



## **ANALYTICAL DEPENDENCE RELATIONS OF CONVERTING GEODETIC COORDINATES INTO UTM COORDINATES RECOMMENDED IN HYDROGRAPHIC WORK**

### **ABSTRACT**

This article presents mathematical dependence relations used for direct conversion of geodetic coordinates into UTM (Universal Transverse Mercator) and inverse conversion recommended by military standards. It mainly focuses on some formulas taken from the National Geospatial-Intelligence Agency publication NGA.SIG.0012\_2.0.0\_UTMUPS — March25, 2014 and additionally from IHO Manual of Hydrography published by the International Hydrographic Office. In order to illustrate application of the presented formulas the article includes some relevant calculation examples on the reference ellipsoid WGS 84 (World Geodetic System of 1984).

**Key words:**

transformation of coordinates, UTM coordinates, WGS 84 ellipsoid.

### **INTRODUCTION**

Syllabi of courses for Marine Hydrographers based on the training Standards of the International Hydrographic Organization (IHO) [10] developed for the subject called ‘Positioning’ include, among others, issues relating to coordinate systems and cartographic projection, which are used for positioning and geodetic (hydrographic) calculations.

Many methods for converting geodetic coordinates into UTM projection have been presented in the literature on navigation and hydrography, e.g. A. Banachowicz,

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J. Urbański (1988) [1], J. Urbański, M. Czapczyk (1988) [12]. The formulas presented differ in the level of complexity and accuracy of calculations and are often referred to as the Krassowski ellipsoid. In professional training of marine hydrographers it is advisable, in addition to proprietary solutions, to make use of ideas recommended by relevant standards and regulations, both national and international. In converting geodetic coordinates into UTM coordinates, among others, applicable are the following regulations:

1. Ordinance by the Council of Ministers — October 15, 2012 on the state system of spatial references. The formulas relating to conversion of PL-UTM coordinates are presented in the article by W. Morgaś and Z. Kopacz (2016) [4].
2. Military Standard NO-02-A019 *Military maps — geodetic coordinate systems, cartographic grids and coordinate grids* [8].
3. Defense Standardization Manual PDNO-06-A072 *Marine Hydrography — Organizations and Rules for Research* [3].

In accordance with the Military Standards, when making and publishing marine charts as well as in geographic information systems the WGS84 geodetic coordinate system together with a coordinate grid calculated in UTM should be used.

Mathematical dependence relations of converting coordinates calculated in one reference system into coordinates in another coordinate system (mostly WGS84) are, among others, presented in the aforesaid military standards. These standards, however, do not comprise mathematical formulas used to convert geodetic coordinates into UTM coordinates and perform the inverse conversion in the same coordinate system, but make references to the DMA TM 8358.2. publication September 18, 1989, which in 2014 was replaced by the NGA.SIG.0012\_2.0.0\_UTMUPS standardization publication [6].

In the mathematical formulas contained in the NGA.SIG.0012\_2.0.0\_UTMUPS standardization publication and in the IHO Manual on Hydrography [2] as in many other publications symbols, for the axis of a coordinate system are used conversely to those used in Poland (Ordinance by CM on the state system of spatial references [9]). In order to avoid misinterpretations alternative symbols are used, i.e.:

- Northing (N) — axis of ordinates overlaps the image of central meridian and is directed towards the North;
- Easting (E): axis of abscissa overlaps the straight-line image of the equator and is directed towards the East.

This article includes mathematical dependence relations together with calculation examples (analogue to [4]) using the Matcad software conversions of geodetic

coordinates into UTM coordinates for the WGS84 ellipsoid following the guidelines contained in:

- National Geospatial-Intelligence Agency standardization publication NGA.SIG.0012\_2.0.0\_UTMUPS, March 25, 2014 [6];
- Manual on Hydrography (MoH) published by the International Hydrographic Bureau (IHB) [2].

