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Investigation of Sandwich Panel Parameters Influence on its Natural Frequencies

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Abstract

In the paper the parametric study of free vibration analysis of sandwich panel is presented. Effects of the face sheet material, as well as those related to the ply-thickness, core thickness and length of the panel are investigated. There are used approaches based on the first order shear deformation theory. Natural frequencies of sandwich panel are calculated by using analytical solution, which is compared with A NSYS results.

Key words: parametric study, free vibration, face sheet, core, natural frequency

1 Introduction

One special group of laminated composites used extensively in engineering applications is sandwich composites. Sandwich panels consist of thin facings, also called skins or face sheets, sandwiching a core, are frequently used instead of solid plates because of their high bending stiffness to weight ratio. Increasing the thickness by adding a core in the middle also increases the bending resistance of the panel [5 & 13].

Commonly sandwich panels are used to covering various types of buildings, where they are subjected not only to static loads, but also to dynamic effects. Dynamic effects can be divided into forced vibrations and natural vibrations, which will be discussed in turn. In this subject area, analytical studies have been conducted by, amongst others, Yu [14 & 15], Ibrahim et al. [3], Ng and Das [9], Kanematsu et al. [4], Chang and Chen [1] and Cheng et al. [2]. Raville and Veng [11] have produced test results as well as analytical results for simply supported plates.

Modern commercial FEM packages are capable of solving the vibration problems for the sandwich panels having wide varieties of the elastic and geometry properties. However, when it comes to the design of the sandwich panels with the fundamental frequency selected as the efficiency criterion, it is still advantageous to have the solution in the analytical form [7 & 13].

2 Problem formulation and analytical solution procedure

The behavior of sandwich panels undergoing deformations may be analyzed by the Kirchhoff hypothesis except assumption, that normals remain perpendicular to the deformed reference plane (first-order shear deformation theory developed by Reissner and Mindlin) [12 & 16].

Consider a rectangular (*a* x *b*) sandwich panel composed of two identical face sheets or face sheets with very small thickness´s difference between each other and orthotropic core. The plate´s middle plane is referred to Cartesian coordinates *x, y, z* and adjacent edges, $x = 0$, a and $y = 0$, b of the panel, are simply supported (SS) (Fig. 1). If the panel is undergoing free undamped vibration, the deflection of the panel $w(x, y, t)$ and rotations of the normals $\overline{\alpha}(x,y,t)$, $\overline{\beta}(x,y,t)$ are given by expressions [5 & 6]:

$$w(x,y,t)=\overline{w}^{\circ}\sin(2\pi ft),\quad \overline{\alpha}(x,y,t)=\overline{\alpha}^{\circ}\sin(2\pi ft),\quad \overline{\beta}(x,y,t)=\overline{\beta}^{\circ}\sin(2\pi ft). \tag{1}$$

To determine unknown functions $\overline{w}^{\circ}, \overline{\alpha}^{\circ}, \overline{\beta}^{\circ}$, that satisfy boundary conditions, equilibrium equations are used.

The governing partial differential equations for a composite panel with mid-plane symmetry are given by [12]:

$$D_{11}\frac{\partial^2\overline{\alpha}}{\partial x^2}+D_{66}\frac{\partial^2\overline{\alpha}}{\partial y^2}+(D_{12}+D_{66})\frac{\partial^2\overline{\beta}}{\partial x\partial y}-k^sA^s_{55}\left(\overline{\alpha}+\frac{\partial w}{\partial x}\right)-I\frac{\partial^2\overline{\alpha}}{\partial t^2}=0, \tag{2}$$

$$(D_{12}+D_{66})\frac{\partial^2\overline{\alpha}}{\partial x\partial y}+D_{66}\frac{\partial^2\overline{\beta}}{\partial x^2}+D_{22}\frac{\partial^2\overline{\beta}}{\partial y^2}-k^sA^s_{44}\left(\overline{\beta}+\frac{\partial w}{\partial y}\right)-I\frac{\partial^2\overline{\beta}}{\partial t^2}=0, \tag{3}$$

