VARIABLE PARAMETER MULTILINEAR MUSKINGUM METHOD: CASE STUDY ON THE DANUBE RIVER

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Abstract

The Muskingum method is based on a linear relationship between a channel's storage and inflow and outflow discharges. The applicability of using travel-time discharge relationships to model the variability of the K parameter in a Muskingum routing model was tested. The new parameter estimation method is based on the relationships between the travel-time parameter (K) and the input discharge for the reach of the Danube River between Devín-Bratislava and Medveďov, which includes the Gabčíkovo hydropower scheme. The variable parametrisation method was compared with the classical approach. The parameter X was taken as the average of its values from a small set of flood waves, K was estimated as a function of the travel-time parameter and discharge, which was optimized for one flood wave. The results were validated using the Nash-Sutcliffe coefficient on 5 floods. The results obtained by these methods were satisfactory and, with their use, one could reduce the amount of data required for calibration in practical applications.

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Key words

- Muskingum method,
- Multilinear model,
- Travel-time discharge relationship,
- Danube River.

1 INTRODUCTION

Many methods have been proposed for predicting the characteristic features of the movement of a flood wave along a river along natural or manmade watercourses. The literature abounds with a broad spectrum of flow routing models (e.g., Liggett and Cunge, 1975; Fread, 1985; Linsley et al., 1986). Flow routing may be generally classified as either lumped or distributed. As the trend in the increased speed computing with decreasing costs continues, the use of distributed routing models for a wider range of applications is steadily increasing, since these models are capable of correctly simulating the movement of the broadest spectrums of hydraulic regimes, wave types, and channel characteristics. Due to their moderate data requirements and operational costs, hydrological routing models are, however, still in use for several practical applications (e.g. flood forecasting). As opposed to distributed models, the flow in these lumped flow routing models is computed as a function of time at one location at the end of a modelled river.

The Muskingum method, which was developed by McCarthy, is a popular two-parameter lumped hydrological flow routing method (McCarthy, 1938). The method is based on an assumed linear relationship between a channel's storage and inflow and outflow discharges; it accounts for prism and wedge storage in the reach modelled. The routing parameters (K, X) in the model were originally empirically derived by calibration using measured discharge hydrographs (e.g., Birkhead and James, 2002; O'Sullivan et al., 2012). In the original procedure, the parameters K and X are assumed to be constant in the reach modelled. It provides reasonably accurate results for moderate-to-slow rising floods propagating through mild-to-steep sloping watercourses (Maidment and Fread, 1993).

As its modest data requirements make it attractive for practical use, the Muskingum method is a popular alternative for routing flood waves through stream reaches. Therefore, various versions of the method, especially methods for estimating its parameters, are well established in the hydrological literature (e.g., McCarthy, 1938; Ponce

and Yevjevich, 1978; Ponce and Theurer, 1982; Ponce and Chaganti, 1994; O'Donnell et al., 1998; Tang, et al., 1999; Birkhead and James, 2002; Al-Humoud and Esen, 2006; Perumal and Sahoo, 2008; Easa, 2013; Karahan et al., 2015; Niazkar et al., 2016; Afzali, 2016; Hamedi et al., 2016; Chulsang et al., 2017; Oliveira et al., n.d.; Asgari et al., 2018; Bai et al., 2018; Farzin et al., 2018; Kang and Zhou 2018; Meyer et al., 2018).

In several previous studies we have shown that the relationship between the travel-time of flood peaks and peak discharges can be used to parameterize multilinear flood routing models based on the state space representation of the classical Kalinin - Miljukov cascade (see, e.g., Szolgay et al., 2007; Danáčová et al., 2015). A similarly structured multilinear version of the Muskingum routing model was proposed in our previous paper (Baláž et al., 2010). It is based on the discrete state space equations for the Muskingum equation, which enables us to more flexibly reflect the changes of parameters governing the system's dynamics (Szolgay, 1982) through a discharge variable parametrisation. The resulting model belonged to the family of multilinear models. An empirical relationship between the K parameter and the discharge was proposed therein, which allowed for the accounting of the nonlinearity of the flood routing process. The Muskingum differential equation was integrated under the assumption of the stepwise constant function of the input discharge. The multilinearity was introduced by allowing the time parameter (K) of the model to vary with the discharge. The disadvantage of such an empirical parametrisation lies in the fact that it needs to be verified under a variety of flow regimes and river morphologies.

In a pilot study by Baláž et al. (2010), the Muskingum model was tested in a river reach on the Morava River. In this study the same multilinear version of the Muskingum model was used on a reach of the Danube that contains the Gabčíkovo hydropower utilisation scheme. The general applicability of using travel-time discharge relationships to model the variability of the K parameter in a Muskingum routing model was further tested in this case study. We have analyzed the relationship between the discharge and travel time of a set of flood waves for the reach of the Danube River between Devín-Bratislava and Medveďov, where the flow regime has been anthropogenically altered by the utilization of hydropower. It is expected that this relationship is implicitly related to flow conditions in the channel and was fitted by empirical relationships between the travel-time of the flood peaks and the inflow to the river reach.