### **MATHEMATICAL DEPENDENCE RELATIONS OF CONVERTING GEODETIC COORDINATES INTO UTM COORDINATES ON THE BASIS OF THE NGA STANDARD [6]**

Universal Transverse Mercator projection is used in geodesy, cartography and geographic information systems (GIS) within the range of geographic latitudes from 84°N to 80°S (in higher latitudes universal polar stereographic (UPS) projection is used).

A coordinate system which should be used in geographic materials dedicated to armed forces is also the UTM coordinate system which:

1. Covers 120 basic zones of Transverse Mercator projection numbered successively from -60 to +60 in the eastern direction, wherein number 1 is assigned to the zone between 180°W and 174°W.
2. The scale factor on the central meridian (*CM*) is equal to 0.9996 which creates two secant lines placed approximately 180 000 m east and west of the central meridian, and the negative numbers for the zones refer to the southern hemisphere.
3. In order to avoid negative values of the coordinates the beginning of the coordinate system in each projection zone must be assigned values of the eastern coordinate on the central meridian  $FE = 500\ 000$  m and values of the southern coordinate on the equator  $FN = 10\ 000\ 000$  m.

#### **Conversion of geodetic coordinates $(\varphi, \lambda)$ into UTM coordinates $(N_{UTM}, E_{UTM})$ — direct solution**

Conversion of geodetic coordinates  $(\varphi, \lambda)$  into Mercator projection coordinates  $(N_{UTM}, E_{UTM})$  is based on the formulas(1-4):

$$\begin{aligned}E_{UTM} &= k_0 f_1 (\varphi, \lambda - \lambda_0) + FE = k_0 E_{TM} + FE \\N_{UTM} &= k_0 f_2 (\varphi, \lambda - \lambda_0) + FN = k_0 N_{TM} + FN\end{aligned}\tag{1}$$

where:

- $N_{\text{UTM}}$  — Northing;
- $E_{\text{UTM}}$  — Easting;
- $\varphi, \lambda$  — coordinates for a geodetic point;
- $\lambda_0$  — geodetic length of the central meridian (CM);
- $k_0 = 0,9996$  — central scale;
- $FN = 0$  (10 000 000 m on the southern hemisphere) — False Northing,  $N_{\text{eq}}$ , equator Northing;

- $FE = 500\ 000\text{m}$  — False Easting, central meridian Easting,  $E_{\text{cm}}$ , CM Easting;
- $Z$  — the number for the projection zone ( $Z = 33$  for  $\lambda_0 = 15^\circ$  E;  $Z = 34$  for  $\lambda_0 = 21^\circ$  E, etc.);
- $u, v, P$  — indirect variables;
- $\chi$  — conformal latitude;
- $R_0$  — meridional isoperimetric radius;
- $N_{\text{TM}}$  and  $E_{\text{TM}}$  — Northern and Eastern Transverse Mecator (TM) coordinates are calculated as follows [6]:

$$E_{\text{TM}} = R_0 \cdot [u + a_2 \cdot \sinh(2u) \cdot \cos(2v) + a_4 \cdot \sinh(4u) \cdot \cos(4v) + a_6 \cdot \sinh(6u) \cdot \cos(6v) + a_8 \cdot \sinh(8u) \cdot \cos(8v) + a_{10} \cdot \sinh(10u) \cdot \cos(10v) + a_{12} \cdot \sinh(12u) \cdot \cos(12v)] \quad (2)$$

$$N_{\text{TM}} = R_0 \cdot [v + a_2 \cdot \cosh(2u) \cdot \sin(2v) + a_4 \cdot \cosh(4u) \cdot \sin(4v) + a_6 \cdot \cosh(6u) \cdot \sin(6v) + a_8 \cdot \cosh(8u) \cdot \sin(8v) + a_{10} \cdot \cosh(10u) \cdot \sin(10v) + a_{12} \cdot \cosh(12u) \cdot \sin(12v)]$$

where:

$$u = \arctan h[(\cos \chi) \cdot (\sin \Delta\lambda)] \quad (3)$$

$$v = \arctan 2[(\cos \chi) \cdot (\cos \Delta\lambda), \sin \chi]$$

$$\cos \chi = \frac{2 \cos \varphi}{(1 + \sin \varphi)/P + (1 - \sin \varphi)/P}; \quad \sin \chi = \frac{(1 + \sin \varphi)/P - (1 - \sin \varphi)/P}{(1 + \sin \varphi)/P + (1 - \sin \varphi)/P} \quad (4)$$

$$P = \exp(e \cdot \arctan h(e \cdot \sin \varphi)) = \left( \frac{1 + e \cdot \sin \varphi}{1 - e \cdot \sin \varphi} \right)^{e/2} \quad (5)$$

Calculating the number for zone Z:

$$Z = \text{floor} \left( \frac{\lambda + 180^\circ}{6^\circ} \right) + 1 \quad (6)$$

If  $\varphi < 0$  to  $Z = -Z$

where

$\text{floor}(x)$  — the function returns the highest integer lower than or equal to x.

Table 1. Parameters of ellipsoid WGS 84 and values of constant UTM factors [6]

Symbol	Value	Symbol	Value
$a$	6 378 137,000 m	$b$	6 356 752.314 245 179 4976
$f$	1/298,257 223 563	$e$	0.081819190842621494335
$R_0$	6 367 449.145 823 415 3093m	$e^2$	0.006 694 379 990 141 3169961
$a_2$	8.377 318 306 244 698 3032 E-04	$b_2$	-8.3773216405794867707 E-04
$a_4$	7.608 527 773 572 489 156 E-07	$b_4$	-5.905870152220365181 E-08
$a_6$	1.197 645 503 242 492 10 E-09	$b_6$	-1.67348266534382493 E-10
$a_8$	2.429 170 680 397 3131 E-12	$b_8$	-2.1647981104903862 E-13
$a_{10}$	5.711 818 369 154 105 E-15	$b_{10}$	-3.787930968839601 E-16
$a_{12}$	1.479 998 027 052 62 E-17	$b_{12}$	-7.23676928796690 E-19

Example 1. Data:  $\varphi = 54^\circ 50'N$ ;  $\lambda = 018^\circ 30'E$ ;  $\lambda_0 = 21^\circ$

Calculation results:

$$P = 1.005 495 751 807 5947$$

$$\cos\chi = 0.578 540 219 674 6492; \sin\chi = 0.815 653 856 865 0361$$

$$u = -0.025 240 928 952 480 35; v = 0.954 307 756 584 6402$$

$$E = -160 630.664 330 011 06 \text{ m}; E_{UTM} = 33 339 433.587 935 72$$

$$N = 6 081 542.197 579 098 \text{ m}; N_{UTM} = 6 079 109.580 700 067$$

$$Z = 34$$

### Conversion of UTM coordinates ( $N_{UTM}, E_{UTM}$ ) into geodetic coordinates ( $\varphi, \lambda$ ) — inverse solution [6]

Conversion of Mercator projection coordinates ( $E, N$ ) into geodetic coordinates ( $\varphi, \lambda$ ) is performed as follows [6]:

$$\begin{aligned} \lambda &= \lambda_0 + \Delta\lambda = \lambda_0 + g_1(E_{TM}, N_{TM}) = \lambda_0 + g_1\left(\frac{E_{UTM} - E_{cm}}{k_0}, \frac{N_{UTM} - N_{eq}}{k_0}\right) \\ \varphi &= g_2(E_{TM}, N_{TM}) = g_2\left(\frac{E_{UTM} - E_{cm}}{k_0}, \frac{N_{UTM} - N_{eq}}{k_0}\right) \end{aligned} \quad (7)$$

where:

$g_1$  and  $g_2$  — functions of inverse conversion of transverse Mercator projection coordinates.