$$k^sA^s_{55}\left(\frac{\partial\overline{\alpha}}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+k^sA^s_{44}\left(\frac{\partial\overline{\beta}}{\partial y}+\frac{\partial^2 w}{\partial y^2}\right)-\rho_m h\frac{\partial^2 w}{\partial t^2}=0\,, \tag{4}$$

where k^s is the transverse shear deformation factor given by value 5/6 and stiffness coefficients are given by:

$$D_{ij}=\frac{1}{2}h^{(2)}\left(C_{ij}^{(3)}-C_{ij}^{(1)}\right),\quad C_{ij}^{(1)}=\sum_{k=1}^{n_1}E_{ij}^{(k)}h^{(k)}\overline{z}^{(k)},\quad C_{ij}^{(3)}=\sum_{k=1}^{n_2}E_{ij}^{(k)}h^{(k)}\overline{z}^{(k)}, \tag{5}$$

where

$$\overline{z}^{(k)}=\frac{1}{2}(z^{(k)}+z^{(k-1)}),\quad A_{ij}^{\,s}=E_{ij}^{\,s}h^{(2)},\quad i,j=4,5, \tag{6}$$

where n_1 and n_2 are the number of layers in the upper (index 1) and the lower (index 3) face sheet respectively (Fig. 1) and $E_{ij}^{\,s}$ are the transverse shear moduli of the core.

For panel mass density per unit area we can write:

$$\rho_m h=\sum_{k=1}^{N}\rho_k(z^{(k)}-z^{(k-1)}),\quad I=\frac{1}{3}\sum_{k=1}^{N}\rho_k(z^{(k)})^3-(z^{(k-1)})^3\,, \tag{7}$$

where ρ_k is the mass density of the k^{th} layer.

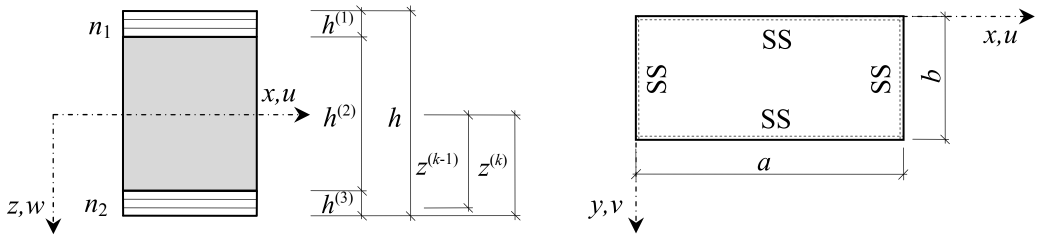

Figure 1: Structure and boundary conditions of the sandwich panel

We adopt the following expressions for $\bar{w}^{\circ}, \bar{\alpha}^{\circ}$ and $\bar{\beta}^{\circ}$ [5 & 7]:

$$\bar{w}^{\circ} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C'_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{8}$$

$$\bar{\alpha}^{\circ} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A'_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \quad \bar{\beta}^{\circ} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B'_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right), \tag{9}$$

where m and n are integers only.

After substituting equations (8), (9) into equations (2), (3), (4) and neglecting rotatory inertia terms we obtain:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \begin{Bmatrix} A'_{mn} \\ B'_{mn} \\ C'_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{10}$$

where

$$\begin{gathered} L_{11} = D_{11}\lambda_m^2 + D_{66}\lambda_n^2 + k^s A_{55}, \quad L_{12} = (D_{12} + D_{66})\lambda_m\lambda_n, \quad L_{13} = k^s A_{55}\lambda_m, \\ L_{22} = D_{66}\lambda_m^2 + D_{22}\lambda_n^2 + k^s A_{44}, \quad L_{23} = k^s A_{44}\lambda_n, \quad L_{33} = k^s A_{55}\lambda_m^2 + k^s A_{44}\lambda_n^2 - \rho_m h \lambda_{mn}, \end{gathered} \tag{11}$$