The travel time parameter K of the model was allowed to change as a function of the input discharge into the reach. Through such a variable parametrization, the model became multilinear (stepwise linear). Since the respective linear submodels change their parameters as a function of the input discharges, the resulting model is nonlinear. Two empirical models of such a relationship were considered. The results of the classic approach of McCarthy were compared with the variable parameter model setups of the model.

2 METHODOLOGY AND DATA

The development of conceptual non-linear, reservoir-type cascade models was one of the approaches used for incorporating non-linearity into the class of hydrological routing models (see, e.g., Laurenson, 1964; O'Connell et al., 1986; Malone and Cordery, 1989; Pekár et al., 2001; Pekárová et al., 2004; Svoboda et al., 2000). These models use a non-linear storage-outflow relationship in conjunction with a lumped continuity equation.

As an alternative to the use of such a relationship, the process models can be assumed to respond linearly to the input at any point in time; however, the model parameters are recalculated as a function of the flow values. These techniques, which are commonly referred to

as "multilinear modelling", usually distinguish different components in the input hydrograph, each of them being subsequently routed through a linear sub-model. The overall output of the non-linear system consists of the outputs from the linear sub-models. Such differing inflow components can be obtained by dividing the input hydrograph into horizontal or vertical segments. The former method is called the amplitude distribution scheme; the latter is known as the time distribution scheme. Kundzewicz (1984) gave an extensive description of the principles of these methods.

2.1 The multilinear version of the Muskingum model

The Muskingum method is a hydrological flow routing model with lumped parameters that describes the transformations of discharge waves in a river bed using two equations. The first one is the continuity equation, and the second equation is the relationship between the storage and the inflow and outflow of the river reach. The model is sufficiently well known; no detailed description is therefore given here (see, e.g., Maidment and Fread, 1993).

The discrete state space representation (Szolgay, 1982) is less well known; therefore, for the sake of completeness, we will show (based on Baláž et al., 2010) how to derive the state equations. The Muskingum equation is adopted as:

$$S(t) = K[XQ^i(t) + (1 - X)Q^o(t)] \tag{1}$$

where S is the storage; K is the time proportionality coefficient; X is a weighting factor (having the range $0 \leq X \leq 0,5$); Q^o is the outflow discharge from the river reach; and Q^i is the inflow discharge from the river reach. In combination with a continuity equation:

$$\frac{dS}{dt} = Q^i(t) - Q^o(t) \tag{2}$$

we obtain the following ordinary differential equation:

$$\frac{dS}{dt} = \frac{1}{K(X-1)}S(t) + \frac{1}{1-X}Q^i(t) \tag{3}$$

which describes the instantaneous change in storage in the river reach. In addition, the output from the reach can be calculated as

$$Q^o(t) = \frac{1}{K(1-X)}S(t) + \frac{X}{1-X}Q^i(t) \tag{4}$$

The last two equations describe the continuous state representation of the Muskingum method where S is the system state variable and Q^o is the system output. The general solution (Szolgay, 1982) is:

$$S(t) = e^{\left(\frac{1}{K(X-1)}\right)t}S_o + \int_0^t e^{\left(\frac{1}{K(X-1)}\right)(t-\tau)}\frac{1}{1-X}Q^i(\tau)d\tau \tag{5}$$

Under the assumption that inputs to the model are considered to be constant during the sampling interval of the length T between the time steps a and $a+1$, the state transition equation will have the form

$$\Phi(a + 1, a) = e^{T/(K(X-1))} \tag{6}$$

and the transition of the input will be calculated as:

$$\Psi(a + 1, a) = K - Ke^{T/(K(X-1))} \tag{7}$$

The discrete state equations then have the form:

$$S(a + 1) = \Phi(a + 1, a) . S(a) + \Psi(a + 1, a) . Q^i(a) \tag{8}$$

$$Q^o(a + 1) = \frac{1}{K(1-X)}S(a + 1) + \frac{X}{1-X}Q^i(a) \tag{9}$$

Since the general applicability of the multilinearity principle has not been proven theoretically for the Muskingum model, additional empirical verification of the variable parameter method is needed under different flow conditions as in former studies in the case of the multilinear Kalinin-Miljukov model (e.g., in Szolgay et al., 2007; Danáčová et al., 2011, Danáčová et al., 2014). Therefore, in this case study, this concept is applied to a river reach with anthropogenically altered flow conditions.