The other symbols are as above.

$$\Delta\lambda = \arctan 2(\cos \nu, \sin u)$$

where:

$$\begin{aligned} u &= \frac{E_{TM}}{R_0} + b_2 \cdot \sinh\left(\frac{2E_{TM}}{R_0}\right) \cdot \cos\left(\frac{2N_{TM}}{R_0}\right) + b_4 \cdot \sinh\left(\frac{4E_{TM}}{R_0}\right) \cdot \cos\left(\frac{4N_{TM}}{R_0}\right) + \\ &\quad + \dots + b_{12} \cdot \sinh\left(\frac{12E_{TM}}{R_0}\right) \cdot \cos\left(\frac{12N_{TM}}{R_0}\right) \\ \nu &= \frac{N_{TM}}{R_0} + b_2 \cdot \cosh\left(\frac{2E_{TM}}{R_0}\right) \cdot \sin\left(\frac{2N_{TM}}{R_0}\right) + b_4 \cdot \cosh\left(\frac{4E_{TM}}{R_0}\right) \cdot \sin\left(\frac{4N_{TM}}{R_0}\right) + \\ &\quad + \dots + b_{12} \cdot \cosh\left(\frac{12E_{TM}}{R_0}\right) \cdot \sin\left(\frac{12N_{TM}}{R_0}\right) \end{aligned} \quad (8)$$

Values of the constants (R and b) for WGS 84 are presented in table 1.

$$\varphi = \arctan 2(\cos \varphi, \sin \varphi)$$

where:

$$\begin{aligned} \sin \chi &= \frac{\sin \nu}{\cosh u} \\ \cos \chi &= \frac{\sinh u}{(\cosh u) \cdot (\sin \lambda)} \quad (9) \\ \text{when: } |\lambda| &\leq 0,01 \quad \text{lub} \quad |\lambda \pm \pi| \leq 0,01 \quad \text{lub} \quad |\lambda \pm 2\pi| \leq 0,01 \\ \text{then: } \cos \chi &= \frac{\sqrt{\sinh^2 u + \cos^2 \nu}}{\cosh u} \end{aligned}$$

$$\cos \varphi = 0,5[(1 + \sin \varphi)/P + (1 - \sin \varphi) \cdot P] \cdot \cos \chi \quad (10)$$

where:

$\sin \varphi$  — the limit of the sequence  $s_1, s_2, s_3, \dots, s_n$ ;

$P$  — the limit of the corresponding to it sequence  $P_1, P_2, P_3, \dots, P_n$  calculated as follows:

$$\begin{aligned} s_1 &= \sin \chi \\ s_{n+1} &= \frac{(1 + \sin \chi) \cdot P_n^2 - (1 - \sin \chi)}{(1 + \sin \chi) \cdot P_n^2 + (1 - \sin \chi)} \\ P_n &= \exp[e \cdot \arctan h(e \cdot s_n)] = \left( \frac{1 + e \cdot s_n}{1 - e \cdot s_n} \right)^{e/2} \\ \lambda_0 &= 6^\circ \cdot |Z| - 183^\circ \end{aligned} \quad (11)$$

Example 2. Data:  $N_{UTM} = 628\ 700,0$ ;  $E_{UTM} = 6\ 068\ 800,0$ ;  $Z = 33$

Calculation results:

$$E_{TM} = 6\ 071\ 228.491\ 396\ 5585; N_{TM} = 128\ 751.500\ 600\ 240\ 09$$

$$u = 0.020\ 231\ 445\ 740\ 401\ 44; v = 0.952\ 687\ 460\ 956\ 6622$$

$$\Delta\lambda = 0.034\ 900\ 407\ 343\ 768\ 06$$

$$\cos\chi = 0.579\ 729\ 422\ 896\ 6682; \sin\chi = 0.814\ 809\ 055\ 072\ 35$$

$$P = 1.005\ 490\ 077\ 954\ 9092; S = 0.816\ 640\ 957\ 353\ 4602$$

$$\lambda = 016^{\circ}59'58.725\ 758\ 707\ 058\ 87"E; \varphi = 54^{\circ}44'59.786\ 350\ 630\ 39292"N$$

$$\lambda_0 = 15^{\circ}$$

### **MATHEMATICAL DEPENDENCE RELATIONS OF CONVERTING GEODETIC COORDINATES INTO UTM COORDINATES BASED ON MOH MATHEMATICAL DEPENDENCE RELATIONS**

Presented below are alternative to NGA algorithms of the transverse Mercator projection contained in IHO Manual of Hydrography (MoH) published by the International Hydrographic Bureau (IHB) [2].

