

# THE ELASTIC UNDRAINED MODULUS $E_{u50}$ FOR STIFF CONSOLIDATED CLAYS RELATED TO THE CONCEPT OF STRESS HISTORY AND NORMALIZED SOIL PROPERTIES

JOANNA STRÓŻYK, MATYLDA TANKIEWICZ

Wrocław University of Science and Technology, Faculty of Civil Engineering,  
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław.

E-mail: joanna.strozyk@pwr.edu.pl, matylda.tankiewicz@pwr.edu.pl

**Abstract:** The paper presents the results of a triaxial test conducted on stiff, consolidated clays. The standard TXCIU procedure (isotropic consolidation and undrained shearing) was applied in the laboratory soil tests. The undrained elastic modulus  $E_{u50}$  was determined from each test.

The  $E_{u50}$  values were determined for soil samples cut out from different depths and tested under different confining pressures. There was a significant scatter of values with depth, and no relationships between  $E_{u50}$  modules or other geotechnical parameters (e.g.,  $c_u$ ) were observed.

This work presents the concept of normalization of  $E_{u50}$  modulus values using modified normalization SHANSEP (Stress History And Normalized Soil Engineering Properties). This method was first proposed for estimating the value of the undrained shear strength  $c_u$  normalizing the parameter relative to the in situ effective vertical stress  $\sigma'_{vo}$  and loading history (overconsolidation stress  $\sigma'_p$  and overconsolidation ratio  $OCR$ ) of the soil.

The study demonstrated that the concept of normalization of soil properties can also be used for testing elastic modulus  $E_{u50}$  of consolidated natural clays and normalized values of geotechnical parameters taking into account the state of stress and load history can be correlated with the value of the overburden pressure.

Key words: *consolidated clays, stress history, normalised elastic modulus, triaxial test*

## 1. INTRODUCTION

The stress–strain characteristics of the soil behaviour are the basis for the assessment of the engineering properties of soil. This knowledge is essential for the estimation of soil response to external overloading. The stress–strain curve obtained, for example, from laboratory tests is the basis to determine soil parameters characterizing the stiffness of the material: the elastic modulus corresponding, in a sense, with Young’s modulus  $E$ , as in the case of soil the stress–strain relationships are not linear and the deformation is of elasto-plastic character [4].

The stiffness modules are among the most elementary geotechnical parameters. They are determined under different stress–strain conditions in drained and undrained tests. For stiff, consolidated clay the basic parameter investigated in standard laboratory test at high strains ( $\varepsilon > 1\%$ ) is the secant undrained modulus  $E_{u50}$  [5], [21] determined from undrained shearing tests, from stress–strain curve (Fig. 1) from the relationship (see [1], [5], [21]):

$$E_{u50} = \frac{\partial \sigma}{\partial \varepsilon_{50}} \quad (1)$$

where  $\delta \sigma$  is the change of vertical stress and  $\delta \varepsilon_{50}$  is the corresponding strain at stress equal 50% of peak strength value.

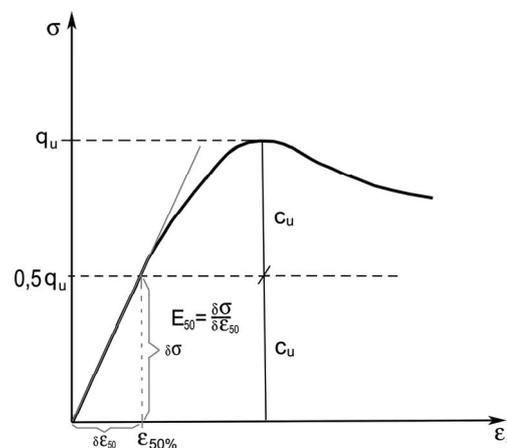


Fig. 1. Derivation of undrained elastic modulus  $E_{u50}$  from non-linear stress–strain relationships

Determining the  $E_{u50}$  modulus for the heavily consolidated clays in the laboratory conditions was found problematic. The difficulties result mainly from the disturbing of the soil structure, unavoidable in the sampling process. The sampling method is of the importance, the tube probe diameter and the depth from which the sample was obtained. It is assumed that the smaller the probe diameter and the greater the depth of sampling, the more disturbed the natural soil structure, and consequently the more difficult the designation of accurate parameters [7], [11]. The most susceptible to structural disturbances are stiff, heavily consolidated clays extracted from great depth, which swell and fracture in the sampling process.

