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CONSTRUCTION OF ESTIMATES IN THE CHOICE OF ALTERNATIVE SOLUTIONS BY USING THE FUZZY UTILITIES

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The problem of multi-criteria choice of alternatives with accounting the multiple stakeholders' preferences has been considered. The model for solution with the composition of fuzzy sets has been described. The mathematical formulation and justification of the solution using a fuzzy utility functions is given. Corresponding numerical calculations and graphic example are presented.

Keywords: multi-criteria alternative choice problem, fuzzy utility function, Fuzicalc program

1. Introduction

As it is well known, management decisions making is a prerequisite for a successful commercial and industrial activity in any sphere of business and economics. It is fully applied to the transport market, the work of enterprises of all forms of transport. The competitive environment of its operation generates and contains all sorts and origins of uncertainty, ambiguity parameters of the market environment, actions and goals of partners, competitors, clients, customers, services, regulatory organizations and so on.

This situation requires the development and application of adequate economic and mathematical tools to model situations, the appropriate methodological support. Fairly efficient tools for this are the methods of the theory of fuzzy sets and fuzzy modelling. Several successful examples of their use in the transport sector are given in our previous publications [1–3].

In particular, as substantiated by the authors earlier, among the decision-making problems in the transport sector the problem of multi-criteria choice from many alternatives occupies an important place. However, in solving these problems, the relevant models reflect the individual interests of specific subjects of the transport sector (transport enterprises, carriers, customers, investors) without construction of common optimal variants for all its. At the same time in practice (for economically viable and socially acceptable solutions) we must take into account the interests of all or several participants of the transport services processes and services (producers, consumers, intermediaries, society).

2. Description of the Problem and its Solution

The validity of the result of solving the problem of multi-alternative choice will be greatly enhanced if the process takes into accounts the assessments and preferences of several parties.

On the one hand, there are experts who represent the interests of the firm, and the contradictions in their estimates consist in the fact that they can use different ways to assess the compliance of products for selected criteria; moreover, they can use the various approaches to the selection of the direct criteria and their importance. Overall, however, their actions and evaluations have the same orientation (direction).

But situation will change, if consumers will present as one or more experts. In this case the coordination of consumer ratings will be impractical (and often impossible), when there is no desire to wishful thinking. The harmonization of the consumer's ratings and evaluations of experts, which represent the interests of the producers, is useful in the sense that a balanced system of estimates can be constructed, and product that satisfies it, may be the best way to comply with the restrictions and the wishes of the producer and desires of the consumer, i.e. competitive market conditions.

The problem is complicated by the fact that the preferences of the parties are likely to be confusing, especially when it comes to new or substantially new products. Just as in many other cases of fuzziness market and decision making conditions, the apparatus of the theory of fuzzy sets allows to achieve some effective results in this area.

2.1. Formalization in Fuzzy Set Theory Terms

In the theory of fuzzy sets such problems can be solved based on the composition of fuzzy criteria ratings and heuristic arguments in the form of linguistic evaluations of the utility. In general, this problem is formulated in [4].

The model of decision-making includes:

- the evaluation of alternatives;
- views of persons, involved in the decision-making process;
- description of the solution process.

As in the previous problems, each alternative product is described by the quality criteria, the extent to which may be given in a fuzzy linguistic form. Linguistic values can be selected from dictionary; the contents of such dictionary can be expanded by modifiers like "very", "more or less", etc. In general case, can be used N criteria of quality d_i, $i = \overline{1, N}$, which form the criteria space $D = \{ d_1, d_2, ..., d_N\}$.

If there is usage of fuzzy evaluations of criteria conformity, it means that the set of linguistic values $L = \{l_j : j = \overline{1, M}\}$ is determined, and each element of this set is associated with a fuzzy set, which is defined by membership function $\mu_{l_j}(z), z \in [0,1]$.

In general, a fuzzy mapping of the set of linguistic evaluations criterion conformity on the set of quality criteria $G: L \to D$, determined at $D \times L$, can be constructed.

However, it is necessary to consider that according to experts, only a few combinations have real meaning from the entire set of possible combinations of quality criteria and evaluations of the criteria conformity and only for these combinations the experts may give their estimates about the rate of usefulness.