#### **Transformation of coordinates $\varphi, \lambda$ into $N_{UTM}, E_{UTM}$ following MoH formulas — direct solution**

Conversion of coordinates  $\varphi, \lambda$  into  $N_{UTM}, E_{UTM}$  following MoH formulas, taking into account adopted changes and noticed errors, can be performed on the basis of the following dependence relations:

$$N_{UTM} = N_0 + k \cdot N_{TM}$$

$$E_{UTM} = E_0 + k \cdot E_{TM}$$

$$N_{TM} = S + \frac{N \cdot \sin\varphi \cdot \cos\varphi}{2} N \cdot \Delta\lambda^2 + \frac{N \cdot \sin\varphi \cdot \cos^3\varphi}{24} (5 - \tan^2\varphi + 9\eta^2 + 4\eta^4) \cdot \Delta\lambda^2 = \dots$$

$$E_{TM} = N \cdot \Delta\lambda \cdot \cos\varphi + \frac{N \cdot \cos^3\varphi}{6} (1 - \tan^2\varphi + \eta^2) \cdot \Delta\lambda^3 + \dots$$

where:

$$\begin{aligned}
S &= \int_0^\varphi M d\varphi = \alpha \cdot \varphi + \beta \cdot \sin 2\varphi + \gamma \cdot \sin 4\varphi + \delta \cdot \sin 6\varphi + \dots & (12) \\
\alpha &= a \left( 1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256} \right) & \beta &= -a \left( \frac{3e^2}{8} + \frac{3e^4}{32} + \frac{45e^6}{1024} \right) \\
\gamma &= a \left( \frac{5e^4}{256} + \frac{45e^6}{1024} \right) & \delta &= -a \left( \frac{35e^6}{3072} \right) \\
M &= \frac{a(1-e^2)}{\sqrt{(1-e^2 \cdot \sin^2 \varphi)^3}} & N &= \frac{a}{\sqrt{1-e^2 \cdot \sin^2 \varphi}} \\
\eta^2 &= e^{12} \cdot \cos^2 \varphi & \Delta\lambda &= \lambda - \lambda_0 \\
Z &= \frac{\lambda_0 + 183}{6} \quad \text{dla } \varphi \leq 0^\circ \quad Z = -Z
\end{aligned}$$

$FN_0$  — False Northing:  $FN_0 = 0$  m on the northern hemisphere,  $FN_0 = 10\ 000\ 000$  m on the southern hemisphere;

$FE_0$  — False Easting:  $FE_0 = 500\ 000$  m;

$S$  — meridional arc from equator to the  $\varphi$  latitude;

$M$  — meridian curvature radius;

$N$  — normal section curvate.

Table 2. Values of factors for equation (12) [2]

No.	Parameter	WGS 84 [m]
1	$\alpha$	6 367 449.146
2	$\beta$	-16 038.509m
3	$\gamma$	16.833
4	$\delta$	-0.022

Calculation results for example 1:

$$S = 6\ 078\ 676.646\ 973 \text{ m}$$

$$N_{\text{UTM}} = 6\ 079\ 109.580 \text{ m}; E_{\text{UTM}} = 34\ 339\ 433.573 \text{ m}; Z = 34$$

### **Conversion of UTM coordinates ( $N_{\text{TM}}$ and $E_{\text{TM}}$ ) into geodetic coordinates ( $\varphi, \lambda$ ) — inverse solution**

First calculated are coordinates ( $N_{\text{TM}}$  and  $E_{\text{TM}}$ ) of the transverse Mercator projection:

$$\begin{aligned} N_{\text{TM}} &= (N_{\text{UTM}} - FN_0)/k_0 \\ E_{\text{TM}} &= (E_{\text{UTM}} - FE_0)/k_0 \end{aligned} \quad (13)$$

The solution requires calculating footpoint latitude  $\varphi_f$  for the point whose distance from the equator is equal to  $N_{\text{TM}}$  i.e.  $B(\varphi_f) = N_{\text{TM}}$  (fig. 1).