2.2 Data

In this paper the river reach on the Danube River with a length of 75 km between the gauging stations Devín-Bratislava (located northeast of the town of Bratislava) and Medveďov (located below the confluence of the Danube and the navigational channel of the Gabčíkovo hydropower scheme) (Fig. 1) was selected. It has an average longitudinal slope of 0.03 %. The mean annual discharge is slightly above 2000 m^3/s, and the 100-year flood discharge is 11000 m^3/s in Bratislava. The annual peak discharges of its floods show a modest upward trend (linear line) since 1876 and range from 3000 m^3/s to 10870 m^3/s (see, e.g. Bačová-Mitková and Halmová, 2014). The natural channel of the river reach includes a system of channels of an inland delta. The bankfull discharge in the natural part is around 2500 m^3/s. The manmade navigational channel with a capacity of approximately 4000 m^3/s is used both for navigation and energy production.
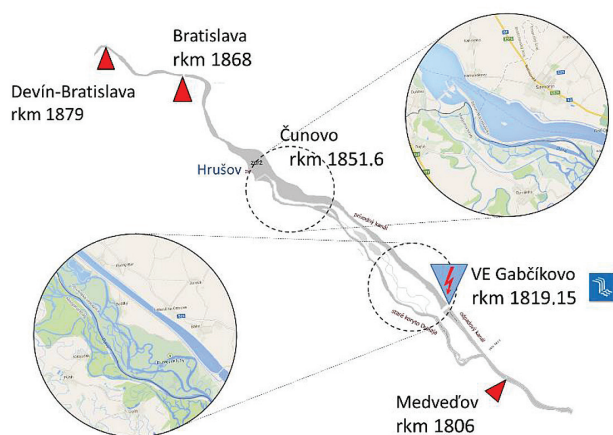

Fig. 1 *The river network of the Danube river with the river reach between Devín-Bratislava, Bratislava and Medveďov (rkm is the river kilometer)*
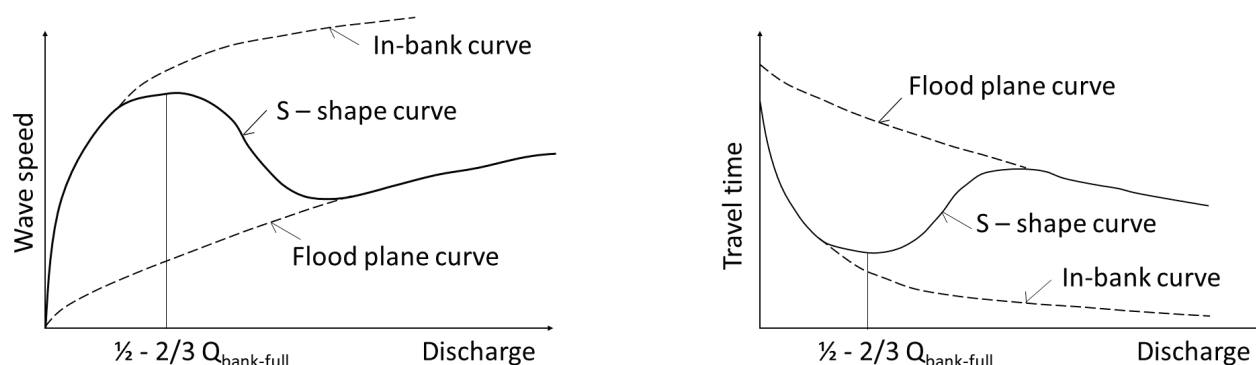
For the analysis of the travel time of the flood waves, we used hourly discharge data from the years 1992 to 2014 (the data were provided by the Slovak Hydrometeorological Institute). The whole dataset represents the altered flow conditions after the completion of the Gabčíkovo hydropower scheme.

2.3 Empirical travel-time and discharge relationship in the Danube reach with the anthropogenically altered flow conditions

Price (1973) postulated a wave speed (alternatively expressed as "travel time") and discharge relation for natural rivers, which can be interpreted as consisting of two power functions, i.e., one for the main channel and the other for overbank flow, which are joined by an S-shaped transition curve, Fig 2. On the left side the wave-speed and discharge are shown, and the travel-time and discharge relationships are shown on the right side.

At first the flood wave-speed increases sharply with an increasing discharge followed by a decrease. The maximum wave-speed usually occurs between one half and two thirds of the bankfull discharge. The slight decrease in wave-speed with the increasing discharge is attributed to an increase in roughness when the floodplains start to be filled by the lateral outflow. The decrease in wave-speed occurs until the point when the floodplain is fully connected to the flow (Price, 1973). Flood wave speed is an important parameter for flood alert and forecasting, decision making and optimization on flood management structures (Skublics et al., 2016). This type of relationship can be similarly interpreted as the travel-time and discharge relationship (see Fig. 2b).

To estimate the travel-time and discharge relationships from flood data, the method used by Wong and Laurenson (1983), which estimates the travel time of the wave as the time between the respective peaks at the inflow and outflow sections, was adopted here. From the discharge waves recorded, those with a peak discharge at Devín-Bratislava exceeding 1800 m^3/s were selected.

Figure 3 contains the basic descriptive statistics of the empirical data processed, the distributions of the instantaneous peak flows, and the peak-to-peak travel times, respectively.