2.2. Example to Illustrate the Existing Problem

For example, consider the situation, represented by Table 1.

Table 1. Fragment of table for evaluating the usefulness (values for the criteria and utilities)

d_1	d_2	d_3	d_4	Utility for consumers	Utility for experts
large	weak	large	small	relatively low	average
good	acceptable	large	small	average	relatively low
good	weak	large	small	relatively low	relatively low

Let us note, first of all, that this example is used only to illustrate the proposed method of solving the problem and, of course, it is highly simplified, and secondly, evaluations of the usefulness also have a linguistic form, i.e. there is a number of linguistic values $P = \{p_k : k = \overline{1, K}\}$, which are formalized by fuzzy sets with membership functions $\mu_{p_k}(z), z \in [0,1]$ (Fig. 1). Triangular membership functions are chosen only for simplicity of graphic constructions.

If we consider several alternatives $A = \{a_i : i = \overline{1, I}\}$, the tables similar to table 1, will be constructed for all alternatives.

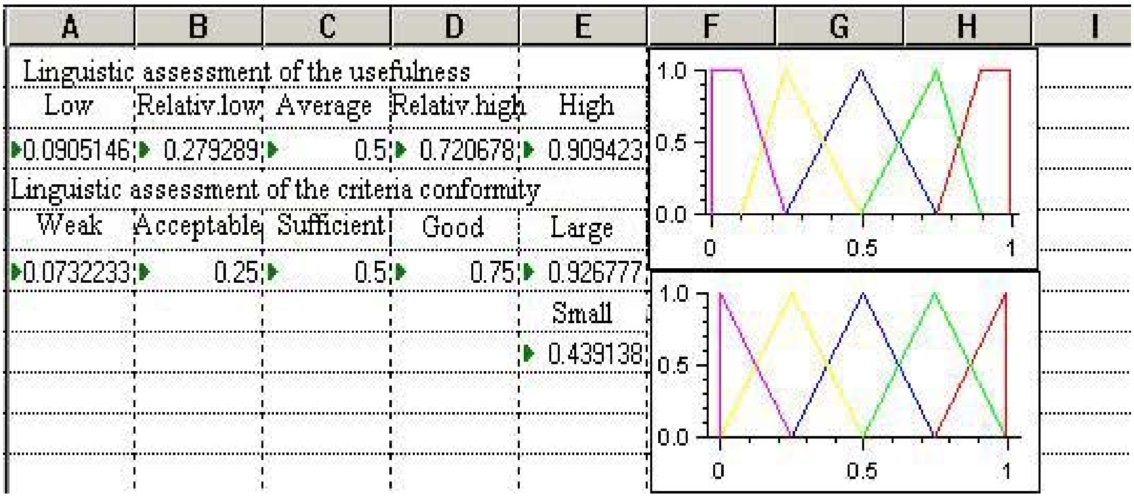

Figure 1. Some linguistic assessments of the usefulness and corresponding fuzzy functions

Let us suppose, that evaluations of the conformity by criteria have been formulated for some alternative a_i. For example, the condition that evaluations by the criterion D_1 are closer to good one for the alternative a_i, show in Fig. 2 and can be written the following way: $F_i\,(D_1) = \{0{,}3\,/\,large,\ 0{,}7\,/\,good\}$.

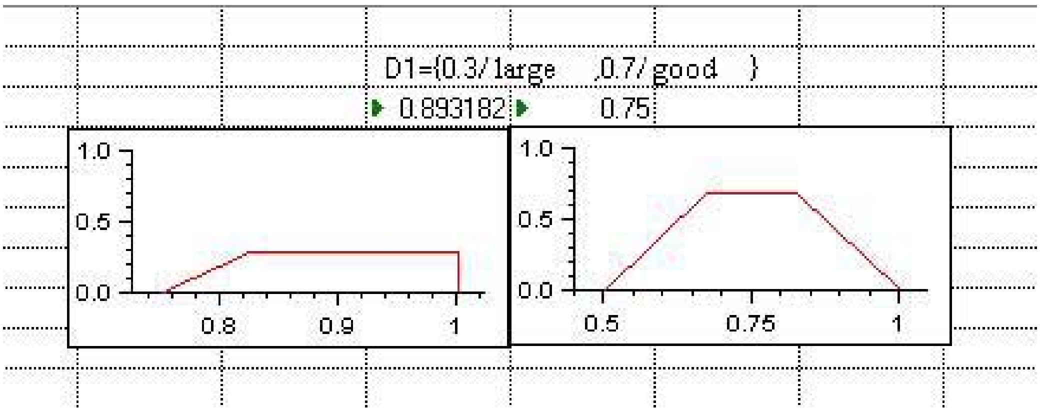

Figure 2. Evaluation of the alternatives for the criterion closer to good than to the excellent

