourism has played increasingly important role in regional and national economies partly due to rapid economic globalization and economic growth in different parts of the world. Tourism has a special character in that it converts some non-traded

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\* Acknowledgements: The author is thankful for the constructive comments by the two anonymous referees. The author also likes to show the gratitude to the financial support from the Grants-in-Aid for Scientific Research (C), Project No. 25380246, Japan Society for the Promotion of Science.

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goods into tradable ones. Foreign tourism is an important source of income and employment in some economies. Tourism has interdependent relations with economic growth and economic activities. This study examines the dynamic interdependence in a general equilibrium framework. Tourism has caused increasingly more attention in the literature of economics (e.g., Sinclair and Stabler, 1997; Hazari and Sgro, 2004; and Hazari and Lin, 2011). Nevertheless, as Chao *et al.* (2009) reviewed, the study of tourism has been mainly static. The necessity of building dynamic models with tourism has been well recognized. There only a few models of growth with tourism on microeconomic foundation. Another important issue is related to economic structural changes with tourism. As tourism uses national resources, development of tourism affects economic structure (e.g., Corden and Neary, 1982; Copeland, 1991; and Oh, 2005). This study studies tourism and economic growth on basis of Uzawa's two-sector growth model in context of a small-open economy (Uzawa, 1963; Galor, 1992; Mino, 1996; Cremers, 2006; Li and Lin, 2008; and Stockman, 2009). A unique contribution of this study to the literature of economic growth with tourism is that we deal with endogenous income and wealth distribution with any number of types of households.

It is well known that economics still needs an analytical framework for properly dealing with issues related to income and wealth distribution and economic growth with microeconomic foundation, even though there are some models in the literature of economic theory (e.g., Burmeister and Dobell, 1970; Jones and Manuelli, 1997). We study growth and inequality with tourism within an integrated framework of the Walrasian general equilibrium and neoclassical growth theories. The Walrasian general equilibrium theory was initially proposed by Walras. Its mathematical sophistication was carried out mainly in the 1950s by Arrow, Debreu and others (e.g., Walras, 1874; Arrow and Debreu, 1954; Gale, 1955; Nikaido, 1956, 1968; Debreu, 1959; McKenzie, 1959; Arrow and Hahn, 1971; Arrow, 1974; and Mas-Colell *et al.*, 1995). The theory is essential for explaining equilibrium of pure economic exchanges with heterogeneous supplies and households. We apply this theory to explain how economic structure and distribution in income and wealth are determined with given wealth. Although the Walrasian theory is mathematically refined, it is not very useful when we deal with dynamic issues related to wealth accumulation. Walras failed in developing a general equilibrium theory with endogenous saving and capital accumulation (e.g., Impicciatore *et al.*, 2012). Over more than hundred years only a few formal models in economics are built to extend the Walrasian theory to include endogenous wealth (e.g., Morishima, 1964, 1977; Diewert, 1977; Eatwell, 1987; Dana *et al.* 1989; and Montesano, 2008). Nevertheless, no study succeeds in solving the common problem of lacking proper microeconomic foundation for wealth accumulation within the Walrasian framework. It has become clear that the Walrasian general equilibrium theory is effective for analyzing issues related to growth and structural change with wealth and income distribution. Nevertheless, the most well-known

growth model with endogenous capital is the Solow one-sector growth model. As the basic model of the neoclassical growth theory the Solow model has been extended and generalized in different ways. The neoclassical growth theory with the Ramsey approach to households directly models endogenous wealth accumulation with microeconomic foundation. Nevertheless, the Ramsey-based approach is not effective to deal with growth with heterogeneous households. Zhang (2013, 2014) integrates the two theories by applying the approach to household behavior proposed by Zhang (1993). The Zhang model integrates the neoclassical growth theory with the Walrasian general equilibrium theory for studying dynamic interactions among growth, wealth and income distribution, and economic structures. It should be noted that some attempts have been made to introduce neoclassical growth theory into the general equilibrium analysis (e.g., Jensen and Larsen, 2005). As reviewed by Shoven and Whalley (1992:1), “Most contemporary applied general models are numerical analogs of traditional two-sector general equilibrium models popularized by James Meade, Harry Johnson, Arnold Harberger, and others in the 1950s and 1960s.” Only a few formal dynamic models explicitly deal with distribution issues among heterogeneous households in the neoclassical growth theory (Solow, 1956; Burmeister and Dobell, 1970; and Barro and Sala-i-Martin, 1995). It should be remarked that as far as the Walrasian general equilibrium theory and the traditional capital theory are concerned, the issues examined by Polterovich’s approach with heterogeneous capital and heterogeneous households (Polterovich, 1977, 1983; Bewley, 1982; Amir and Evstigneev, 1999) are quite similar to the model in this study. Polterovich’s approach also tries to integrate the Walrasian general equilibrium theory and capital theory. The main difference is in modeling household behavior, in addition to human capital dynamics and ethnic externalities. Polterovich’s approach to household is basically based on the Ramsey model, while this study is based on Zhang’s approach. Zhang (2013) recently integrates the two theories by applying an alternative approach to household behavior proposed by Zhang (1993). This study introduces tourism into an analytical framework of synthesizing the Solow-Uzawa growth and Walras-Arrow equilibrium models. The model is also partly based on a growth model with tourism for a small open economy by Zhang (2012). We organize the rest of the paper as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the model. Section 4 examines effects of changes in some parameters on the economic system over time. The appendix proves the main results in Section 3.

