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# Geometry and Modeling of the Oval Strand of $n_0$+($2n_0$+4)+$n_2$ Type

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**Abstract**

A wire rope is a structure composed of many individual wires. The wires form two major structural elements: the strand core and the layers of the strand. There may be different shapes of strands in the rope depending on the shape of strand core. The paper deals with the mathematical geometric modeling of the oval wire strand created of $n_0 + (2n_0 + 4) + n_2$ wires. The number and diameter of the core wires are the initial data. The mathematical representation is in form of parametric equations of the wire axes. The equations are implemented in the Pro/Engineer Wildfire V5 software for creating the geometrical model of the strand.

**Key words:** wire rope, strand of a rope, oval strand, geometrical model

## 1 Introduction

The utilization of advanced technology for design of steel ropes and detection their properties obtain considerable attention recently. Different analytical methods including the finite element method are used. Majority is devoted to simple wire strands where strands are constructed from one or more layers of circular wires helically laid around a circular straight core wire [1], [2]. Few papers are devoted to the ropes formed from the strands other than a circular cross section. In this paper, one type of the oval strand is analyzed. We will describe its geometric construction and derive the mathematical expression of the wire axes. Using the mathematical expression we will construct geometric model of the strand.

## 2 Geometry of the strand

Let the considered oval strand [3] be created by $n_0$ wires lying next to one another of the core, $n_1 = 2n_0 + 4$ wires in the 1$^{st}$ layer and $n_2$ wires in the 2$^{nd}$ layer (Fig.1). The points of the centerlines of core wires lie on the line segment and the points of the centerlines of layer wires lie in part on the line segments and partly on the circular arcs in the cross section (Fig.2). We assume that the wires in the core and in the $1^{st}$ layer have the diameter $\delta_1$, in the $2^{nd}$ layer diameter $\delta_2$. There is the gap $\Delta_j$ between the wires. The wires of both layers have the right- hand pitch and the winding angle is $\alpha_j$.

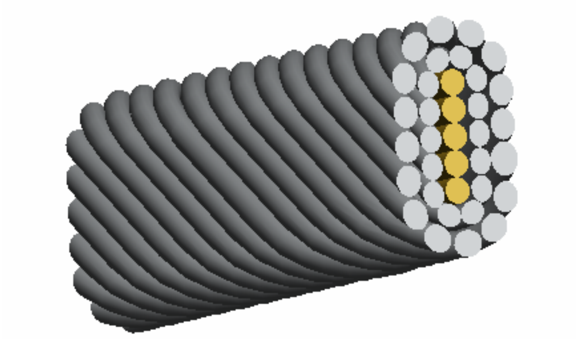

Figure 1: Oval strand of $n_0 + (2n_0 + 4) + n_2$ type

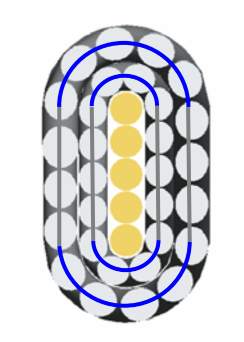

Figure 2: Cross-section of the strand

The surface generated by the wire can be formed by translation of the circle whose center is on the wire axis and the circle lies at the normal plane of this curve. Therefore, it is sufficient to derive a mathematical expression of the wire axes curves.

# 3 Mathematical expression of the wire axes

## 3.1 The wires of the $1^{st}$ layer

Let the right-hand Cartesian coordinate system $(O; x, y, z)$ be placed so that the $z$ axis is identical with the axis $o_s$ of the strand and the $x$ axis is perpendicular to the line $OX$ (Fig.3).