Similar estimates can be constructed according to all criteria: $F_i(D_2) = \{0{,}6/acceptable,\ 0{,}4/weak\}$, $F_i\,(D_3) = \{0{,}4/sufficient,\ 0{,}6/large\}$, $F_i\,(D_4) = \{1/small\}$.

The aim is to find the estimate of the usefulness for each alternative, which will have the greatest level of conformity. In the known studies [5] this problem has been solved as follows. The convolution of estimates of the criteria conformity has been computed for each set of table 1 by using the operation *min*:

$$w_q = \bigcap_{jq} \mu_{l_{jq}}(z) = \min_{j_q}[\ \mu_{l_{jq}}(z)], \tag{1}$$

$$V_q(E_h) = \min[\mu_{p_q}(z), w_q] = \min[\mu_{p_q}(z), \min[\mu_{l_{jq}}(z)]], \tag{2}$$

where q is the set number in table 1; h is the expert group number. For considered example $h = 1, 2$.

Then, the cumulative estimate over all values of q has been calculated for each expert group:

$$R(E_h) = \bigcup_q V_q(E_h) = \max_q\ [V_q(E_h)]\ . \tag{3}$$

The resulting estimation is a fuzzy set that represents an integrated estimate for each category of experts. At next stage of solving the problem we will need to find the value of usefulness, which has the highest degree of truth for all experts (expert groups). This phase will be discussed later.

Let us return to equations (1) and (2), which represent the traditional approach, used in various papers [4, 5]. There should be noted a number of disadvantages of this approach.

The first is the fact that multi criteria problem essentially boils down to one-criterion, because of everything will be determined by evaluation with the minimum value of $\mu_{l_{jq}}(z)$, and all the remaining estimates have no significance.

The second disadvantage is that the type of used membership functions is irrelevant too. This occurs because of the fact that the fuzzy set is essentially replaced by a clear α-subset, while performing an operation *min*, which is used to formalize the operation intersection.

This, in turn due to the fact that it is not ruled out the possibility of empty intersections in (1), which obviously will take place for the sample (Table 1). At the same time, if the different types of membership functions are used, we will obtain different decision-making situations. In addition, the estimates of criteria conformity and evaluations of the usefulness can have various membership functions, that is essential for the situations of decision-making.

2.3. Proposed Approach to Solving

Let us consider a method for finding estimates of usefulness that is free of these shortcomings. It is easy to show that the equation (2) can be represented by the following way:

$$V_q(E_h) = \min\{\min[\mu_{p_q}(z), \mu_{l_{1q}}(z)], \min[\mu_{p_q}(z), \mu_{l_{2q}}(z)], \ldots \min[\mu_{p_q}(z), \mu_{l_{jq}}(z)]\}. \quad (4)$$

For the construction of each particular convolution $\min[\mu_{p_q}(z), \mu_{l_{jq}}(z)]$ we propose to use the operation, which has been defined in [3, 6] and called as "the shadow of a fuzzy set" $Sh(\widetilde{A}, \widetilde{A})$, where $\widetilde{A}, \widetilde{A}$ are the fuzzy sets. In our case:

$$\widetilde{A} = (\mu_{l_{jq}}(z)/z), \widetilde{B} = (\mu_{p_q}(z)/z). \quad (5)$$

The result of this operation is shown in Fig. 3, where the membership function of evaluation $\{0.3/big\}$ of the criteria compliance is represented by green colour (a), the membership function of evaluation of the usefulness which means $\{quite\ low\}$ is represented by blue colour (b) , "the shadow of a fuzzy set" $Sh(\widetilde{A}, \widetilde{A})$ is represented by red colour (c).