## The Growth Model with Tourism

Like Chao *et al.* (2009), we consider a small-open economy that produces two goods: an internationally traded good (called industrial good) and a non-traded good (called services). Domestic households consume both goods, while foreign tourists consume

only services. There is a single good, called industrial good, in the world economy and the price of the industrial good is unity. Capital depreciates at a constant exponential rate,  $\delta_k$ . The households hold wealth and land and receive income from wages, land rent, and interest payments of wealth. Land is used by the residents and service sector. All markets are perfectly competitive and capital and labor are completely mobile between the two sectors. Capital is perfectly mobile in international market. The population is classified into  $J$  groups, each group with fixed population,  $\bar{N}_j$ . Let  $N$  stand for labor services used for production. The full employment of labor implies

$$N = \sum_{j=1}^J h_j \bar{N}_j, \quad (1)$$

where  $h_j$  are the levels of human capital of group  $j$ .

### *Industrial Sector*

The industrial sector uses capital and labor as inputs. We use subscript index,  $i$  and  $s$ , to denote respectively the industrial and service sectors. Let  $K_j(t)$  and  $N_j(t)$  stand for the capital stocks and labor force employed by sector  $j$ ,  $j = i, s$ , at time  $t$ . We use  $F_j(t)$  to represent the output level of sector  $j$ . The production function of the industrial sector is

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \quad (2)$$

where  $A_i$ ,  $\alpha_i$ , and  $\beta_i$  are parameters. The rate of interest,  $r^*$ , is fixed in international market. The wage rate,  $w(t)$ , is determined in domestic market. The marginal conditions are

$$r_\delta = \alpha_i A_i k_i^{-\beta_i}(t), \quad w(t) = \beta_i A_i k_i^{\alpha_i}(t), \quad (3)$$

where  $k_i(t) \equiv K_i(t) / N_i(t)$  and  $r_\delta \equiv r^* + \delta_k$ . As  $r^*$  is fixed, equation (3) implies

$$k_i = \left( \frac{\alpha_i A_i}{r_\delta} \right)^{1/\beta_i}, \quad w = \beta_i A_i k_i^{\alpha_i}. \quad (4)$$

Hence, we can treat  $k_i$  and  $w$  as functions of  $r^*$  and  $A_i$ .

### Service Sector

The service sector employs three inputs, capital  $K_s(t)$  labor force  $N_s(t)$  and land  $L_s(t)$  to produce services. We specify the production function as

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t) L_s^{\gamma_s}(t), \quad \alpha_s, \beta_s, \gamma_s > 0, \quad \alpha_s + \beta_s + \gamma_s = 1, \quad (5)$$

where  $A_s$ ,  $\alpha_s$ ,  $\beta_s$  and  $\gamma_s$  are parameters. We use  $p(t)$  and  $R(t)$  to represent respectively the price of services and the land rent. The marginal conditions are

$$r_\delta = \alpha_s A_s p(t) k_s^{\alpha_s-1}(t) l_s^{\gamma_s}(t), \quad w = \beta_s A_s p(t) k_s^{\alpha_s}(t) l_s^{\gamma_s}(t), \quad R(t) = \gamma_s A_s p(t) k_s^{\alpha_s}(t) l_s^{\gamma_s-1}(t), \quad (6)$$

where

$$k_s(t) \equiv \frac{K_s(t)}{N_s(t)}, \quad l_s(t) \equiv \frac{L_s(t)}{N_s(t)}.$$

From (6) we imply

$$k_s = \frac{\alpha_s w}{\beta_s r_\delta}. \quad (7)$$

### Full Employment of Capital and Labor

The total capital stocks utilized by the small-open economy,  $K(t)$  is distributed between the two sectors. Full employment of labor and capital implies

$$K_i(t) + K_s(t) = K(t), \quad N_i(t) + N_s(t) = N.$$

The above equations imply

$$k_i N_i(t) + k_s N_s(t) = K(t), \quad N_i(t) + N_s(t) = N. \quad (8)$$

Solve (8)

$$N_i(t) = (K(t) - k_s N) k_0, \quad N_s(t) = (k_i N - K(t)) k_0, \quad (9)$$

where  $k_0 \equiv (k_i - k_s)^{-1}$ . We require  $k_0 \neq 0$ .

### *Demand Function of Foreign Tourists*

Following Schubert and Brida (2009), we use an iso-elastic tourism demand function as follows

$$D_T(t) = a y_f^\phi(t) p^{-\varepsilon}(t), \quad (10)$$

where  $y_f(t)$  denotes the disposable income of foreign countries,  $\phi$  and  $\varepsilon$  are respectively the income and price elasticities of tourism demand.