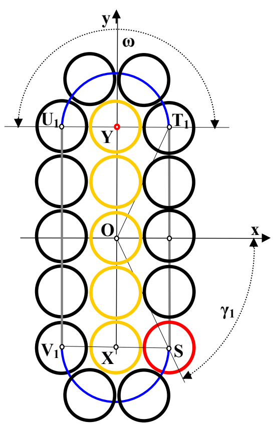


Figure 3: Cross-section of the $1^{st}$ layer

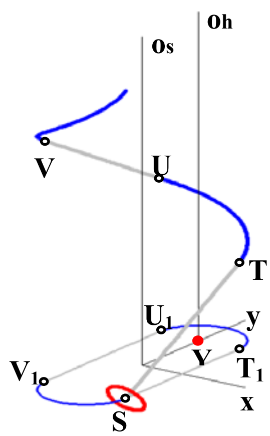


Figure 4: Wire axis in the $1^{st}$ layer

Let $S$ be the point of the curve, which will be expressed. The part $ST$ of the curve is a line segment $l$, the part $TU$ is a part of cylindrical helix $h$. We will derive the equations of both parts using the angle of rotation around the z axis as a parameter $\psi$. Let us mark $\gamma_1$ the angle between the x axis and the line $OS$. The size of them is given by the formula

$$\gamma_1 = \arctan\frac{(n_0 - 1)(\delta_1 + \Delta_0)}{2\delta_1}. \tag{1}$$

The parametric equations of the part $ST$ have the form

$$x_{l_1}(\psi) = \delta_1, \tag{2}$$

$$y_{l_1}(\psi) = \delta_1 \tan(\psi - \gamma_1), \tag{3}$$

$$z_{l_1}(\psi) = \frac{(n_0 - 1)(\delta_1 + \Delta_0) + 2\delta_1 \tan(\psi - \gamma_1)}{2\tan(\alpha_1)}, \tag{4}$$

where $\psi \in \langle 0; 2\gamma_1 \rangle$.

The part $TU$ of the curve of wire axis is the cylindrical helix with axis $o_h$, which is parallel to the z-axis and goes through the point $Y$ (Fig.4). The coordinates of the point $Y$ are

$$x_Y = 0, \quad y_Y = \frac{(n_0 - 1)(\delta_1 + \Delta_0)}{2}, \quad z_x = 0. \tag{5}$$

By following from (1), (2), (3), (4), (5) and the smoothly connection between the line segment and the helix segment $TU$ has the parametric equations

$$x_{h_1}(\psi) = \delta_1 \cos(\psi - 2\gamma_1), \tag{6}$$

$$y_{h_1}(\psi) = \frac{(n_0 - 1)(\delta_1 + \Delta_0) + 2\delta_1 \sin(\psi - 2\gamma_1)}{2}, \tag{7}$$

$$z_{h_1}(\psi) = \frac{(n_0 - 1)(\delta_1 + \Delta_0) + \delta_1(\psi - 2\gamma_1)}{\tan\alpha_1}, \tag{8}$$

where $\psi \in \langle 2\gamma_1; 2\gamma_1 + \pi \rangle$.

These two segments are repeatedly appear in the subsequent section of the curve, where they are turned by the angle $\kappa = k\pi$ and shifted by the height

$$h_{\Delta_1} = \frac{(n_0 - 1)(\delta_1 + \Delta_0) + \delta_1\pi}{\tan\alpha_1}. \tag{9}$$

Wires of the layer are the same surfaces. Axes of all wires in the layer are identical curves. Each wire in the layer is given in the previous shifted by a certain size $h_{w_1}$ in the axial direction. This size is depended on the one pitch length $h_1$ which can be calculated from the relationship

$$h_1 = 2h_{\Delta_1}\,. \tag{10}$$

Then the size of displacement is

$$h_{w_1} = \frac{(n_0 - 1)(\delta_1 + \Delta_0) + \delta_1\pi}{(n_0 + 2)\tan\alpha_1}\,. \tag{11}$$

So, the curve of any wire axis of the 1st layer can be expressed by parametric equations

$$x_1(\psi) = x_{s_1}(\psi)\cos\kappa - y_{s_1}(\psi)\sin\kappa\,, \tag{12}$$

$$y_1(\psi) = x_{s_1}(\psi)\sin\kappa + y_{s_1}(\psi)\cos\kappa\,, \tag{13}$$

$$z_1(\psi) = z_{s_1}(\psi) + kh_{\Delta_1} + jh_{w_1}\,, \tag{14}$$

in which for $s=l$ we use the equations (2) - (4), $\psi \in \langle 0;2\gamma_1\rangle$ and for $s=h$ we use the equations (6) - (8), $\psi \in \langle 2\gamma_1;2\gamma_1+\pi\rangle$ and $j = 0,\ldots, 2n_1 + 3$.

