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TECHNICAL NOTE

PREDICTION OF STATIC LIQUEFACTION BY NOR SAND CONSTITUTIVE MODEL

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Abstract: The paper gives a short description of unstable behaviour of saturated sand under undrained monotonic loading. Constitutive model Nor Sand capable to describe static liquefaction is presented. The model is based on critical state soil mechanics and assumes associated flow rule. Hardening law incorporates the state parameter proposed earlier by Been and Jefferies. Results of numerical simulations of undrained element tests have been presented and discussed.

Key words: *static liquefaction, Nor Sand, element test simulation*

1. INTRODUCTION

Youd et al. [34] give the definition of liquefaction after Marcuson [19] as "The act or process of transforming any substance into a liquid". In cohesionless soils, the transformation from a solid state to a liquefied state is a consequence of increased pore pressure and reduced effective stress. Liquefaction occurs when the total stress remains constant and the pore pressure increases such that the normal effective stress becomes zero. It also occurs when the pore pressure remains constant and the total stress decreases such that the normal effective stress becomes zero.

Liquefaction may be of static or dynamic nature according to specific loading paths. It may occur in the following situations:

(a) when the hydraulic gradient of an upward current in saturated soil or an upward gas current equalizes the gravity forces ("boiling sands"),
(b) contacts between sand grains are lost due to soil vibrations (earthquakes) for saturated loose or even medium dense sands,
(c) rapid deviatoric loading is applied to a saturated very loose sand ("quicksands").

In the past decades liquefaction of soils has been extensively studied both experimentally and theoretically. Experimental works on static liquefaction originate from the works of Castro [1] and Castro and Poulos [7], a review article by Ishihara [12] as well as more recent papers by Yamamuro and Lade [28], [32], [33], Świdziński [27], Sawicki and Świdziński [20]. Theoretical works aim at formulating elasto-plastic constitutive equations capable of describing the behaviour of granular materials in undrained conditions. Constitutive models exhibiting this feature are of different origins (arbitrarily classified):

- elasto-plastic models based on Critical State Soil Mechanics assumptions and the works of the group of researchers from Cambridge (Cam-clay models), e.g., [1], [14], [15], [20];
- Lade's model with a double plastic potential ([17]);
- generalized plasticity models ([21], [22], [35], [36]);
- incremental octo-linear model ([9]);
- incrementally non-linear model ([8]);
- hypoplasticity models (e.g., [16], [28]).

In the above works if liquefaction is to be simulated within the framework of elasto-plasticity two conditions must be met:

(1) a model must reproduce hardening/softening of soil, and
(2) the flow rule must be non-associated.

More recently Jefferies and Been [15] showed that unstable behaviour of sands can be modelled within the framework of critical state mechanics of soils with the classical assumption of normality in elasto-plastic formulation. The aim of the paper is to show that static liquefaction can be predicted by the model Nor Sand proposed by Jefferies [14] without having recourse to the non-associated flow rule. The model assumptions and formulation are given in the following sections of the article. Results of numerical simulations of element tests in undrained triaxial conditions that confirm the ability to modelling the static liquefaction are presented.

2. TRIAXIAL COMPRESSION TESTS

Most frequently used device for testing soils is conventional triaxial apparatus with a cylindrical soil specimen. Stress and strain conditions are assumed to be axi-symmetrical. During testing the radial stress σ_3' remains constant and is exerted on the sample by means of the confining pressure in the cell. The only stress component that changes is the axial stress σ_1'. The increase and decrease of vertical pressure are caused by the moving piston (Fig. 1).

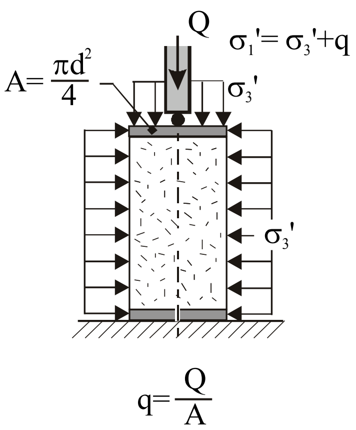

Fig. 1. Scheme of a sample in a triaxial apparatus

One of the major advantages of the triaxial apparatus is the control provided over drainage from the sample. When no drainage is required (i.e., in undrained tests), solid end caps are used at the top and the bottom of a sample. When drainage is required, the end caps are provided with porous plates and drainage channels. It is also possible to monitor pore-water pressures during a test. In drained conditions a sample is free to change its volume without pore-water pressure generation and total stresses σ are equal to effective stresses σ'. In undrained conditions the volume of a sample remains unchanged and for fully saturated sample the relationship between total stress, effective stress and pore-water pressure u is given by Terzaghi's principle

$$\sigma = \sigma' + u \, . \qquad (1)$$

In the above formula, the excess of pore-water pressure as well as compressive stresses are positive.