### *Behavior of Domestic Households*

Let  $L$  and  $R(t)$  respectively stand for the fixed land and land rent. In this study, for simplicity of analysis we assume the land equally owned by the population. The land rent income per household  $\bar{r}(t)$  is

$$\bar{r}(t) = \frac{L R(t)}{\bar{N}}, \quad (11)$$

where  $\bar{N}$  is the total population

$$\bar{N} = \sum_{j=1}^J \bar{N}_j.$$

Households choose lot size, consumption levels of industrial goods and services, and saving. This study models behavior of households with the approach proposed by Zhang (1993). The current income is

$$y_j(t) = r^* \bar{k}_j(t) + h_j w + \bar{r}(t), \quad (12)$$

where  $r^* \bar{k}_j$  is the interest income,  $h_j w$  the wage income, and  $\bar{r}(t)$  the land rent income. The disposable income consists of the current income and the value of the household's wealth

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \quad (13)$$

The household spends the disposable income on the lot size, consumption of services, consumption of industrial goods, and saving. The budget constraint is

$$R_j(t) l_j(t) + p_j(t) c_{sj}(t) + c_{ij}(t) + s_j(t) = \hat{y}_j(t). \quad (14)$$

We assume that utility level,  $U_j(t)$  of the household is dependent on  $l_j(t)$ ,  $c_{sj}(t)$ ,  $c_{ij}(t)$  and  $s_j(t)$  as follows

$$U_j(t) = \theta_j l_j^{\eta_{0j}}(t) c_{sj}^{\gamma_{0j}}(t) c_{ij}^{\xi_{0j}}(t) s_j^{\lambda_{0j}}, \quad \eta_{0j}, \gamma_{0j}, \xi_{0j}, \lambda_{0j} > 0,$$

in which  $\eta_{0j}$ ,  $\gamma_{0j}$ ,  $\xi_{0j}$ , and  $\lambda_{0j}$  are a typical household's utility elasticity of lot size, services, industrial goods, and saving. We call  $\eta_{0j}$ ,  $\gamma_{0j}$ ,  $\xi_{0j}$ , and  $\lambda_{0j}$  household  $j$ 's propensities to leisure time, to consume housing, to consume services, to consume industrial goods, and to hold wealth, respectively. Maximizing  $U_j(t)$  subject to the budget constraint implies

$$l_j(t) = \frac{\eta_j \hat{y}_j(t)}{R(t)}, \quad c_{sj}(t) = \frac{\gamma_j \hat{y}_j(t)}{p(t)}, \quad c_{ij}(t) = \xi_j \hat{y}_j(t), \quad s_j(t) = \lambda_j \hat{y}_j(t), \quad (15)$$

where

$$\gamma_j \equiv \rho_j \gamma_{0j}, \quad \xi_j \equiv \rho_j \xi_{0j}, \quad \lambda_j \equiv \rho_j \lambda_{0j}, \quad \rho_j \equiv \frac{1}{\eta_{0j} + \gamma_{0j} + \xi_{0j} + \lambda_{0j}}.$$

According to the definition of  $s_j(t)$ , the wealth accumulation of household  $j$  is

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \quad (16)$$

This equation implies that the change in wealth is the saving minus dissaving.

### *Full Use of Land and Demand of and Supply for Services*

Land is used for the residential use and service production

$$\sum_{j=1}^J l_j(t) \bar{N}_j + L_s(t) = L. \quad (17)$$

The equilibrium condition for services is

$$\sum_{j=1}^J c_{sj}(t) \bar{N}_j + D_T(t) = F_s(t). \quad (18)$$

The national wealth is equal to the sum of the wealth owned by all the households in the country

$$\sum_{j=1}^J \bar{k}_j(t) \bar{N}_j = \bar{K}(t). \quad (19)$$

### Trade Balance

We use  $E(t)$  to denote the balance of trade. We have

$$E(t) = r^* (\bar{K}(t) - K(t)). \quad (20)$$

We have thus built the dynamic growth model with endogenous wealth, consumption, and tourism.

### The Dynamics of the National Economy

The system has many variables and variables are connected to each other in nonlinear relations. The following lemma shows how to follow the motion of the dynamic system for given initial conditions.