and

$$\lambda_m = \frac{m\pi}{a}, \quad \lambda_n = \frac{n\pi}{b}, \tag{12}$$

$$A'_{mn} = \left[\left(L_{12}L_{23} - L_{22}L_{13}\right)C'_{mn}\right]/Q, \quad B'_{mn} = \left[\left(L_{12}L_{13} - L_{11}L_{23}\right)C'_{mn}\right]/Q. \tag{13}$$

For nonzero deflections equation (10) is satisfied when the determinant of the matrix, comprised of elements L_{ij}, for i, j = 1, 2, 3, is zero. At this condition λ_{mn} stand for eigenvalues of equation (10) and the lowest value is of interest. It can be easily found to be:

$$\lambda_{mn} = \left[QL_{33} + 2L_{12}L_{23}L_{13} - L_{22}L_{13}^2 - L_{11}L_{23}^2\right]/\rho_m h Q, \quad Q = L_{11}L_{22} - L_{12}{}^2. \tag{14}$$

The natural frequencies are given by:

$$f_{mn} = \sqrt{\lambda_{mn}} / 2\pi. \tag{15}$$

3 Numerical results

The natural frequencies have been calculated for a set of sandwich panels having length dimension a = 2 m to 3.8 m with substep 0.1 m, constant width dimension b = 1.15 m, thicknesses of the panel h = 0.04 m to 0.12 m with substep 0.02 m and various types of face sheets with total constant thickness $h^{(1)}$ = 0.0004 m in the upper and $h^{(3)}$ = 0.0006 m in the lower face sheet.

The face sheets of the panel are made of carbon fiber reinforced plastic (CFRP) with [0/90/0] (Fig. 2), $[0]_3$ fiber orientation angle, respectively and zinc coated steel (ZCS) with the following properties: E_s = 210 GPa, v_s = 0.3, ρ_k = 7850 kg/m^3. It is neglected thickness of zinc coat in calculations. Ply material properties [8] of CFRP used in the analysis are given as E_1 = 139.23 GPa, E_2 = 12.26 GPa, G_{12} = 4.28 GPa, v_{12} = 0.26, ρ_k = 1508 kg/m^3.

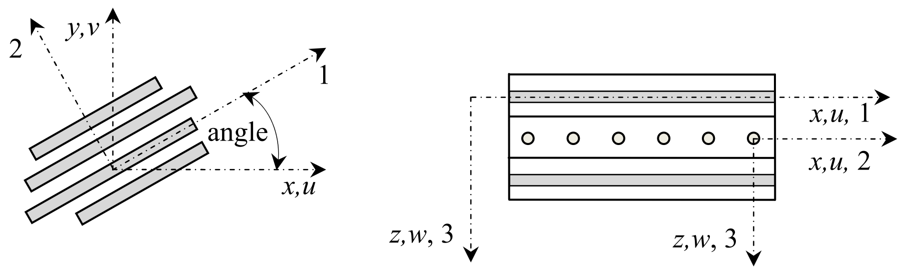

Figure 2: Notation of fiber orientation angle and composition of CFRP [0/90/0] face sheets

Material properties of the PUR core with thickness $h^{(2)}$ = 0.04 m are given as E_{PUR1} = 2.9 MPa, G_{PUR12} = 4.3 MPa, v_{PUR12} = 0.3, ρ_{PUR} = 43 kg/m^3.

The validation of the analytical (ALT) analyses has been performed using finite element software ANSYS (ANS). The panel was discretized using the laminated shell elements SHELL181 with the size of 50 mm x 50 mm.