These problems have been extensively described in the subject literature for years [3], [7], [8], [11], [17]. Various methods of dealing with such situations have also been widely described and applied in engineering practice. One of the most common procedures is based on the reconsolidation technique of soil [3] and procedures of normalization involving the preconsolidation stress  $\sigma'_p$  [6], [12] and overconsolidation ratio  $OCR$  defined as (see [11])

$$OCR = \frac{\sigma'_p}{\sigma'_{vo}} \quad (2)$$

where  $\sigma'_p$  is the preconsolidation stress and  $\sigma'_{vo}$  is the effective vertical in situ stress.

The normalization technique is known as SHANSEP procedure (Soil History And Normalized Soil Engineering Properties) developed at MIT [9], [18]. This procedure is used to estimate the undrained geotechnical parameters in situ. It allows geotechnical parameters obtained from the laboratory tests to be converted to in situ values. The procedure can be summarized in a few steps [7], [9]:

- stress history should be precisely established (i.e., preconsolidation stress  $\sigma'_p$  and overconsolidation ratio  $OCR$ ),
- a series of laboratory consolidated undrained shear tests should be performed with the reconsolidation technique,
- overconsolidation of soil should be the effect of mechanical overloading,
- geotechnical parameters should be expressed in terms of normalized soil parameters and relationship between parameter and  $\sigma'_p$  versus  $OCR$  should be established,
- normalized geotechnical parameter is the undrained shear strength  $c_u$  normalized from the equation

$$\frac{c_u}{\sigma'_{vc}} = S \cdot (OCR)^m \quad (3)$$

where  $c_u / \sigma'_{vc}$  is the normalized undrained shear strength ( $\sigma'_{vc}$  – effective vertical consolidation stress),  $S$  is the undrained shear strength ratio for normally consolidated clay ( $OCR = 1$ ) and  $m$  is an exponent.

This paper presents the results of laboratory tests of undrained elastic modulus  $E_{u50}$  conducted on stiff heavy consolidated clays taken from depths of 100 m below terrain level (b.t.l.). The value of  $E_{u50}$  modulus was determined according to the procedure described in the standards [5], [21]. The laboratory test results showed a great variation of  $E_{u50}$ , so an attempt was made to normalize the value of  $E_{u50}$  implying a modified procedure of SHANSEP. There is little experience in application of this procedure to the  $E_{u50}$  value [2], [10] even though the normalization concept has been widely described in the literature [9], [10], [13], [14].

The normalized values of geotechnical parameters were shown correlated to the  $OCR$  ratio. Due to the relatively high value of the estimated effective vertical stress in situ, it was considered that the consolidation of the soil examined is consolidation *sensu stricto* [16] (the mechanical loading is the main reason of consolidation) and the preconsolidation stress  $\sigma'_p$  is equal to the effective vertical stress in situ  $\sigma'_{vo}$ .

## 2. MATERIALS AND METHODS

The soil samples tested were cut out from boreholes in the region of Lower Silesia. A series of unconfined and confined compression tests were conducted on seven intact soil samples taken from 100 m to 287 m below terrain level. Soil type and basic geotechnical parameters were estimated in accordance with the PN-EN ISO 14688: 2006 [20] and PKN-CEN ISO/TS 17892: 2009 [21]. The obtained data are presented in Table 1.

The soils have been classified as clay, silty clay or sandy clay, all stiff in consistency. The bulk density  $\rho$  of the soil samples ranged from 1.52 g/cm<sup>3</sup> to 2.21 g/cm<sup>3</sup> and natural water content  $w$  varied from 13.8% to 36.5%.