#### Lemma

The motion of the economic system with  $J$  types of households is governed the following  $J$  nonlinear differential equations

$$\begin{aligned} \dot{R}(t) &= \Omega_1 \left( R(t), \{\bar{k}_j(t)\} \right), \\ \dot{\bar{k}}_j(t) &= \Omega_j \left( R(t), \{\bar{k}_j(t)\} \right), \quad j = 2, \dots, J, \end{aligned} \quad (21)$$

where  $\Omega_j$  are functions of  $R(t)$  and  $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \dots, \bar{k}_J(t))$  given in the appendix. We determined the time-invariant variables  $k_i$ ,  $w_j$ , and  $k_s$  as functions of  $r^*$  and  $A_i$ . All the other variables are given as functions of  $R(t)$  and  $\{\bar{k}_j(t)\}$  by the following procedure:  $\bar{k}_1(t)$  by (A13)  $\rightarrow \hat{y}_j(t)$  by (A4)  $\rightarrow p(t)$  by (A10)  $\rightarrow l_j(t)$ ,  $c_{ij}(t)$ ,  $c_{sj}(t)$ ,  $s_j(t)$  by (15)  $\rightarrow K(t)$  by (A12)  $\rightarrow K_i(t)$  and  $K_s(t)$  by (A1)  $\rightarrow N_i(t)$  and  $N_s(t)$  by (9)  $\rightarrow D_T(t)$  by (10)  $\rightarrow \bar{K}(t)$  by (19)  $\rightarrow L_s(t)$  by (A2)  $\rightarrow F_i(t)$  by (2)  $\rightarrow F_s(t)$  by (5).

The lemma shows how to follow the motion of the economic system once we know the initial conditions of the system and the rate of interest in the global market. For simulation, we specify the parameter values

$$\begin{aligned} \bar{N}_1 = 2, \quad \bar{N}_2 = 3, \quad \bar{N}_3 = 5, \quad h_1 = 3, \quad h_2 = 1, \quad h_3 = 0.5, \quad A_i = 1.5, \quad A_s = 1, \quad \alpha_i = 0.3, \quad \alpha_s = 0.3, \\ \beta_s = 0.6, \quad r = 0.06, \quad L = 10, \quad a = 1, \quad y_f = 4, \quad \varphi = 1.8, \quad \varepsilon = 1.2, \quad \lambda_{01} = 0.8, \quad \xi_{01} = 0.15, \\ \gamma_{01} = 0.06, \quad \eta_{01} = 0.08, \quad \lambda_{02} = 0.7, \quad \xi_{02} = 0.15, \quad \gamma_{02} = 0.07, \quad \eta_{02} = 0.06, \quad \lambda_{03} = 0.65, \\ \xi_{03} = 0.18, \quad \gamma_{03} = 0.08, \quad \eta_{03} = 0.05, \quad \delta_k = 0.05. \end{aligned} \quad (22)$$



The rate of interest is fixed at 6 per cent and the population 10. It should be noted that some empirical studies show that income elasticity of tourism demand is well above unity (Syriopoulos, 1995; and Garín-Múños, 2007). According to Lanza *et al.* (2003), the price elasticity is in the range between 1.75 and 7.36 and income elasticities are in the range between 1.75 and 7.36. We choose the initial conditions,

$$R(0) = 0.6, \quad \bar{k}_2(0) = 3.5, \quad \bar{k}_3(0) = 2.1.$$

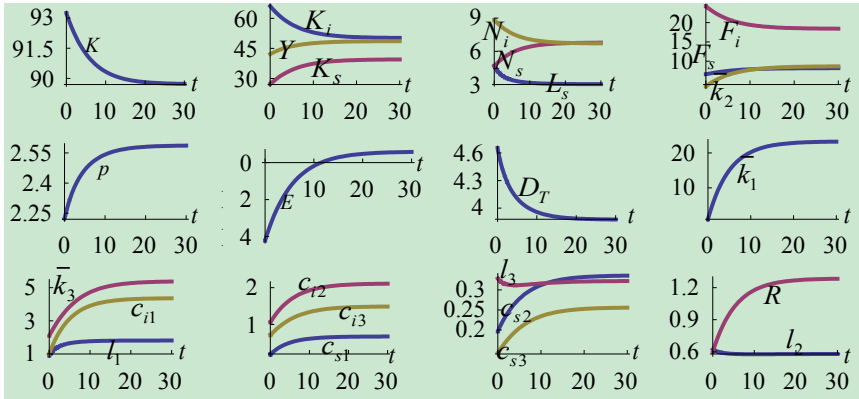
The values of the time-invariant variables are

$$k_i = 7.48, \quad k_s = 5.82, \quad w_1 = 5.76, \quad w_2 = 1.92, \quad w_3 = 0.96. \quad (23)$$

We plot the motion of the dynamic system in Figure 1, in which the national product is given by

$$Y(t) \equiv F_i(t) + p(t)F_s(t) + \sum_{j=1}^J l_j(t)\bar{N}_j.$$

Figure 1. The motion of the economic system



The labor share of the service sector rises as time passes. The price of the consumer good rises over time. The wealth per capita, lot size, the consumption levels of industrial and consumer goods of group 1 are augmented. The wealth per capita, the consumption levels of industrial and consumer goods of groups 2 and 3 are augmented, but the groups' lot sizes are reduced. The simulation in Figure 1 shows that the system moves towards a stationary state. We confirm the equilibrium values of the variables as follows

$$\begin{aligned}
Y = 48.63, K = 89.72, \bar{K} = 99.62, E = 0.59, D_T = 3.88, R = 1.29, p = 2.59, F_s = 8.22, \\
F_i = 18.4, N_i = 6.71, N_s = 6.79, K_i = 50.19, K_s = 39.53, L_s = 3.03, \bar{k}_1 = 23.30, \bar{k}_2 = 8.70, \\
\bar{k}_3 = 5.39, c_{i1} = 4.37, c_{i2} = 2.11, c_{i3} = 1.49, c_{s1} = 0.68, c_{s2} = 0.34, c_{s3} = 0.26, l_1 = 1.81, \\
l_2 = 0.58, l_3 = 0.32.
\end{aligned} \tag{24}$$