In Fig. 3a), the effects of thickness h for face sheets CFRP $[0]_3$ with length dimension a = 2 m on natural frequencies at various natural mode shape (NMS) are presented. In Fig. 3b), the effects of length dimension a for face sheets CFRP $[0]_3$ with thickness h = 0.08 m on natural frequencies at various natural mode shape (NMS) are presented. Fig. 3 shows that the natural frequency increases with the increasing natural mode shape, thickness h and decreasing the length dimension a.

In Fig. 4a), two lamination schemes of faces sheets, [0/90/0] and $[0]_3$, are taken in to investigate of the fiber orientation angle effect on natural frequencies at NMS 1 at various length dimension a and thickness h. The results show that used scheme $[0]_3$ decreases the natural frequencies at every length a and both thicknesses against to used scheme [0/90/0]. Difference between analytical and numerical analyses is higher for scheme [0/90/0] and thicker panel. In Fig 4b), the natural frequencies at NMS 1 of the sandwich panel with length

a = 2.9 m for various panel thicknesses and face sheets are shown. The length was chosen as the medium value from its range. Increasing the panel thickness increases the frequencies (also Fig. 3a) and Fig. 4a)) and difference between ALT and ANS analyses. The lower frequencies are obtained for sandwich with ZCS and $[0]_3$ face sheets.

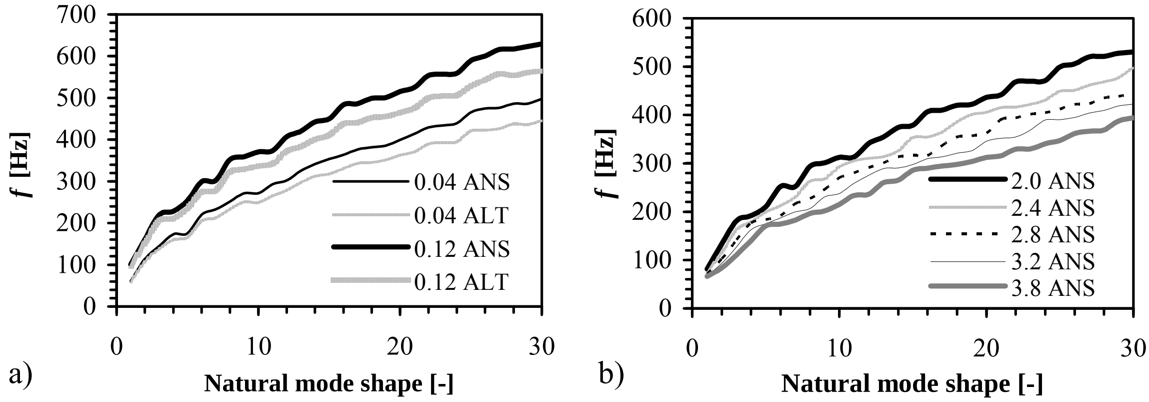

Figure 3: Natural frequencies for face sheets CFRP $[0]_3$;
a) for a = 2 m, b) for h = 0.08 m