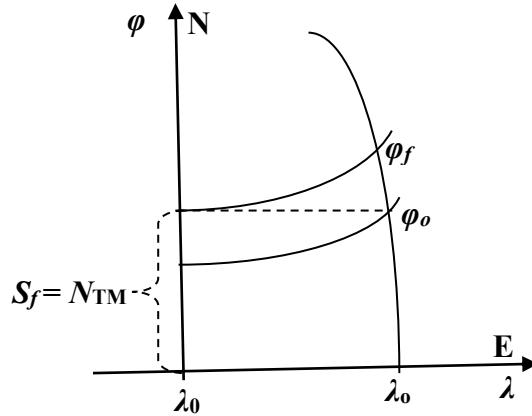


Fig. 1. Relations between latitudes  $\varphi_o$  and  $\varphi_f$  [own work]

This latitude is usually calculated from the formula (12) by means of iteration, adopting  $\varphi_{f0} = N_{\text{TM}}/\alpha$  as the initial value.

$$\begin{aligned} \varphi_{f0} &= N_{\text{TM}} \\ S_i &= \alpha \cdot \varphi_{fi} + \beta \sin 2\varphi_{fi} + \gamma \sin 4\varphi_{fi} + \delta \sin 6\varphi_{fi} \\ \dots \\ \varphi_{fi} &= \varphi_{f0} + (N_{\text{TM}} - B_i)/\alpha \\ S_i &= \alpha \cdot \varphi_{fi} + \beta \sin 2\varphi_{fi} + \gamma \sin 4\varphi_{fi} + \delta \sin 6\varphi_{fi} \\ \text{STOP} \quad \text{when} \quad N_{\text{TM}} - S_i &\leq \varepsilon \quad \varphi_f = \varphi_n \end{aligned} \quad (14)$$

where

$\varepsilon$  — low value adopted as calculation accuracy.

The troublesome iteration-based calculation of the geodetic latitude ( $\varphi_f$ ) for the point on the central meridian at the distance equal to  $N_{\text{TM}}$  can be substituted

by a direct calculation from the formula presented e.g. in Admiralty Manual of Navigation Vol. 1 The Principles of Navigation formula A4.14 [11], as follows:

$$\varphi_1 = \frac{8011 n^5}{2560} \sin 10\Theta_1 + \frac{1097 n^4}{512} \sin 8\Theta_1 + \left( \frac{151 n^3}{96} - \frac{417 n^5}{128} \right) \sin 6\Theta_1 + \\ + \left( \frac{21 n^2}{16} - \frac{55 n^4}{32} \right) \sin 4\Theta_1 + \left( \frac{3n}{2} - \frac{27 n^3}{32} + \frac{269 n^5}{512} \right) \sin 2\Theta_1 + \Theta_1$$

where: (15)

$$\Theta_1 = \frac{N_{TM}}{b \cdot (1+n) \cdot \left( 1 + \frac{5n^2}{4} + \frac{81n^4}{4} \right)}$$

$$n = \frac{a-b}{a+b}$$

Calculation of geodetic coordinates ( $\varphi$  and  $\lambda$ ) is performed as follows:

$$\varphi = \varphi_f - \frac{\tan \varphi_f}{2} \left( \frac{E_{TM}^2}{M_f \cdot N_f} \right) + \frac{\tan \varphi_f}{24} \left( 5 + 3 \tan^2 \varphi_f + \eta_f^2 - 9 \eta_f^2 \cdot \tan^2 \varphi_f \right) \cdot \left( \frac{E_{TM}^4}{M_f \cdot N_f^3} \right) + \dots$$

$$\Delta \lambda = \frac{E_{TM}}{N_f \cdot \cos \varphi_f} - \left( \frac{1 + 2 \tan^2 \varphi_f + \eta_f^2}{6 \cos \varphi_f} \right) \cdot \left( \frac{E_{TM}}{N_f} \right)^3 + \dots$$

$$\lambda = \lambda_0 + \Delta \lambda$$

(16)