Test results are usually represented by the invariants of stress and strain, which in axi-symmetric conditions take the form:

- mean stress $p' = \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3') = \frac{1}{3}(\sigma_1' + 2\sigma_3')$,
- deviatoric stress $q = \sigma_1' - \sigma_3'$,
- volumetric strain $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_1 + 2\varepsilon_3$,
- shear strain $\varepsilon_s = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$.

In undrained conditions $\varepsilon_v = 0$, so $\varepsilon_s = \varepsilon_1$.

Conventional triaxial compression can be represented by a straight total stress path with a slope $\eta = q/p' = 3$ in the stress invariant space (Fig. 2). In undrained conditions increasing pore-water pressure bends an effective stress path towards failure condition (critical stress line).

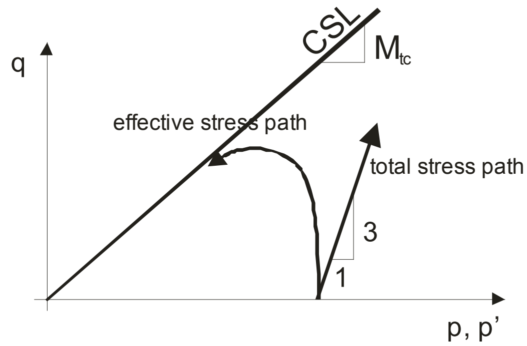

Fig. 2. Stress path in conventional triaxial compression

3. STATIC LIQUEFACTION

The critical state was defined by Roscoe et al. ([23]) as the state in which soil continues to deform at constant stress and constant void ratio. Jefferies and Been [15] point out that not only the volume of soil element is constant but also there is no tendency to change this state. The term *critical state* was firstly introduced by Casagrande in 1936 with reference to a critical void ratio. Next, Castro [4] conducted triaxial tests on loose samples which resulted in well-defined steady state at the end of the tests. Castro termed the relationship between the critical void ratio and the mean stress a *steady state line*.

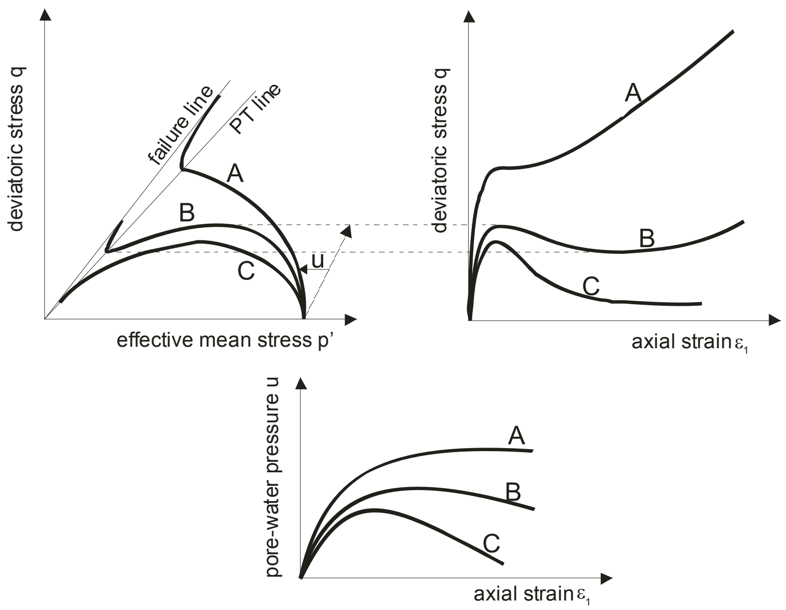

Fig. 3. Static liquefaction

Poulos [23] gave the formal definition of the steady state line: The steady state of deformation for any mass of particles is that state in which the mass is continuously deforming at constant volume, constant normal effective stress, constant shear stress and constant velocity.

Despite the discussion whether the critical state and steady state lines are the same [1], [4], [23], [26], Been et al. [2] found that for practical purposes their equivalence could be assumed.

Undrained shearing is carried out in triaxial apparatus usually after isotropic consolidation. Typical behaviour of sand in triaxial undrained compression is given in Fig. 3. Conclusions that follow from this figure are also applicable to description of the undrained behaviour during shearing after anisotropic consolidation.

During shearing in drained conditions the volume of the sample varies depending on its initial state. Loose sand decreases its volume whereas dense sand contracts a little at the onset of shearing and then dilates. Thus, sand can be contractive or dilative. As Świdziński points out in [27], the terms *loose* and *dense* are not quite adequate since behaviour of sand depends not only on the initial void ratio but also on the effective mean pressure. Thus, terms *contractive* or *dilative* are more appropriate.