It can be seen that about half of the peak flows (70 waves) exceeded the capacity of the navigation channel. The distribution of the peak-to-peak travel times is asymmetrical; the average travel time was 12.2 hours for all the 143 events analysed.
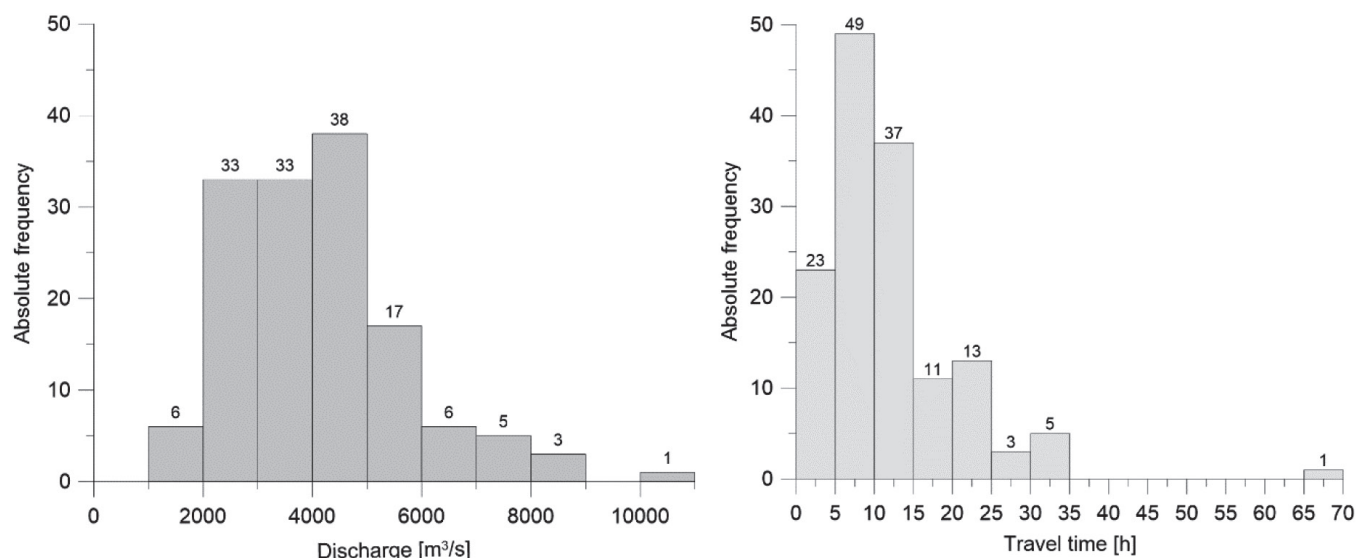

Fig. 2 *The general shape of the wave speed (travel time) and discharge relationship according to Price (1973)*

Fig. 3 *The histograms show the absolute frequency of the peak discharge and travel time in the river reach for all the discharge waves (period 1992-2014)*
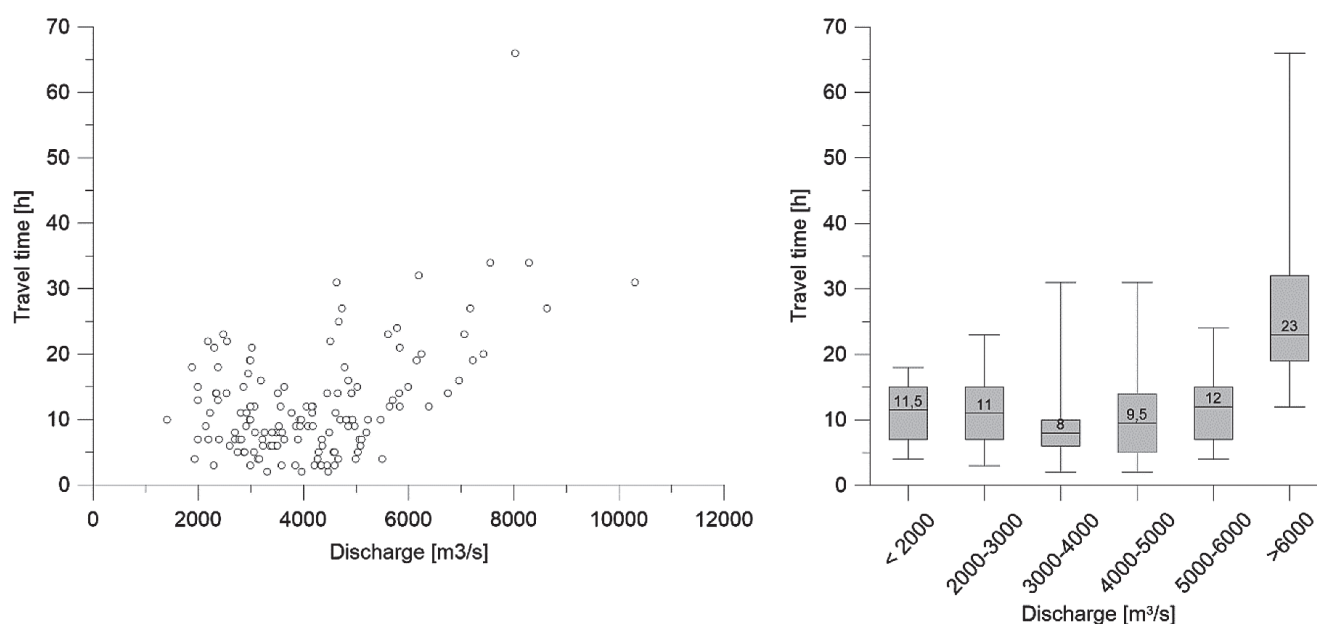