### 3.2 The wires of the 2nd layer

The procedure for deriving the equations of wire axis in the second layer will be similar to the first layer of wires.

For the angle between the x axis and the line $OS$ is valid relationship

$$\gamma_2 = \arctan\left[\frac{(n_0 - 1)(\delta_1 + \Delta_0)}{3\delta_1 + \delta_2}\right] \tag{15}$$

The curve of any wire axis of the 2nd layer is expressed by parametric equations

$$x_2(\psi) = x_{s_2}(\psi)\cos\kappa - y_{s_2}(\psi)\sin\kappa\,, \tag{16}$$

$$y_2(\psi) = x_{s_2}(\psi)\sin\kappa + y_{s_2}(\psi)\cos\kappa\,, \tag{17}$$

$$z_2(\psi) = z_{s_2}(\psi) + kh_{\Delta_2} + jh_{w_2}\,, \tag{18}$$

in which for $s=l$ we use the equations (19) - (21), $\psi \in \langle 0;2\gamma_2\rangle$

$$x_{l_2}(\psi) = \frac{3\delta_1 + \delta_2}{2}\,, \tag{19}$$

$$y_{l_2}(\psi) = \frac{(3\delta_1 + \delta_2)}{2}\tan(\psi - \gamma_2)\,, \tag{20}$$

$$z_{l_2}(\psi) = \frac{(n_0 - 1)(\delta_1 + \Delta_0) + (3\delta_1 + \delta_2)\tan(\psi - \gamma_2)}{2\tan\alpha_2}, \tag{21}$$

and for $s = h$ we use the equations (22) - (24), $\psi \in \langle 2\gamma_2; 2\gamma_2 + \pi \rangle$.

$$x_{h_2}(\psi) = \frac{(3\delta_1 + \delta_2)}{2}\cos(\psi - 2\gamma_2), \tag{22}$$

$$y_{h_2}(\psi) = \frac{(n_0 - 1)(\delta_1 + \Delta_0) + (3\delta_1 + \delta_2)\sin(\psi - 2\gamma_2)}{2}, \tag{23}$$

$$z_{h_2}(\psi) = \frac{2(n_0 - 1)(\delta_1 + \Delta_0) + (3\delta_1 + \delta_2)(\psi - 2\gamma_2)}{2\tan(\alpha)}. \tag{24}$$

The height of the segment displacement is determined by the formula

$$h_{\Delta_2} = \frac{2(n_0 - 1)(\delta_1 + \Delta_0) + (3\delta_1 + \delta_2)\pi}{2\tan\alpha_2} \tag{25}$$

and the shift of the wire in the layer has the size

$$h_{w2} = \frac{2h_{\Delta_2}}{n_2}. \tag{26}$$

where $n_2$ is the number of the wires in the 2$^{nd}$ layer.

## 4 Geometrical model of the strand

The parametric equations enable to define the geometry and position of an arbitrary wire in the strand. Consequently, they are sufficient to design the geometrical 3D model of the strand of type $n_0 + (2n_0 + 4) + n_2$ [4]. As an example, we create the model of the strand of (5+14+15) type. Basic strand parameters are listed in Table 1.

Table 1: The parameters of the strand of (5+14+15) type

| | $n$ | $\alpha_j$ (°) | $\delta_j$ (mm) | $\Delta_j$ (mm) |
|---|---|---|---|---|
| Core | 5 | 0 | 1,18 | 0,02 |
| First layer | 14 | 15 | 1,18 | 0,04 |
| Second layer | 15 | 15 | 1,62 | 0,09 |

Derived mathematical equations are implemented in the Pro/ENGINEER Wildfire 5.0 software [5]. Models of individual wires of the 1$^{st}$ layer are formed gradually (Fig.5). The 2$^{nd}$ layer is formed in the same way (Fig.6).