Depending on the initial state of sand three types of *undrained* behaviour are observed (Fig. 3). Dense sand (initially in the dilative state – curve A) exhibits constant growth of the deviatoric stress (the strength increases) under monotonic loading. It is accompanied by the initial growth of pore-water pressure followed by subsequent drop. Local minimum of the deviatoric stress is not observed. Instead, a point of inflection on the stress path is observed.

Curve B represents undrained behaviour of initially loose (dilative) sand. In these cases stress paths exhibit local peak of deviatoric stress accompanied by the increase of pore-water pressure and a drop of the effective mean stress. Then, after a small drop of deviatoric stress, abrupt turn of the stress path towards increase of the strength occurs. The pore-water pressure drops at the same time (effective mean stress rises). The minimum value of the deviatoric stress corresponds to the so called phase transformation [12], [28], [29]. Under the phase transformation transition from contractancy to dilatancy takes place (PT line in Fig. 3). This kind of behaviour is called limited liquefaction. The state of stress corresponding to the local minimum of deviatoric stress is called the quasi-steady state [2].

Curve C in Fig. 3 represents static liquefaction. It occurs in basically undrained shearing of contractive (loose) sands. During monotonic loading pore-water pressure increases and the effective mean stress drops. The stress path passes the peak deviatoric value (peak strength) and then goes down. Both deviatoric and effective mean stress eventually reach residual values during constantly increasing deformation.

4. CRITICAL STATES AND NORMAL COMPRESSION LOCI

In the early formulations of elasto-plastic description of soil behaviour Mohr–Coulomb criterion was viewed as a yield surface. Normality rule is applied in this case which means that a plastic strain increment vector is directed normal to the yield surface. In such a case dilation angle is equal to friction angle which implies large dilatancy. This dilatancy is many times greater than what is measured in real soils.

It has been well recognized that stress ratios at failure (q_f/p'_f or τ_f/σ'_f) form a line called Mohr–Coulomb failure criterion. At the beginning the Mohr–Coulomb criterion was viewed as a yield surface. Such treatment did not take into account the fact that normal (or isotropic) compression also produces irrecoverable (plastic) strains. Drucker et al. [10] recognized that the Mohr–Coulomb criterion was not a yield surface and a yield surface must intersect the normal compression line (NCL).

On the other hand, Casagrande [4], using shear box tests, found that loose sands contracted and dense sands expanded their volume eventually reaching approximately the same void ratio at large strains when sheared under the same normal stress. Casagrande termed this void ratio as the critical state void ratio. Critical void ratio distinguishes the mode of behaviour soil exhibits. Sand looser than corresponding critical density is in contractive state whereas sand denser than critical density is in dilative state. The relationship between the critical void ratio and the mean effective pressure is called the critical state locus (CSL) and is presented in Fig. 4. Contractive state is distinguished from dilative state by means of the state parameter

$$\psi = e - e_c \,. \tag{2}$$

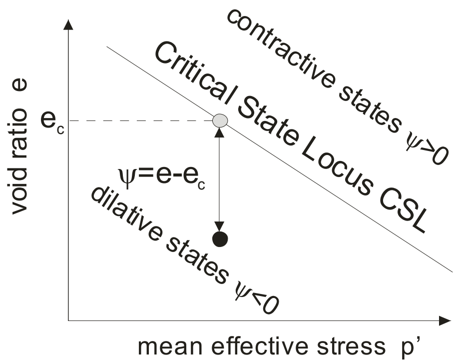

Fig. 4. Critical state locus

Roscoe et al. [24] defined critical state as the state in which soil continues to deform at constant volume and constant stresses. This state can be described by two conditions: dilatancy changes disappear and this is permanent state. These conditions can be expressed mathematically as

$$\dot{\varepsilon}_v = 0 \,, \tag{3a}$$

$$\ddot{\varepsilon} = 0 \,. \tag{3b}$$

Based on the concept of the critical void ratio Critical State Soil Mechanics has been formulated. It assumes that at the onset of critical state shear strain ε_s occurs without any further changes in mean effective stress p' nor deviatoric stress q nor void ratio e.

Critical state locus can be approximated by a line with the equations

$$e_c = \Gamma - \lambda \ln p'_c \,, \tag{4a}$$

$$q_c = Mp'_c \,, \tag{4b}$$

where the subscript c denotes critical state, M, Γ, λ are material constants (Fig. 5).

Although there are numerous experimental results showing that critical states do not undergo such simple description like (4a), for most practical engineering problems the description of CSL given by (4a) is sufficient as long as the mean effective stress does not exceed 500 kPa.