Fig. 4 *Empirical peak travel-time and input peak discharge in the Devín-Bratislava - Medveďov river reach (period 1992-2014), and box plot of the data for the categories selected (1ˢᵗ and 3ʳᵈ quartiles, minimum, median and maximum travel times)*

Figure 4 shows the empirical relationship between the peak-to-peak travel-time and input peak discharge in the Devín-Bratislava and Medveďov river reach (period 1992-2014) and a box plot of the data for the categories selected (1ˢᵗ and 3ʳᵈ quartiles, minimum, median and maximum travel times).

The pattern predicted by the Price model was not clearly reproduced by the raw travel time data. The variability of the travels times is high. This may be due to complicated flood attenuation in the pilot reach caused by the interaction of several factors such as the division of the flow between the natural river and the navigational channel at the weir in Hrušov, the operating rules of this weir and those of the power station (these not only depend on the flow conditions and flow forecasting but also on stock market-driven energy production), the differing attenuation of the partial flood waves in the natural river

and the navigation channel, and the specific attenuation in the natural river due to the interplay of the main river bed and the inland delta. Interestingly, when categorizing the discharges and interpreting the travel times as box-plotted data, the median values reveal a pattern similar to the Price model. Note that since it is impossible to interpret the Price concept of a bankfull discharge in the system, the minimum value of the medium travel time is around the capacity of the navigational channel, which is, on the average, expected to carry almost all the flows below 4000 m³/s. With a necessary degree of caution, one could interpret this as a threshold corresponding to the bankfull discharge in a natural river in the Price model, since flows above this threshold are routed through the natural river.

Based on experience with the multilinear Kalinin Miljukov model and with the Muskingum model, this behavior indicates (Szolgay
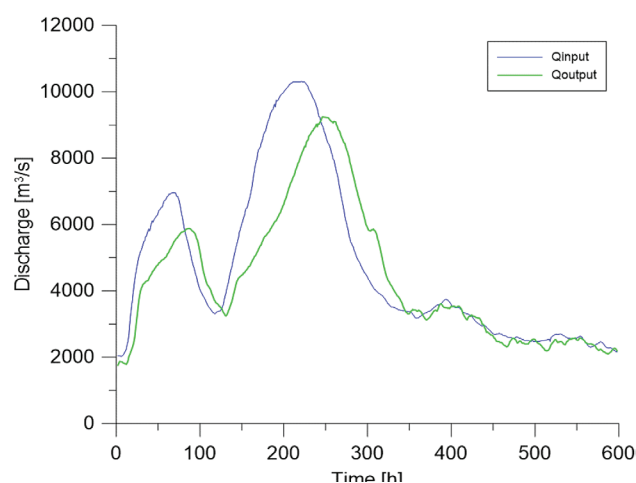
Tab. 1 *The dataset for validating the Muskingum model, parameters K and X and validation model efficiency*

No.	Flood wave	Peak discharge [m³/s]	Parameter X [-]	Parameter K [s]	Nash-Sutcliffe [-]
1	1995/08/25 - 1995/09/13	5832	0.011	39473.7	0.933
2	1998/03/14 - 1998/03/29	4646	0.095	38111.9	0.942
3	2000/03/05 - 2000/03/30	5189	0.145	31144.6	0.936
4	2002/08/06 - 2002/08/31	10310	0.0179	96133,6	0,921
5	2004/01/11 - 2004/01/24	4864	0.0875	28377,0	0.954
6	2005/07/09 - 2005/07/19	6741	0.2379	6847.4	0.892
7	2007/09/03 - 2007/09/18	7550	0.1689	81420.3	0.982
8	2008/08/15 - 2008/08/20	4780	0.4144	98210.6	0.945
9	2008/12/21 - 2008/12/29	3183	0.296	31646.1	0.946
10	2012/06/12 - 2012/06/18	5403	0.2438	40126.7	0.930
11	22/01/1999 - 20/02/1999	2992	0.375	2427.4	0.994
12	01/12/1997 - 07/01/1998	3398	0.117	5961.5	0.996

et al., 2007; Baláž et al., 2010; Danáčová et al., 2015) that we can try to adopt a similar methodology of introducing multilinearity for the Muskingum model on this river with anthropogenically modified flood regime, namely, by also using the relationship between the travel time parameter K of the model and the peak flow of the input hydrographs as the parametrization for the multilinear Muskingum model in this river reach with an anthropogenically altered flow regime.