A series of laboratory tests were performed in the standard triaxial apparatus. For each soil unconfined compression and confined compression tests (under 3 different confined pressures) were performed. The unconfined test was conducted as described in PKN-CEN

Table 1. Characteristic of soil specimens

Sample	Type of soil [ISO]	Depth below terrain level $z$ [m b.t.l.]	Effective stress in situ $\sigma'_{vo}$ [MPa]	Bulk density $\rho$ [g/cm <sup>3</sup> ]	Natural water content $w$ [%]	Consistency [ISO]
A1	Cl	100	1.98	2.21	13.8	stiff
A2	Cl	104	2.08	1.99	23.4	stiff
A3	Cl	155	3.06	1.80	36.5	stiff
A4	siCl	212	4.20	1.52	19.9	stiff
A5	saCl	217	4.30	1.85	22.7	stiff
A6	Cl	268	5.31	2.00	23.4	stiff
A7	siCl	287	5.66	1.83	25.8	stiff

ISO/TS 17892: 2009 [21]. The confined compression tests were carried out following the TXCIU procedure (triaxial isotropic consolidation undrained shearing) [5]. Following isotropic consolidation with confining pressure,  $\sigma_3$  samples were sheared, without drainage allowed, until failure point. Soil strength was established as a peak deviatoric stress ( $\sigma_1 - \sigma_3$ ). For each soil three specimens were tested with different confining pressure  $\sigma_3$ , which also means with different starting OCR ratio estimated from equation (2), where preconsolidation pressure  $\sigma'_p$  was equally effective in situ vertical stress  $\sigma'_{vo}$ . After the tests the undrained shear strength  $c_u$  and undrained elastic secant modulus  $E_{u50}$  for each sample and for each confined pressure were estimated. For all test samples  $E_{u50}$  modulus was estimated from stress–strain curves from relation (1) (see Fig. 1) at strain  $\varepsilon$  in range from 0.64 up to 4.33%.

### 3. RESULTS

The unconfined compression test results are presented in Table 2. The obtained  $c_u$  values range from 30.7–531.3 kPa. Values of undrained secant modulus  $E_{u50}$  vary from 4.0 MPa to 16.7 MPa. To correlate the values of test results to in situ values, a procedure of

normalization was performed [7], [9]. Both  $c_u$  and  $E_{u50}$  values were divided by the effective vertical stress in situ  $\sigma'_{vo}$  (Table 1). The normalization procedure in the case of determined undrained shear strength  $c_u$  (3) value showed that not every soil follows the SHANSEP concept [7], [19]. The aim of the unconfined test was to pre-check the possibility to normalize geotechnical parameters of the test soils. The relationships between normalized parameter are presented in Fig. 2. Correlations can be observed between undrained shear strength and the undrained elastic modulus  $E_{u50}$ . There is a high coefficient of determination  $R^2$  for polynomial regression. The investigation confirms the nature of relationship and shows that examined soil can be submitted to the normalization procedure.

The results of triaxial tests have been compiled in Table 3. The OCR ratio, defined as a quotient of applied confining pressure  $\sigma_3$  and in situ stress  $\sigma'_{vo}$  ranges from 5.0 to 28.4. Estimated undrained secant modulus  $E_{u50}$  (1) varies from 7.5 MPa to 52.3 MPa. When analysing the  $E_{u50}$  value according to the vertical in situ stress (Fig. 3) there are no relationships. A correlation can be seen in Fig. 4, where the  $E_{u50}$  modulus versus consolidation pressure  $\sigma'_{vc}$  is presented. It gives us a proper reason to normalize  $E_{u50}$  parameter according to consolidation stress.

Table 2. Results of unconfined compression tests

Sample	In situ stress $\sigma'_{vo}$ [MPa]	Undrained shear strength $c_u$ [kPa]	Normalized shear strength $c_u/\sigma'_{vo}$ [-]	Undrained modulus $E_{u50}$ [MPa]	Axial strain $\varepsilon_{50}$ [%]	Normalized modulus $E_{u50}/\sigma'_{vo}$ [-]
A1	1.98	531.3	0.268	16.7	2.86	8.43
A2	2.08	185.6	0.090	6.6	2.46	3.17
A3	3.06	218.7	0.071	6.0	3.35	1.96
A4	4.20	56.0	0.013	5.7	0.90	1.36
A5	4.30	30.7	0.007	4.0	0.74	0.93
A6	5.31	107.1	0.020	15.2	0.64	2.86
A7	5.66	329.8	0.058	7.4	4.33	1.30