The shares of the two sectors' national output are not very different. The national economy is in trade surplus. The service sector employs almost the same number of workers as the industrial sector, even though the service sector uses less capital than the industrial sector. We call groups 1, 2, and 3, respectively, as the rich class (RC), middle-income class (MIC), and low-income class (LIC). The RC, MIC and LIC's per capita wealth, income and lot size are ranked from high to low in the long term, irrespective of their initial relative positions. It is straightforward to calculate the three eigenvalues as follows

$$\{-0.27, -0.24, -0.19\}.$$

The equilibrium point is stable. The stability is important. It guarantees that we can effectively conduct comparative dynamic analysis.

### Comparative Dynamic Analysis

The previous section plots the motion of the variables. It is also important to see how exogenous changes such as global economic conditions affect dynamics of the small open economy. This section examines how changes in some parameters affect the national economy. We introduce a variable,  $\bar{\Delta}x(t)$ , to stand for the change rate of the variable,  $x(t)$  in percentage due to changes in the parameter value.

#### *A Rise in the Rate of Interest in the Global Market*

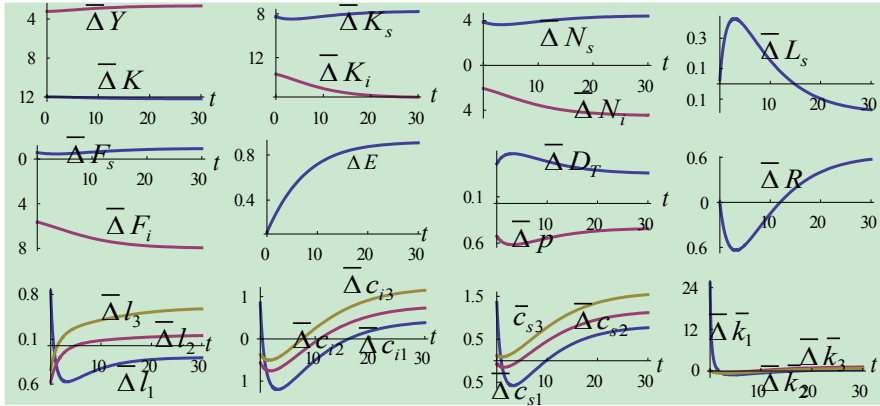
First, we examine the case that the rate of interest is changed as follows:  $r^* : 0.06 \Rightarrow 0.07$ . The capital intensities and wage rates are reduced as follows

$$\bar{\Delta}k_i = \bar{\Delta}k_s = -11.69, \bar{\Delta}w_1 = \bar{\Delta}w_2 = \bar{\Delta}w_3 = -3.66.$$

As the cost of capital is increased, the capital intensities of the two sectors and the wage rates of the three groups are all reduced. The values of the time-dependent variables are plotted in Figure 2. The two sectors to use less capital. The capital stock employed by the economy is thus reduced. The national wealth is lessened. The price of services is reduced in association with expanded tourism. The fall in the price of

services attracts more tourists. The land rent is reduced initially and subsequently increased. The output of the industrial sector is reduced as a consequence of the increased production cost, while the output level of the service sector is increased as a net result of increased demand of the tourists and lowered production cost. Some of the labor force is shifted from the industrial sector to the service sector. The GDP falls over time. The trade balance is improved. The changes in the lot sizes vary over time and the change directions are different among the three groups. The lot size, consumption levels of industrial goods and services, and wealth of the RC's per household are increased initially. The lot size is reduced in the long term. The consumption levels of industrial goods and services are increased in the long term. The wealth of the RC's is affected slightly in the long term. This occurs as the rise in the RC's income from wealth dominates the fall in the wage income in the initial stage. Nevertheless, over time the reduction in the wage income becomes dominant in the long term. The wealth of the MIC's and LIC's per households are slightly affected.