2.4 Parametrisation of the multilinear Muskingum model by the travel-time parameter *K* – peak discharge relationships

For the following validation of the proposed concept for variable parametrization of the model, the two parameters of the Muskingum model were separately fitted to the 12 waves by genetic optimization. The range of the parameter X (from 0.11 to 0.414) and parameter K (from 2427.39 to 98210.6) were obtained, see Table 1.

The Nash-Sutcliffe coefficient was used as the objective function (Nash-Sutcliffe, 1970). It was also subsequently used throughout this study for all the optimization tasks.

$$R = \frac{\sum_{t=1}^{N}\left(Q_{o(t)}-\overline{Q}_o\right)^2 - \sum_{i=1}^{N}\left(Q_{o(t)}-Q_{s(t)}\right)^2}{\sum_{t=1}^{N}\left(Q_{o(t)}-\overline{Q}_o\right)^2} \qquad (10)$$

where:
- $\overline{Q}_o$ - is the mean value of the observed discharge [m³.s⁻¹]
- $Q_{s(t)}$ - is the simulated discharge [m³.s⁻¹]
- $Q_{o(t)}$ - is the observed discharge [m³.s⁻¹]

The following two variable parammetrisation approaches were then attempted (similarly as in Baláž et al., 2010): First, several alternative piecewise linear relationships of the $K = f$ (inflow) relationship were tested. These were composed from a preset number of non-overlapping linear segments of equal lengths. For this procedure the discharge range observed was first divided into intervals of the same length. Five to 12 intervals were used. For each of the respec-

Fig. 5 *The flood wave used for model calibration (2.8.2002 - 15.9.2002)*
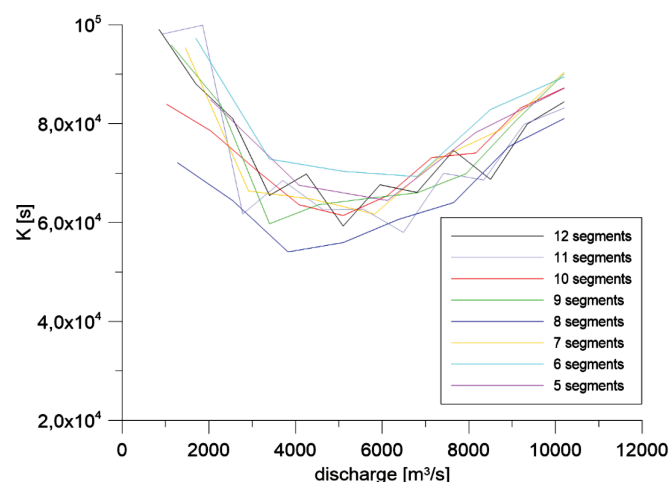
Fig. 6 *Piecewise linear relationships between the K parameter and inflow fitted by optimizing the multilinear model performance*

tive sets, the linear segments were chained, and an optimal chain was selected for simulations on one large calibrated flood wave (2.8.2002 - 15.9.2002, Fig. 5) with the help of a genetic algorithm (GA) using the Nash Sutcliffe criterion. This calibration procedure based on a genetic algorithm was used in several hydrology studies, e.g., Sleziak et al., (2016); Valent and Paquet (2017); Sleziak et al., (2018).

Similarly, as in Baláž et al., (2010), for practical reasons, we constrained the search space from above and below by the maximum K of 100000s and minimal K of 2000s from the previous calibration exercise (see Tab. 1). The results are shown in Fig. 6. The X parameter remained constant and was taken as the average X from the previous calibration on the set of 12 waves (Tab. 1).

Tab. 2 *The Nash-Sutcliffe coefficient for the piecewise linear relationships between the K parameter and inflow*

Piecewise linear relationships of	Nash - Sutcliffe [-]	Piecewise linear relationships of	Nash - Sutcliffe [-]
5 segments	0.9596	9 segments	0.9652
6 segments	0.9631	10 segments	0.9606
7 segments	0.9681	11 segments	0.9476
8 segments	0.9652	12 segments	0.9662

For the subsequent validation, a relationship composed of 7 segments was selected based on the Nash-Sutcliffe coefficient and the shape of the relationship (see Tab. 2).

In the second variable parametrization approach, as an attempt to generalize these relationships into one final mathematical description of the $K=f$ (inflow) relationship for the reach, a 5th degree polynomial was fitted into the breakpoints of the chains of linear segments by the least squares method. The result is shown in Fig. 7.