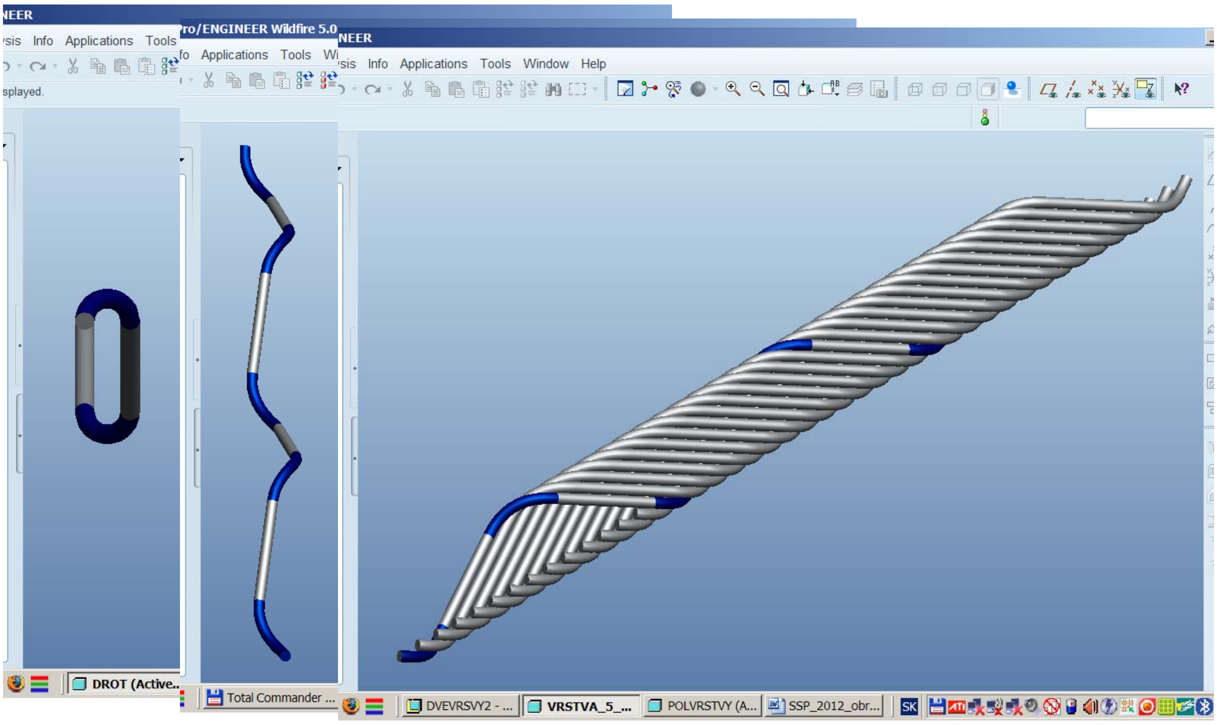


Figure 5: Modeling of the 1$^{st}$ layer wires

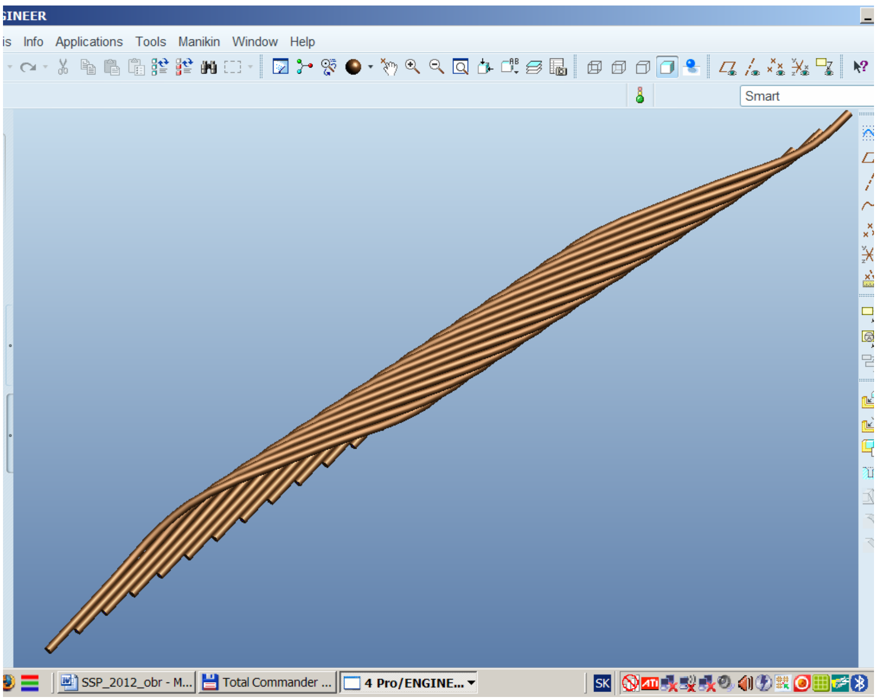


Figure 6: Model of the 2$^{nd}$ layer

To create models of core wires (Fig.7) we use equations

$$x_c(t) = 0 \tag{27}$$

$$y_c(t) = \frac{(n_0 - 1)(\delta_0 + \Delta_0)}{2} 0 + i(\delta_0 + \Delta_0) \tag{28}$$