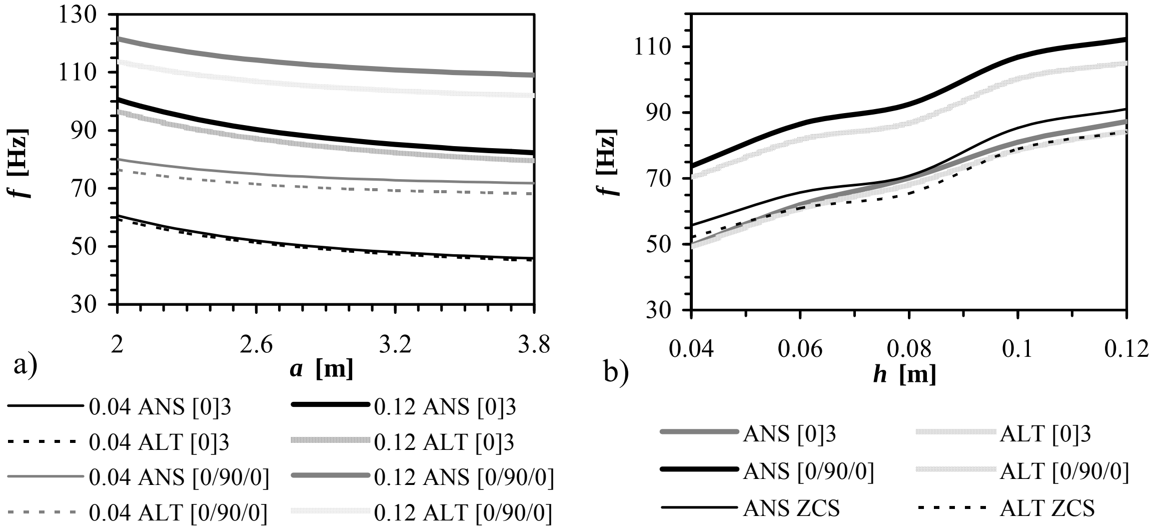

Figure 4: Natural frequencies at natural mode shape 1;
a) for face sheets CFRP [0/90/0] and $[0]_3$,
b) for face sheets CFRP [0/90/0], $[0]_3$ and ZCS (a = 2.9 m)

The natural frequencies of the sandwich panel having length dimension a = 2 m and 3.8 m, respectively and thickness h = 0.04, 0.08, 0.12, respectively at fundamental (i.e. the 1^{st}), 15^{th} and 30^{th} natural mode shape are presented in Table 1 and Table 2.

Table 1: Natural Frequencies (*f*) of the Sandwich Panel (*a* = 2 m)

Face sheets	NMS	*m, n* [-]			*f* [Hz]					
		h [m]			0.04		0.08		0.12	
		0.04	0.08	0.12	ANS	ALT	ANS	ALT	ANS	ALT
[0/90/0]	1	1, 1	1, 1	1, 1	80.08	76.39	100.36	94.12	121.58	113.74
$[0]_3$					60.63	59.34	81.61	78.63	100.66	96.57
ZCS					60.23	56.44	76.29	70.52	98.08	90.59
[0/90/0]	15	4, 3	4, 3	4, 3	365.25	330.65	392.01	355.12	463.54	420.49
$[0]_3$		6, 1			353.28	317.76	378.75	345.41	450.30	410.71
ZCS		4, 3			219.77	197.61	263.32	237.86	336.05	304.13
[0/90/0]	30	8, 2	2, 5	2, 5	514.89	458.07	544.21	486.21	642.30	574.88
$[0]_3$		2, 5			497.87	445.09	530.81	475.85	629.13	564.62
ZCS		2, 5			303.37	269.09	362.31	323.27	462.20	413.24

Table 2: Natural Frequencies (*f*) of the Sandwich Panel (*a* = 3.8 m)

Face sheets	NMS	*m, n* [-]			*f* [Hz]					
		h [m]			0.04		0.08		0.12	
		0.04	0.08	0.12	ANS	ALT	ANS	ALT	ANS	ALT
[0/90/0]	1	1, 1	1, 1	1, 1	71.80	68.23	90.03	84.33	109.09	101.99
$[0]_3$					45.84	45.21	65.83	63.94	82.29	79.51
ZCS					54.12	50.37	68.74	63.44	88.42	81.59
[0/90/0]	15	4, 3	4, 3	4, 3	272.55	248.74	297.33	270.72	352.38	321.12
$[0]_3$					247.79	229.31	285.61	261.98	340.32	312.09
ZCS					167.61	150.91	201.52	182.18	257.28	233.02
[0/90/0]	30	2, 5	2, 5	2, 5	373.43	337.72	400.31	362.39	473.29	429.05
$[0]_3$					360.57	327.38	394.50	355.55	465.87	420.58
ZCS					224.36	201.58	268.73	242.60	342.93	310.18

The percentage differences between analytical and numerical solution of the natural frequencies shown in Table 1 and Table 2 are calculated in Table 3.