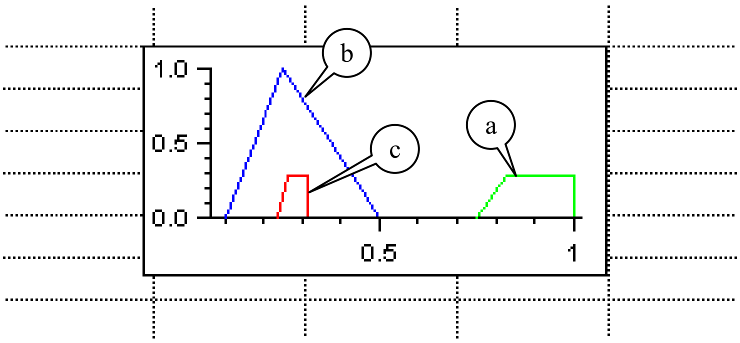

Figure 3. Result of operation of fuzzy sets shadow construction

Using this operation allows taking into account the influence of all evaluations of the criteria conformity, and not just the effect of estimates, which have the minimal values. As a result it also reflects the nature of the selected membership functions, in addition the problem of empty intersections is solved too, because according to definition, $Sh(\widetilde{A}, \widetilde{B}) = \varnothing$ only if $\widetilde{A} = \varnothing$ and $\widetilde{B} = \varnothing$ or $\widetilde{A}$ and $\widetilde{B}$ are orthogonal [3, 6]. In this case the equation (4) is reduced to the following form:

$$V_q(E_h) = \min_{j_q}[Sh_{j_q}(\widetilde{A}, \widetilde{B})],$$

where the fuzzy sets $\widetilde{A}$ and $\widetilde{B}$ are determined by equation (5).

The integrated estimate $R(E_h)$ for each category of experts (Fig. 4) is obtained from equation (3).

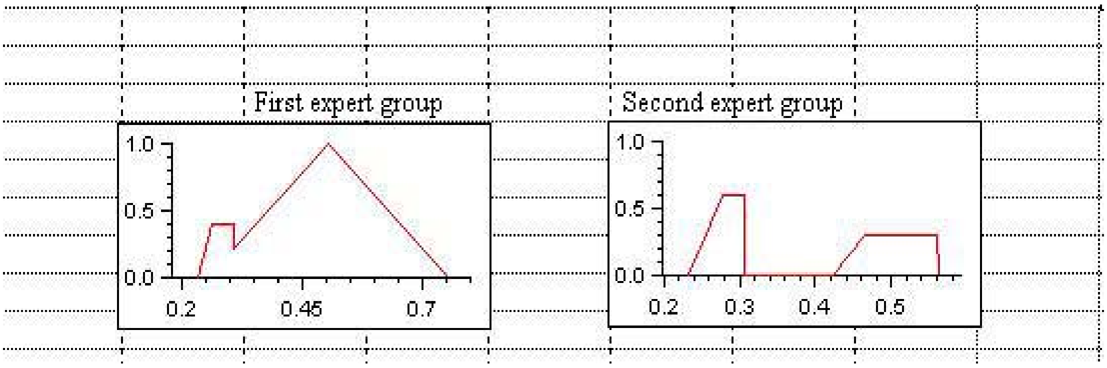

Figure 4. Integrated experts' estimates of usefulness

It is obvious, that a concerted fuzzy estimate of usefulness can be found as the intersection: $R_c = R(E_1) \cap R(E_2)$, corresponded curve is represented by red colon in Fig. 5.

