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# PIERRE LIARDET (1943-2014)

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Pierre Liardet was a unique person, as a mathematician, as a friend and also as a collaborator. In papers like this often the life and work of the deceased is explained in some detail; see for example the nice and well-written obituary on Pierre's life and work by Guy Barat, Peter Grabner and Peter Hellekalek ([BGH]), or Pierre's own obituary of Gérard Rauzy ([L55, L56]). Here I hardly will do this and take a more personal approach and will speak about this complex and unique man as I remember him.



FIGURE 1. Pierre's picture on his own webpage.

When I met Pierre for the first time in 1986 he was already an accomplished if not famous mathematician. I had just started my Ph.D.-work with Henk Jager,

who was visiting Pierre during a sabbatical year in Marseille. What immediately struck me, was the openness and hospitality of Pierre and his wife Josy. The expression "our house is your house" definitely applied to both of them, and this did not change over the years. What also struck me, was the scope of Pierre's knowledge; Pierre became famous as a Ph.D.-student of Gérard Rauzy for solving a conjecture by Serge Lang in algebraic number theory (see [L5]), and Lang even attended his Ph.D. defence. But Pierre was also a medical doctor, who worked at night at SAMU<sup>1</sup> in Marseille. Pierre studied medicine after he finished his Ph.D. in mathematics, and even got a Ph.D.-degree in medicine.

Henk Jager visited Pierre because he wanted to extend his knowledge on ergodic theory, a field in which Pierre had become a leading specialist after switching from algebraic number theory. A few year before this visit, Henk-together with Wieb Bosma and Freek Wiedijk-had solved a conjecture of Hendrik Lenstra (and-as turned out later–Wolfgang Döblin) on the almost sure distribution of the so-called "approximation coefficients". The proof was based on the natural extension of the regular continued fraction expansion, as studied by Shunji Ito, Hitoshi Nakada and Shigeru Tanaka, and in 1986-87 Pierre and Henk were able to adapt this approach in a very original way. In a way this paper ([L17]) testifies of many of Pierre's favourite subjects: uniform distribution, natural extensions and skew products, and continued fractions.

Pierre invited me to visit Marseille in May and June 1987. At that time he and Gérard Rauzy were busy organizing a large conference on ergodic theory and number theory<sup>2</sup> at the CIRM in Luminy. As a beginner I had no idea of all the people invited; a large group of Japanese mathematicians (Shunji Ito, Yuji Ito, Hitoshi Nakada, Makoto Mori, Teturo Kamae), from Austria (Peter Hellekalek, Gerhard Larcher), Holland (Michel Dekking, Mike Keane), from the US (Doug Lind), and of course from all over France! Due to the atmosphere of openness and friendliness before, during and after this conference and all the subsequent conferences which Pierre organized over the years I made friendships and collaborations which have lasted until today. These conferences were not just about mathematics; for example, after the 1987 conference there was a large party where Hitoshi and I, together with Brigitte Mossé played Bach, and the next day quite a number of us participated in a demonstration in Marseille against the FN. Often during these conferences there were large groups of people visiting Pierre and Josy in Saint Cannat, where we had wonderful dinners in their garden.

<sup>&</sup>lt;sup>1</sup>Service d'Aide Médicale Urgente.

<sup>&</sup>lt;sup>2</sup>Arithmetics and Coding Systems, Marseille-Luminy, June 1987.