$$z_c(t) = 2th_{\Delta_2} \tag{29}$$

By composition of created models and cutting to required length we obtain the geometrical model of the strand.

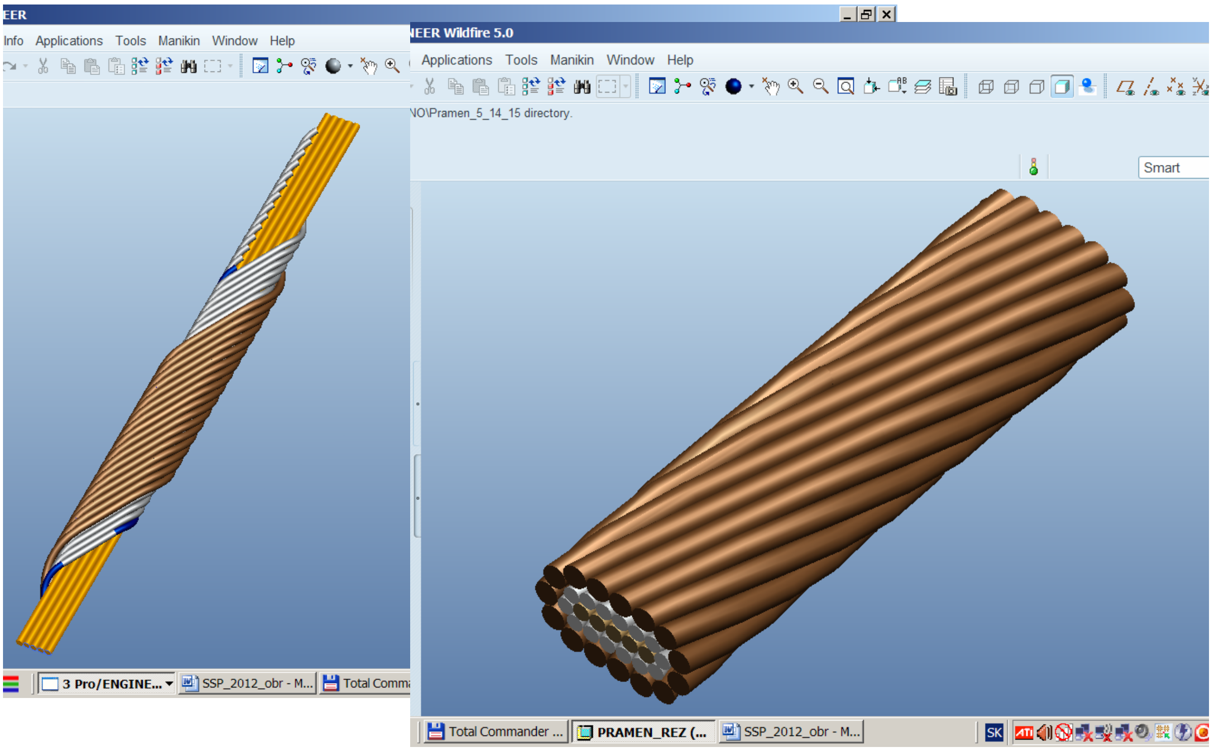


Figure 7 The geometrical model of the strand (5+14+15)

## 5 Conclusion

Based on the geometry of an oval wire strand of type $n_0 + (2n_0 + 4) + n_2$ were derived the geometric parametric equations of the wire axis. The mathematical expression of the curves allowes us to create a geometrical model of the strand using appropriate software. The Pro/ENGINEER Wildfire 5.0 software for modeling has been used. The mathematical geometric model can be implemented in a finite element program and applied to solve complex analysis of strands and ropes behaviour.

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