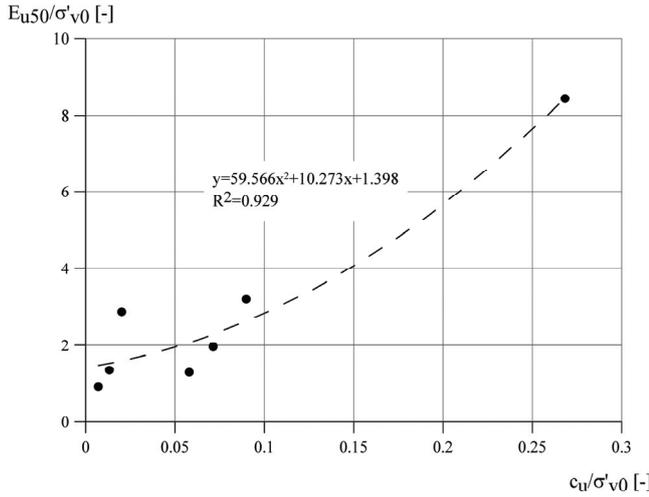


Fig. 2. Normalized undrained modulus  $E_{u50}$  versus normalized undrained shear strength  $c_u$

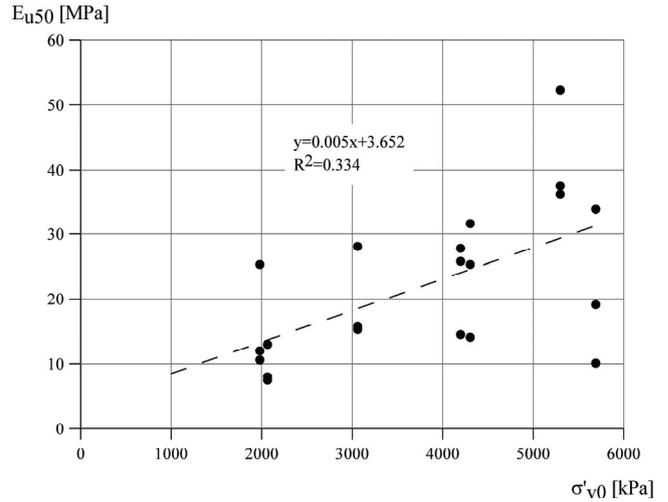


Fig. 3. The undrained modulus  $E_{u50}$  versus effective vertical stress in situ  $\sigma'_{v0}$

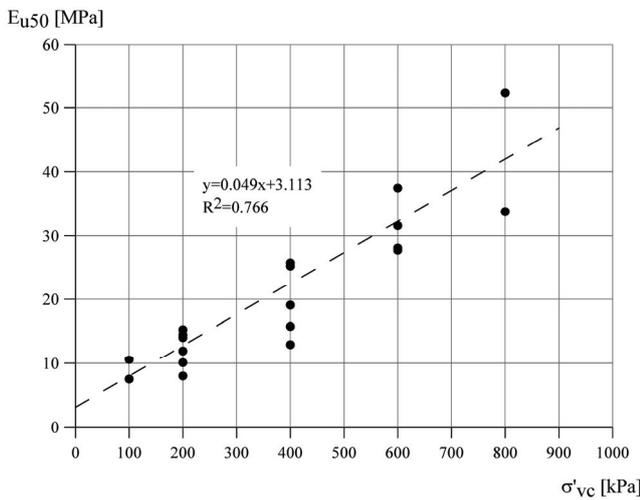


Fig. 4. The undrained modulus  $E_{u50}$  versus effective vertical consolidation stress in situ  $\sigma'_{vc}$

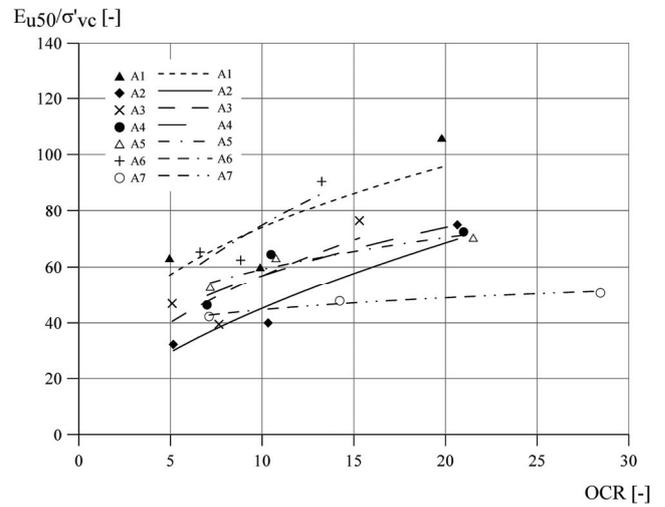


Fig. 5. The normalized undrained modulus  $E_{u50}$  versus OCR ratio

The normalized secant modulus  $E_{u50}$  value was established by the procedure described earlier. The relation of normalized secant modulus  $E_{u50}$  with OCR ratio for each soil sample is presented in Fig. 5.