Shortly after the '87 conference, Pierre and Josy's daughter became my girlfriend, and due to this I saw Pierre even more often; in the years which followed I visited Marseille (and in particular their home in Saint-Cannat) regularly, and often for several months on end (the longest period was in 1989–90, when I was appointed in Luminy for six months and shared Gérard Rauzy's office). Very quickly Pierre and I started to work on a project, and in doing so I got to know Pierre's personality very well. In a (at that time) recent paper on continued fractions I had found a 'gap' in the proof of the result. Pierre and I quickly decided that the result should hold (and actually could be generalized), but that the approach of the author would not lead to a proof. So we tried several other ways to get the result. For several months Pierre came up with various ideas, which I 'took down' with counter-examples; the last time this happened was during Christmas breakfast, were I mentioned a counter-example to an approach we had been working on for weeks. At first Pierre seemed hurt and a bit down (it was a stupid thing of me to do this at Christmas breakfast), but then he blurted out a vague outline of a possible other approach. After almost two days of hard work I could explain to him after dinner that he was completely correct! We wrote a paper and submitted this to a British journal. After a few months we received a letter from the journal containing an almost unintelligible handwritten referee-report of about two lines and a photocopy of the original paper, with the request to first read the original paper (in our paper we did not mention the gap in the proof). Pierre was livid, and immediately sent the journal an email in which he expressed the opinion that 'they' were complete idiots. It happened so fast that the email was sent before I could react. After this email I thought there was no point in explaining to this particular journal why we had written our paper. So we sent it to another journal, where the paper was rather promptly accepted. However, one day we were having tea in Josy's garden with some of the editors of this journal, and we were told that there is a page-fee; on the spot Pierre withdrew the paper! For me this was a rather unpleasant development; I was about to finish my Ph.D.-thesis, and therefore needed accepted papers to get at least a post-doc position. Pierre wanted to re-write the paper, but this time it was my turn to make a decision, and we sent the paper to the same journal were the original paper had appeared.

Pierre was a passionate and intense man; not only about doing math, but also in his love for his wife, children and grandchildren, or his dedication to his friends. This of course does not imply that it was always easy to be with Pierre. Like many other Frenchmen at that time Pierre had a very personal way of driving a car (meaning: very fast). During one of the conferences in Luminy Yuji Ito, Doug Lind, Michel Dekking and I had to sign some paperwork, so Pierre drove

us down from Luminy to St. Charles in his Golf. On the way to St. Charles, Yuji was quietly sitting next to Pierre, while I was sitting in the middle on the backseat. Before we left St. Charles to go back to Luminy, Michel thought that it was perhaps better if Doug would sit in the middle; in this way it would be easier for both of them to have a conversation. After two minutes Doug puts his face in his hands, and shouts "I can't stand this." I look at Michel, who shrugs his shoulders (both me and Michel had been in the car of many French mathematician (Gérard Rauzy, Christian Maudiet, François Blanchard), so we were quite used to this). At that moment Pierre turns around, and asks me: "Ça va?" To which I answer "Oui Pierre, ça va très bien!" and at the same time I see we're doing about 100 km an hour in the inner city of Marseille. In fact this intensity extended to everything Pierre did, whether it was mathematics, repairing his car, walking in the mountains (which he loved, and did very often after he was told he was critically ill), or trying to safe their cat which has been poisoned by some of the neighbours.



FIGURE 2. Pierre's picture on some webpage on which he briefly describes his own life.

Apart from intensity there is always originality and beauty in what he did. This is obvious to anyone familiar with his famous papers, but is certainly also true for the lesser known ones, such as his papers with Pierre Stambul ([L31, L36]). Pierre curiosity never was really satisfied, and over the years he 'dived' into many new subjects. Apart from working in ergodic theory, dynamical systems and number theory, Pierre also became interested in cryptology, which he liked a lot and which offered him the opportunity to work with his oldest son Pierre-Yvan.



FIGURE 3. Pierre and Josy during my Ph.D.-defence in Amsterdam, April 23, 1990.

Christine and I separated in 1990, and in the years that followed my life literarily changed directions; I became a post-doc in College Park and Seattle, and met Karma in New Haven during a conference, where Mike Keane offered me a job in Delft. As a consequence I saw much less of Pierre. For me, working with Pierre was like going to a party; he was always full of ideas, and always willing to listen and if necessary to change his opinions and views. It therefore saddened me, that I saw Pierre much less frequently than before; Karma and I visited Marseille for a month at the end of the 1990's, and Pierre (together with Josy, and once with Christine and her daughter Lucy in 2002) visited Delft a number of times. Unfortunately, together with Karma I wrote only one other

paper with Pierre, and it grieves me that it will never be possible to work with, or even talk to and see this beautiful man, who fought bravely against death, even when he knew that this was a fight which could not be won.

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