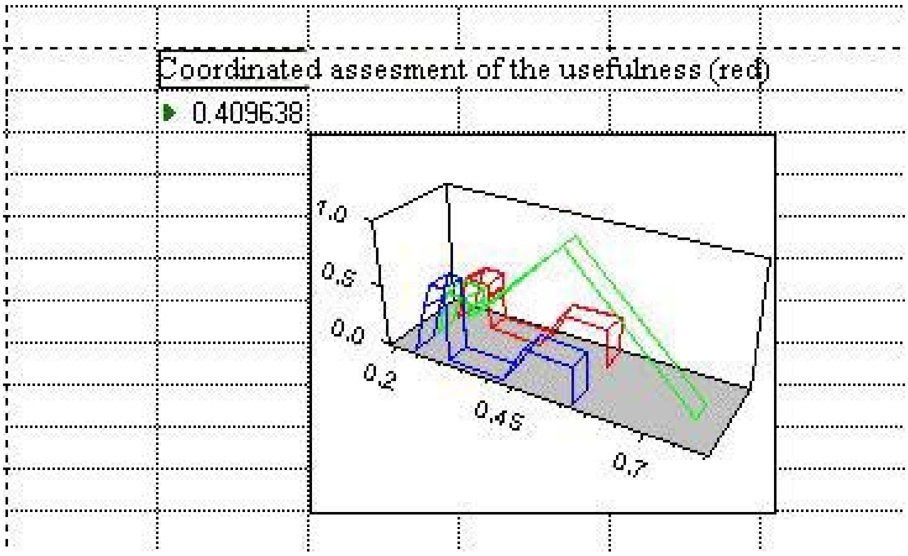

Figure 5. Estimate of usefulness coordinated by two groups of experts

2.4. Interpretation of Results

Due to the fact that the evaluation R_c should be correlated with one of used utility estimates, the comparison procedure must be performed, which can be realized as the intersection $\widetilde{S}_k = p_k \cap R_c$ for all $p_k \in P$. Each of the resulting fuzzy sets will represent the estimate of the degree truth of the conformity between consistent assessment of the usefulness and the initial estimates of the usefulness.

In order to make a final decision it is necessary to compare the obtained fuzzy sets. A comparison of fuzzy sets can be done by calculating the value of their capacities by using α-partitions. This process is a quite time-consuming procedure. The definition of the value of membership function at the point, which corresponds to the coordinates of its centre of gravity, would be simpler. Due to the fact that this value can be interpreted as an estimate of the truth, the truth degree of performance of some assessment of the utility will be determined too.

In table 2 the second line represents the coordinates of the centres of gravity of the sets $\widetilde{S}_k$, the third line is the corresponding values of the membership functions, the fourth line is the composition of the coordinates of the centre of gravity and importance of membership, and the fifth line is the result of the operation *EffPeak*, conducted in the program for computing with fuzzy functions and variables, namely *FuziCalc* [7].

All of these results indicate that the estimation of the usefulness, which has the value "*average*", will be the most closely to the expert evaluation under the considered conditions.

Table 2. Results of calculations

Low	Relativ.low	Average	Relativ.high	High
0.238613	0.320294	0.46943	0.542177	0.23
0.0764228	0	0.31	0.168762	0
0.0182355	0	0.145523	0.0914989	0
0.236652	0.25707	0.560548	0.559399	-0.314472

Finally, Fig. 6 represents the membership functions from the Table 2, which correspond to the second line of this table.

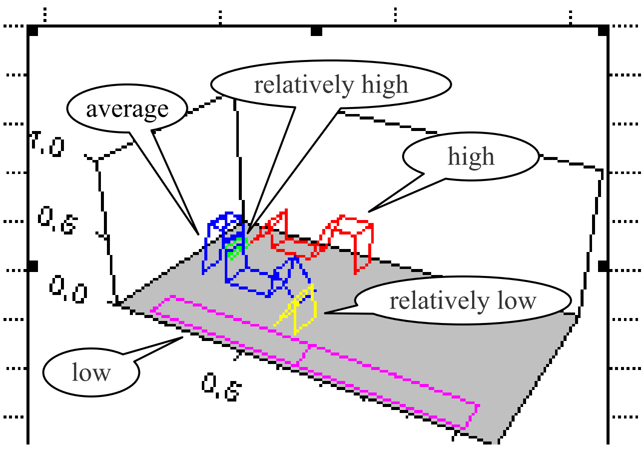

Figure 6. Membership functions correspond to the second line of table

If there are analyzed the several alternatives, the discussed procedures will be performed similarly and that alternative, which has the highest estimate of the utility will be chosen as the best one.

3. Conclusions

Thus, effective and simply approach that allows taking into account the interests (often contradictory) of different aspects of economic activity in the investment decision-making in market conditions has been proposed and justified.

The formulation of the corresponding problem of multi-criteria choice is described. The theoretical basis and practical sequence of these problems solutions using a specialized spreadsheet FuzzyCalc for computing with fuzzy numbers has been presented.

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