This relationships can be expressed by the power functions modified SHANSEP equation (see [10])

$$(E_{u50}/\sigma')_{KOC} = (E_{u50}/\sigma')_{NC} \cdot OCR^n \quad (4)$$

where  $(E_{u50}/\sigma')_{KOC}$  and  $(E_{u50}/\sigma')_{NC}$  are respectively normalized modulus for overconsolidated (KOC) and normally consolidated soil (NC), and  $n$  is exponent of equation (4). Table 4 presents the values of those parameters and the coefficient of determination  $R^2$  for all the samples.

The value of  $(E_{u50}/\sigma')_{NC}$  parameter (corresponding to parameter  $S$  from original SHANSEP equation) for the test soils is between 11.26 and 33.19 and for  $n$  parameter the range is 0.129–0.603. The relationships show a good agreement: the coefficient  $R^2$  ranges from 0.658 to 0.956. The parameter  $n$  is characterized by a significant scatter of values. As observed in previous work based on soils with artificial structure consolidated in laboratory [2], [10], [15], this exponent may depend on the range of strain at the stiffness being estimated. In the case where the strain is about 1% and more the  $n$  value was close to 0.99 [10]. In presented test results (performed on natural soil consolidated under heavy overburden stress) (Table 4, Fig. 5) the  $n$  values are scattered, but always lower than 0.99 even at strain  $\varepsilon$  higher than 1%.

Table 3. Results of triaxial compression tests

Sample	In situ stress $\sigma'_{vo}$ [MPa]	Confining pressure $\sigma'_3$ [kPa]	OCR [-]	Modulus $E_{u50}$ [MPa]	Normalized modulus $(E_{u50}/\sigma')_{KOC}$ [-]
A1	1.98	100	19.8	10.6	106.0
		200	9.9	12.0	60.0
		400	5.0	25.3	63.3
A2	2.08	100	20.8	7.5	75.0
		200	10.4	8.0	40.0
		400	5.2	13.0	32.5
A3	3.06	200	15.3	15.3	76.5
		400	7.7	15.8	39.5
		600	5.1	28.1	46.8
A4	4.20	200	21.0	14.5	72.5
		400	10.5	25.8	64.5
		600	7.0	27.8	46.3
A5	4.30	200	21.5	14.1	70.5
		400	10.7	25.3	63.3
		600	7.2	31.6	52.7
A6	5.31	400	13.3	36.2	90.5
		600	8.8	37.5	62.5
		800	6.6	52.3	65.4
A7	5.68	200	28.4	10.1	50.5
		400	14.2	19.1	47.8
		800	7.2	33.8	42.3

Table 4. Parameters of modified SHANSEP equation

Sample	$(E_{u50}/\sigma')_{NC}$	$n$	$R^2$
A1	31.43	0.373	0.674
A2	11.26	0.603	0.922
A3	17.91	0.501	0.658
A4	23.60	0.382	0.834
A5	32.95	0.254	0.915
A6	23.77	0.498	0.738
A7	33.19	0.129	0.956

#### 4. CONCLUSIONS

The normalization procedures provide relevant tools to evaluate the geotechnical parameters of heavy consolidated, stiff, natural clay. As has been shown not only undrained shear strength parameter  $c_u$  can be normalized using normalization concept. The stiffness parameter such as  $E_{u50}$  can also be normalized taking into account stress history and the in situ stress. There are some problems in the estimation of undrained secant modulus  $E_{u50}$  values for natural soil samples in laboratory. The stiff, heavy consolidated soil samples are unlikely to be disturbed by the sampling process and estimated in laboratory decreasing the values of geotechnical parameters. In the case of such soil, the laboratory test results usually show scattered parameter values, and show poor relation or no relation be-

tween stiffness and consolidation stress. This result cannot be related to the in situ stress-strain conditions. The normalization procedure provides a very useful tool to estimate the in situ parameter from laboratory tests. Although some correlation has been found, further work seems to be necessary to achieve a more reliable correlation applicable in geotechnical design.

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