Figure 2. A rise in the rate of interest in international markets

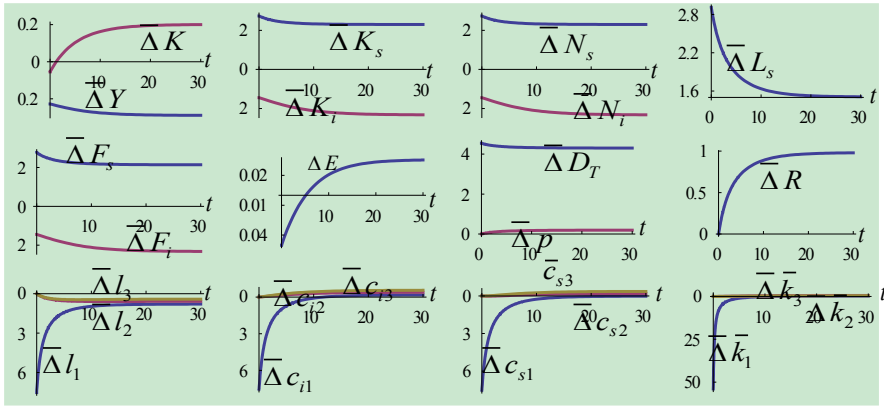


### *An Improvement in the Global Economic Conditions*

We now specify the following change in the global economic conditions:  $y_f : 4 \Rightarrow 4.1$ . The capital intensities and wage rate are not affected  $\bar{\Delta}k_i = \bar{\Delta}k_s = \bar{\Delta}w_j = 0$ . As the global economic condition is only reflected in the income in foreign countries and the rate of interest is assumed constant, the home country's capital intensities and wage rates (which are affected only by the rate of interest in the global markets) are not affected. As people in other countries have more income, the country under consideration tends to attract more foreign tourists. As the number of tourists is increased, the price of services is slightly increased. Some workers shift their jobs from the industrial sector to the service sector. The industrial sector uses less capital

input and the service sector uses more capital input. The net result is that the national economy use mores capital. The trade balance is slightly affected. The trade balance is initially deteriorated and subsequently improved. The output level of the service sector is enhanced and the output level of the industrial sector is reduced. It should be noted that the net impact on the national income is negative. The industrial sector's output is reduced more than the increase in the service sector's output level and the price of services is slightly affected. The labor, capital, and land inputs used by the service sector are increased. The consumption and wealth levels of the three groups are slightly affected. Hence, the improved global economic conditions have negative impact on the GDP mainly by causing economic structural changes in the economy.

Figure 3. An improvement in the global economic conditions



#### *An Improvement in the RC's Human Capital*

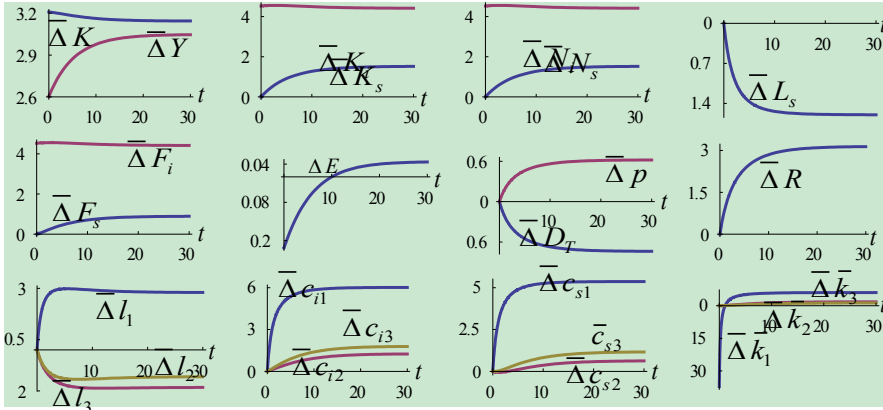
We now examine the following change in the RC's human capital:  $h_1 : 3 \Rightarrow 3.2$ . The RC's wage is increased and the wage rates of the MIC and the LIC and the capital intensities are not affected

$$\bar{\Delta} w_1 = 6.67, \quad \bar{\Delta} k_i = \bar{\Delta} k_s = \bar{\Delta} w_2 = \bar{\Delta} w_3 = 0.$$

The increase in the total labor input results in the increases in the two sectors' inputs. The two sector's capital inputs and the total capital employed by the economy are also increased. The national output and the output levels of the two sectors are increased. As the price of services is increased, the number of tourists is reduced. Some of the land is shifted from the service use to the residential use. Although the RC's lot size is increased, the lot sizes of the other two groups are lessened. The land rent is increased. The consumption levels of the industrial goods and services of the

three groups are increased. The trade balance is slightly affected. The trade balance is initially deteriorated and subsequently improved.

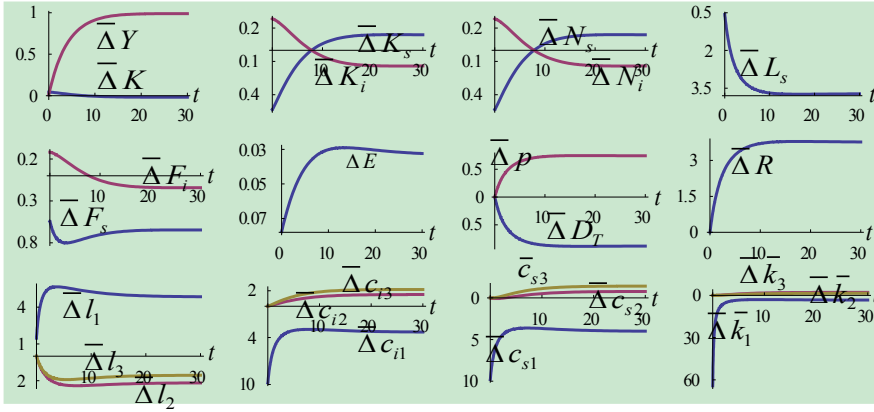
Figure 4. An improvement in the rich class' human capital