In conclusion, one can see that when comparing the trends in Figs. 4, 6, and 7, similar patterns can be identified between those of the medians of the travel times in the reach and the K parameter
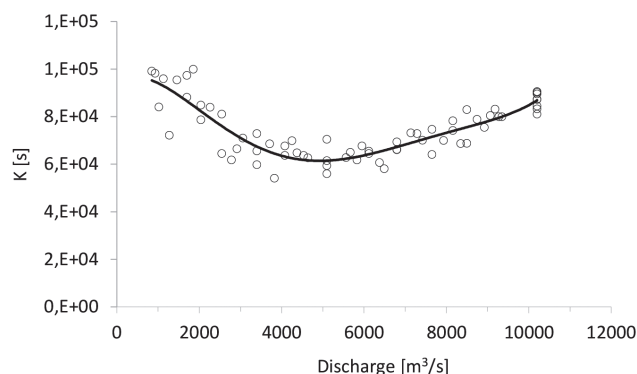

Fig. 7 *Generalization of the K=f (inflow) relationships by a 5th degree polynomial fitted into the breakpoints of the set of chains of the linear segments*

changes with the discharges in Figs. 6 and 7. This shows that similarly as on natural rivers, the K parameter can be taken as an indicator of the travel time; on the other hand, the patterns observed in the changes of the travel time could be expected to be reflected in the $K=f$ (inflow) in rivers with an altered flood regime.

3 VALIDATION OF THE VARIABLE PARAMETRISATION OF THE MUSKINGUM MODEL

For the verification of the principle, which focused on the empirical proofing of the applicability of the model's variable parametrization on a river with an anthropogenically altered flood regime, we used five randomly selected of flood waves in the middle range of peak discharges with diverse shapes (see Tab. 3). The following parametrization approaches were compared in the course of the model's validation:

Tab. 3 *Results: Nash-Sutcliffe values for schemes using the Muskingum model. The arrows indicate changes in the Nash Sutcliffe criterion*

No.	Flood wave	Peak discharge [m³/s]	Classical calibration method		Multilinear model	
			Optimal X and K for each flood	Average X and K for the set calibration wave (from optimal model)	Average X and variable K (7 segments)	Average X and variable K (smoothing polynomial fitted)
					(using GA)	(using GA)
			Column "A"	Column "B"	Column "C"	Column "D"
1	22/01/1999 - 20/02/1999	2992	0.985	0.960	0.958	0.962
2	01/12/1997 - 07/01/1998	3398	0.996	0.965	0.971	0.975
3	21/11/1993 -25/02/1994	4151	0.99	0.989	0.986	0.992
4	08/08/1995 -10/10/1995	5238	0.996	0.984	0.991	0.992
5	26/06/1997 - 19/08/1997	6576	0.997	0.995	0.996	0.996
Average values of the Nash-Sutcliffe coefficients			**0.993**	**0.978**	**0.980**	**0.983**

- the optimal X and K values were separately estimated for every flood wave (see Tab. 2, column "A") by the classical calibration approach as implemented in Valent, (2011). This parameter set served as a baseline representing the optimal model performance achievable for each wave separately (the standard classical calibration).

- X and K were taken as the average of all the parameter pairs from this baseline set (in Tab. 3, Column "B"). These values represent the usual practical situation in the application of the Muskingum model (e.g., for an engineering study for design flood wave attenuation or in practical forecasting applications), where the optimal parameters for a given flood event are not known, and the average parameters for the given reach have to be used in the simulation (here, averages from the 12 calibration flood waves were used).

- as the first alternative of the discharge-dependent variable parametrization, when $K=f$ *(inflow)* is expected to hold, a piecewise linear relationship consisting of 7 segments was chosen, and the parameter X was taken as the average from the 12 flood waves calibrated (Column "C" of Tab. 3).

- for the second alternative of the variable parametrization, a fitted 5th order smoothing polynomial was chosen, and the parameter X was again taken as the average of the 12 calibration flood waves (Column "D" of Tab. 3).

As can be seen in Tab. 3, the application of the average parameters (as expected) degrades the performance of the model when compared to the case-by-case optimal performance (which is, however, generally not achievable in a practical applications). The use of the discharge variable parametrization of the wave travel time parameter (K) results in an increased level of accuracy of the model, when compared to the "averaging engineering" scheme. Both variable parameter models outperform that model on each wave and come closer to the optimal solution of the case-by-case parametrization.

It is also important to note that even if the accuracy of the fit for simulated discharges was not the best, the visual comparison clearly shows that the match of the timing of the intermediate peaks is better than in the case of the average model (constant travel time) (see Fig. 8). In principle, both the multilinear models based on the variable travel-time discharge relationships performed slightly better than the optimal "engineering" linear model.

Another advantage of the proposed approach is that the $K=f$ (inflow) relationship was calibrated on one wave, and a limited amount of waves were needed for determining the X parameter. This could be set, in the worst case, according to the experience of the modeler for a given river when just one flood would be available.

Therefore, based on the results, one can conclude that both proposed variable parametrization approaches are applicable to rivers with an anthropogenically altered flow regime and could therefore be further developed.