Table 3: Differences between analytical and numerical solution

Face sheets	NMS	Differences [%]					
		a = 2 m			*a* = 3.8 m		
	h [m]	0.04	0.08	0.12	0.04	0.08	0.12
[0/90/0]	1	4.83	6.63	**6.89**	5.23	6.76	**6.96**
$[0]_3$		2.17	3.79	**4.24**	1.39	2.96	**3.50**
ZCS		6.72	8.18	**8.27**	7.44	8.35	**8.37**
[0/90/0]	15	**10.46**	10.39	10.24	9.57	**9.83**	9.73
$[0]_3$		**11.18**	9.65	9.64	8.06	9.02	**9.05**
ZCS		**11.21**	10.70	10.50	**11.07**	10.62	10.41

[0/90/0]		**12.40**	11.93	11.73	**10.57**	10.46	10.31
$[0]_3$	30	**11.86**	11.55	11.43	10.14	**10.95**	10.77
ZCS		**12.74**	12.08	11.85	**11.30**	10.77	10.56

It follows from the comparison of the data shown in Table 3 that the differences between the natural frequencies calculated using the ALT solution and those obtained from the ANS analysis do not exceed 12.74 % for panel with ZCS face sheets, h = 0.04 m and a = 2 m at its 30th mode shape. The less differences between values at the same mode shape, are obtained for panel with [0/90/0] and $[0]_3$ face sheets.

In Fig. 5, the natural frequencies at various natural mode shapes of sandwich panel with thickness h = 0.08 m and length a = 3 m for various types of face sheets are shown. The lower frequencies are obtained for ZCS face sheets, what is related to their stiffness [10].

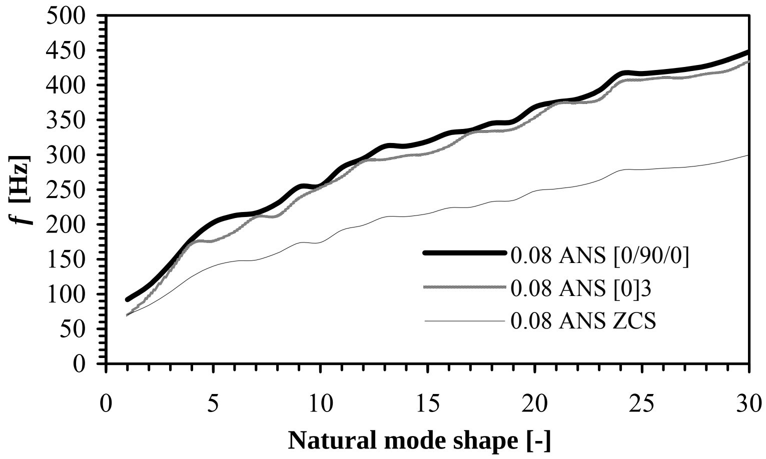

Figure 5: Natural frequencies for various types of face sheets (a = 3 m)

4 Conclusions

The analytical and numerical solution of free vibration of the simply supported sandwich panel with variable length, thickness and type of face sheets was carried out and compared. It has been shown that analytical solution provides an efficient way of calculation of the natural frequencies with sufficient accuracy. The best agreement between analytical and numerical solution was obtained at the first natural mode shape and the lowest value of thickness. Influence of the panel´s length on the difference between natural frequencies at the same mode shape has been not significant. The difference has been increased with increasing value of mode shape. On the other side, the difference between solutions was lower at 15th and 30th mode shape for longer panel. It was shown, that difference has been increased with increasing thickness of the panel at fundamental mode shape. Another phenomenon has occurred at 15th and 30th mode shape for the panel with length of 2 m - increasing thickness reduces differences between both solutions.

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