#### *A Rise in the RC's Propensity to Use Lot Size*

We now examine what happens to the economy when the RC increases its propensity to consume housing as follows:  $\eta_{01} : 0.08 \Rightarrow 0.1$ . The wage rates and the capital intensities are not affected. As the RC increases its propensity to consume housing, its lot size is increased. This causes the land rent to be increased. The rise in the land rent reduces the lot sizes of the other two groups. The CR's wealth and consumption levels of industrial goods and services are reduced. The other two groups increase their wealth and consumption levels of industrial goods and services. In association with increased land rent the price of services is increased. The increased price lessens the number of tourists. Less land is used by the service sector and the output of service sector is reduced. The service sector initially reduces its capital and labor inputs and increases the inputs in the long term. This occurs because of the substitutions among the land, capital and labor as demonstrated in the plots. The total capital employed by the economy is slightly affected. The GDP is enhanced over time.

Figure 5. The rich class increases its propensity to consume housing



## Conclusions

This paper was concerned with how tourism interacts with inequalities in income and wealth and growth of a small open economy. We built a two-sector and heterogeneous-households growth model in an integrated Walrasian-general equilibrium and neoclassical-growth theory for an economy in a perfectly competitive economy. We treat wealth accumulation and land distribution between housing and supply of services as endogenous variables. The economy consists of one service sector which supplies non-traded services and one industrial sector which produces traded goods. We showed that the motion of the economy with  $J$  types of households is given by  $J$  nonlinear differential equations. We simulated the motion of the system with three groups of households. We also conducted comparative dynamic analysis with regards to the rate of interest, the price elasticity of tourism, the global economic condition, and the rich class' human capital, and the rich class' propensity to consume housing. Our comparative dynamic analysis provides some insights into the complexity of dynamics of small open economies which are constantly affected by (exogenous) global economic conditions. For instance, an improvement in the global economic conditions have the following impact: the country attracts more foreign tourists; the price of services is increased and the trade balance is slightly affected.; some workers shift their jobs from the industrial sector to the service sector; the industrial sector uses less capital input and the service sector uses more capital input; the economy use mores capital; the output level of the service sector is enhanced and the output level of the industrial sector is reduced; the consumption and wealth levels of the three groups are slightly affected; the improved global economic conditions have negative impact on the GDP. This implies that if the price of traded goods and rate of interest

are invariant in the global market, a rise in the real income of other countries will not benefit the small open economy in terms of national economic growth.

## APPENDIX

### *Proving the Lemma*

We determined  $k_i$ ,  $w$ ,  $k_s$  as functions of  $r^*$  and  $A_i$ . From  $K_j = k_j N_j$  and (8), we have

$$K_i = (K - k_s N) k_0 k_i, \quad K_s = (k_i N - K) k_0 k_s, \quad (\text{A1})$$

where we omit time variable in expressions. From (6), we solve

$$R = \frac{w_s N_s}{L_s}, \quad (\text{A2})$$

where we also use  $l_s = L_s / N_s$  and  $w_s \equiv w \gamma_s / \beta_s$ . Inserting (A2) in (17) implies

$$\sum_{j=1}^J l_j \bar{N}_j + \frac{w_s N_s}{R} = L. \quad (\text{A3})$$

From the definition of  $\hat{y}_j$ , we get

$$\hat{y}_j = (1 + r^*) \bar{k}_j + h_j w + \frac{RL}{\bar{N}}. \quad (\text{A4})$$

Equation (A4) and  $l_j = \eta_j \hat{y}_j / R$  in (15) implies

$$l_j = \frac{(1 + r^*) \eta_j \bar{k}_j + \eta_j h_j w}{R} + \frac{\eta_j L}{\bar{N}}. \quad (\text{A5})$$

Inserting (A5) in (A3) implies

$$\sum_{j=1}^J \bar{n}_j \bar{k}_j + w_s N_s = \eta_0 R - \bar{\eta}_0, \quad (\text{A6})$$

where

$$\bar{n}_j \equiv (1 + r^*) \eta_j \bar{N}_j, \quad \bar{\eta}_0 \equiv w \sum_{j=1}^J \eta_j \bar{N}_j h_j, \quad \eta_0 \equiv \left( 1 - \frac{1}{\bar{N}} \sum_{j=1}^J \eta_j \bar{N}_j \right) L.$$

From  $r_\delta = \alpha_s p F_s / K_s$  and (16) we have

$$\sum_{j=1}^J c_{sj} \bar{N}_j + D_T = \frac{r_\delta K_s}{\alpha_s p}. \quad (\text{A7})$$