4 DISCUSSION AND CONCLUSION

As one of the spatially lumped hydrological models that describe the routing of discharge waves in riverbeds, the Muskingum method is widely used in engineering studies and in hydrological forecasting. The method assumes a linear relationship between a channel's storage and inflow and outflow discharges; it accounts for prism and wedge storage. The two parameters K and X (travel-time related and storage related) in the model have traditionally been considered to be constant during the simulation and are usually derived empirically off-line by calibrating the storage discharge relationship using measured discharge hydrographs (e.g., Birkhead and James, 2002).

Since the Muskingum model was introduced by McCarthy (McCarthy, 1938), numerous studies have been published both on the properties of the method and its parametrization. In this study the relationship between the travel-time of flood peaks and the peak discharge was taken as the basis for the parametrization of the model. This relationship was empirically described for 143 floods from the period 1992-2014 on a reach of the Danube between Devín-Bratislava and Medveďov, where the flow regime has been anthropogenically altered by the utilization of hydropower. The travel-time parameter K of the model was allowed to change as a function of the input discharge into the reach. Through such a variable parametrization, the model became multilinear, e.g., stepwise linear. Since the respective linear submodels change their parameters as a function of the input discharges, the resulting model is nonlinear. Two empirical models of such a relationship were considered.

In the first model several alternatives of the piecewise linear relationships of $K=f$ (inflow) were tested that were composed of a preset number (five to twelve) of non-overlapping linear segments of equal

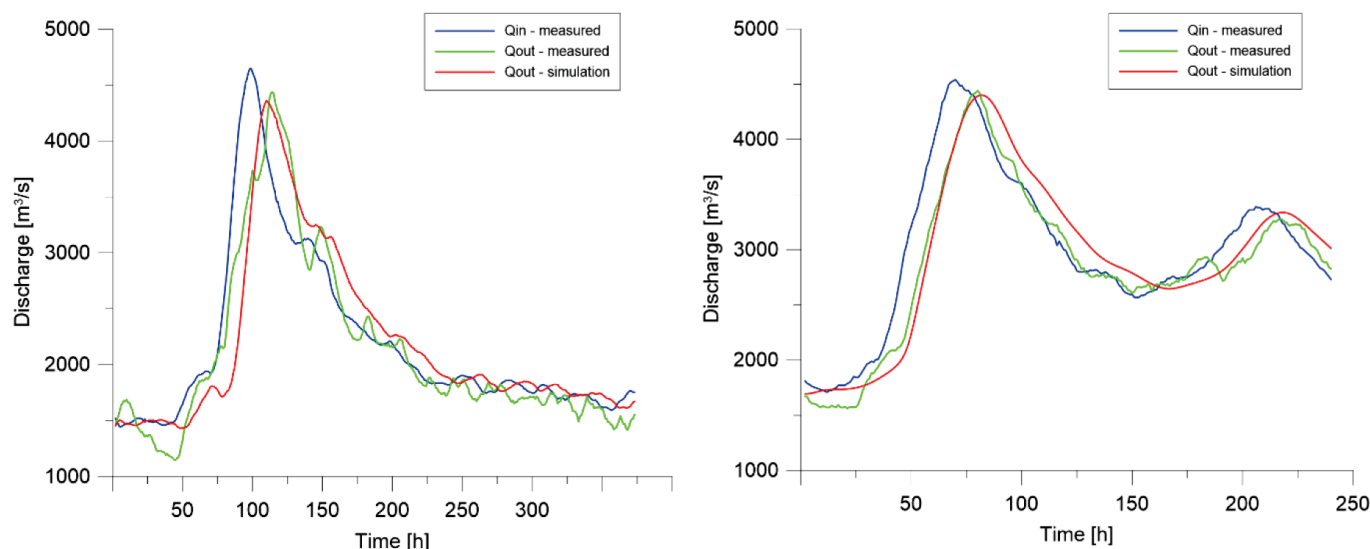**Fig. 8** *Examples of two simulated flood attenuations in the Devín-Bratislava – Medveďov river reach (X = 0,185; variable K = f (inflow))*

length. An optimal chain was selected by simulating one large flood wave calibrated (2.8.2002 - 15.9.2002, Fig. 5) with the help of a genetic algorithm (GA) using the Nash-Sutcliffe criterion. The chain with the best NS value (7 segments) was selected for a subsequent comparison/validation exercise for the model.

In the second model, in an attempt to generalize these segmented chains, a 5th degree polynomial was fitted into all the breakpoints of these chains of linear segments by the least squares method and used in the validation.

The results of the classic approach of McCarthy were compared with the variable parameter model setups of the model. The results obtained with the multilinear versions were satisfactory. Their advantage is that they respect the nonlinear character of the routing process and are only relied upon because of the advantage of the small amount of data that is required for their calibration. In a limiting case, even on reaches without any discharge data and where recorded stage

data are available for a reasonable amount of floods, the model can be set up and used. Based on our previous studies, e.g., Baláž et al. (2010), this was empirically proven until now only on natural rivers. Based on the results of this study, one can conclude that the proposed travel-time based parametrisation approach could also be applicable on rivers with an anthropogenically altered flow regime.

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