Calculation results for example 2:

$$N_{TM} = 6\ 071\ 228.491\ 396\ 5585; E_{TM} = 128\ 751.500\ 600\ 240\ 09$$

$$MoH: \varphi_f = 54^\circ 45' 59.133\ 195\ 790\ 045'; AMoN: \varphi_f = 54^\circ 45' 59.132\ 200\ 906''$$

$$MoH: \lambda = 016^\circ 59' 58.724\ 251''E; \varphi = 54^\circ 44' 59.787\ 327''N; \lambda_0 = 15^\circ$$

$$AMoN: \lambda = 016^\circ 59' 58.724\ 202''E; \varphi = 54^\circ 44' 59.786\ 332''N; \lambda_0 = 15^\circ$$

## GEOTRANS APPLICATION PROGRAM

Of many available in the Internet free of charge programs used to convert geodetic coordinates into UTM coordinates and UTM coordinates into geodetic ones MSP GEOTRANS 3.5 [5] was used to make calculations in both examples. The results are presented below:

Calculation accuracy = 1 m

Calculation results for example 1:

Data:  $\varphi = 54^{\circ}50'N$ ;  $\lambda = 018^{\circ}30'$ ;  $\lambda_0 = 21^{\circ}$

Results:  $E = 339433.588$ ;  $N = 6\ 079\ 109.581$ ;  $Z = 34$

Calculation results for example 2

Data:  $E = 628700,0$ ;  $N = 6068800$ ;  $S = 33$

Results:  $\lambda = 16^{\circ}59'58.7258''E$ ;  $\varphi = 54^{\circ}44'59.7864''N$

## CONCLUSIONS

The recommended military standards taken from NGA.SIG.0012\_2.0.0\_UTMUPS, March 25, 2014, are characterized by the highest accuracy but because of the complexity of calculations they can be applied with support of specialized software for scientists or engineers (e.g. Mathcad, MATLAB and others).

The formulas taken from MoH are less accurate but much Simple and can be applied to practical calculations with electronic calculators:

- when solving single problems;
- when the methods for avoiding iteration calculations proposed in this article have been taken into account.

Computer software available free of charge, e.g. GEOTRANS provide sufficient accuracy for practical needs but they should be used after verification with the presented algorithms.

The presented solutions can be used in training hydrographers and developing computer software dedicated for purposes in navigation and hydrographic work.

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## **ZALEŻNOŚCI ANALITYCZNE PRZEKSZTAŁCENIA WSPÓŁRZĘDNYCH GEODEZYJNYCH NA WSPÓŁRZĘDNE UTM ZALECANE W PRACACH HYDROGRAFICZNYCH**

### **STRESZCZENIE**

W artykule przedstawiono zależności matematyczne do bezpośredniego przekształcenia współrzędnych geodezyjnych na współrzędne płaskie odwzorowania poprzecznego Merkatora UTM (*Universal Transverse Mercator*) i zamiany odwrotnej zalecane postanowieniami norm obronnych.

W szczególności opisano formuły zaczerpnięte z dokumentu normalizacyjnego National Geospatial-Intelligence Agency NGA.SIG.0012\_2.0.0\_UTMUPS z 25.03.2014 r. oraz dodatkowo podręcznika IHO *Manual of Hydrography* wydanego przez Międzynarodowe Biuro Hydrograficzne. Dla zilustrowania zastosowania zaprezentowanych wzorów zamieszczono stosowne przykłady obliczeniowe na elipsoidzie odniesienia WGS 84 (*World Geodetic System of 1984*).

**Słowa kluczowe:**

przekształcanie współrzędnych, współrzędne UTM, elipsoida WGS 84.