Inserting  $c_{sj} = \gamma_j \hat{y}_j / p$  in (A7) implies

$$\sum_{j=1}^J \gamma_j \hat{y}_j \bar{N}_j + p D_T = \frac{r_\delta K_s}{\alpha_s}. \quad (\text{A8})$$

Insert (A4) into (A8)

$$\sum_{j=1}^J \bar{\gamma}_j \bar{k}_j + \bar{\gamma}_0 + p D_T = \frac{r_\delta K_s}{\alpha_s}, \quad (\text{A9})$$

where we also use (10) and

$$\bar{\gamma}_j \equiv (1 + r^*) \gamma_j \bar{N}_j, \quad \bar{\gamma}_0(R) \equiv \sum_{j=1}^J \gamma_j \bar{N}_j \left( h_j w + \frac{RL}{N} \right).$$

We assume  $\varepsilon \neq 1$ . It is straightforward to check that we can easily analyze the case of  $\varepsilon = 1$ . From (6) we have

$$p = p_0 R^{\gamma_s}, \quad (\text{A10})$$

where we also use  $l_s = w_s / R$  from (A2) and

$$p_0 \equiv \frac{w}{\beta_s A_s k_s^{\alpha_s} w_s^{\gamma_s}}.$$

Insert (A10) in (A9)

$$\sum_{j=1}^J \bar{\gamma}_j \bar{k}_j + \bar{\gamma}_0 + a p_0^{1-\varepsilon} y_f^\varphi R^{\gamma_s(1-\varepsilon)} = \frac{r_\delta K_s}{\alpha_s}. \quad (\text{A11})$$

Substitute  $N_s = (k_i N - K) k_0$  from (9) into (A6) and  $K_s = (k_i N - K) k_0 k_s$  from (A1) into (A11) respectively yields

$$\begin{aligned} \sum_{j=1}^J \bar{n}_j \bar{k}_j + (k_i N - K) w_s k_0 &= \eta_0 R - \bar{\eta}_0, \\ \sum_{j=1}^J \bar{\gamma}_j \bar{k}_j + \bar{\gamma}_0 + a p_0^{1-\varepsilon} y_f^\varphi R^{\gamma_s(1-\varepsilon)} &= (k_i N - K) \bar{k}_0, \end{aligned} \quad (\text{A12})$$



where  $\bar{k}_0 \equiv r_\delta k_0 k_s / \alpha_s$ . From (A12), we solve

$$\bar{k}_1 = \Omega\left(R, \{\bar{k}_j\}\right), \quad (\text{A13})$$

where

$$\Omega\left(R, \{\bar{k}_j\}\right) \equiv \frac{(\eta_0 R - \bar{\eta}_0) \bar{k}_0 - w_s \bar{\gamma}_0 k_0 - k_0 a w_s p_0^{1-\varepsilon} y_f^\varphi R^{\gamma_s(1-\varepsilon)} - \sum_{j=2}^J (\bar{k}_0 \bar{n}_j + k_0 w_s \bar{\gamma}_j) \bar{k}_j}{\bar{k}_0 \bar{n}_1 + k_0 w_s \bar{\gamma}_1}.$$

The following procedure shows how to find all the variables as functions of  $R$  and  $\{\bar{k}_j\}$ :  $\bar{k}_1$  by (A13)  $\rightarrow \hat{y}$  by (A4)  $\rightarrow p$  by (A10)  $\rightarrow l_j, c_{ij}, c_{sj}$  by (15)  $\rightarrow K$  by (A12)  $\rightarrow K_i$  and  $K_s$  by (A1)  $\rightarrow N_i$  and  $N_s$  by (9)  $\rightarrow D_T$  by (10)  $\rightarrow \bar{K}$  by (19)  $\rightarrow L_s$  by (A2)  $\rightarrow F_i$  by (2)  $\rightarrow F_s$  by (5).

From this procedure and (16), we have

$$\dot{\bar{k}}_1 = \Omega_0\left(R, \{\bar{k}_j\}\right) \equiv s_1 - \bar{k}_1, \quad (\text{A14})$$

$$\dot{\bar{k}}_j = \Omega_j\left(R, \{\bar{k}_j\}\right) \equiv s_j - \bar{k}_j, \quad j = 2, \dots, J. \quad (\text{A15})$$

Taking derivatives of (A13) with respect to time implies

$$\dot{\bar{k}}_1 = \frac{\partial \Omega}{\partial R} \dot{R} + \sum_{j=2}^J \Omega_j \frac{\partial \Omega}{\partial \bar{k}_j}, \quad (\text{A16})$$

where we use (A18). We do not provide the expression of the partial derivatives because they are tedious. Equaling the right-hand sides of (A17) and (A19), we get

$$\dot{R} = \Omega_1\left(R, \{\bar{k}_j\}\right) \equiv \left( \Omega_0 - \sum_{j=2}^J \Omega_j \frac{\partial \Omega}{\partial \bar{k}_j} \right) \left( \frac{\partial \Omega}{\partial R} \right)^{-1}. \quad (\text{A17})$$

We proved the lemma.

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