Equation (4b) assumes the critical stress ratio M is constant. In fact it is not true, and M has to be treated as a function of the Lode angle θ which takes into account the influence of the intermediate principal stress. Frequently used Mohr–Coulomb and Matsuoka–Nakai failure criteria use triaxial compression conditions as the basis for evaluation of $M(\theta)$. In such a case $M(\theta)$ depends on M_{tc} determined in triaxial compression. In (4b) M is equivalent to M_{tc}. Critical stress ratio and critical friction angle are related by the well known relationship

$$M_{tc} = \frac{6 \sin \phi'_c}{3 - \sin \phi'_c} \,. \tag{5}$$

Early critical state models Cam clay and Granta Gravel assumed that both critical state locus (CSL) and normal compression locus (NCL) are parallel straight lines (Fig. 5). N, Γ and λ are intrinsic soil parameters. The parameters N and Γ are associated with the stress p' = 1 kPa by convention.

Nevertheless, Cam clay and Granta Gravel are not able to reproduce dilation and yielding of real sands. According to Jefferies and Been [15], this inability arises from the assumption that a yield surface intersects the critical state line.

Ishihara et al. [12] proposed the existence of infinity of normal compression loci (NCL) in $e - p'$ plane. Normal compression lines may not be parallel to the critical state line neither be straight (Fig. 6). From the possible infinity number of NCLs only one is correct for a given arrangement of particles and a void ratio.

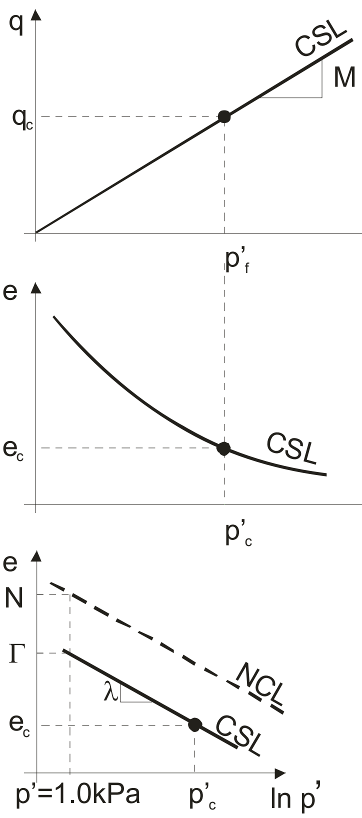

Fig. 5. Critical State Locus and Normal Compression Locus in Cam clay

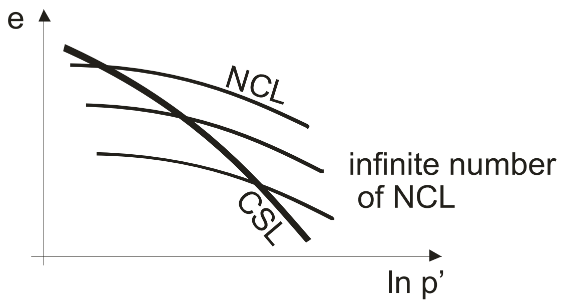

Fig. 6. Critical State Locus and Normal Compression Loci according to Ishihara et al. ([12])

5. CRITICAL STATE MODEL NOR SAND

Nor Sand [14], [15] assumes a yield surface and stress-dilatancy relationship known from Cam clay but decouples hardening from CSL which gives the model capability to reproduce real sand behaviour. Decoupling consists in using similar equation for the yield surface as for Cam clay but not using in it the mean stress at the critical state p'_c nor the critical stress ratio M. Instead, Nor Sand yield surface incorporates p'_i and M_i which correspond to the image state. The image state refers to the condition where dilatancy vanishes temporarily ($\dot{\varepsilon}_v = 0$) and soil behaviour converts from contractive to dilative. This is the peak point of the logarithmic spiral known from the yield surface of Cam clay.

Yield surface is given by the equation resembling Cam clay surface with M_i and p'_i instead of critical values of M and p'_c

$$\eta = M_i \left(1 - \ln \frac{p'}{p'_i}\right) \tag{6}$$

where $\eta = q/p'$.

Stress-dilatancy relationship is assumed after Cam clay model

$$D^p = M_i - \eta \tag{7}$$

where D^p is the plastic dilatancy

$$D^p = \frac{\dot{\varepsilon}_v^p}{\dot{\varepsilon}_s^p} \tag{8}$$

and $M_i = q_i/p'_i$ evolves with strain rather than being a material property.

Stress ratio at image state M_i is related to the state parameter according to formula proposed by Li et al. [18]

$$M_i = M\left(1 - \frac{|\psi_i|}{M_{tc}}\right) \tag{9}$$

where M is assumed as the average of critical stress ratios for Mohr–Coulomb and Matsuoka–Nakai conditions

$$M = 0.5(M_{MC} + M_{MN}), \tag{10}$$

$$M_{MC} = \frac{3\sqrt{3}}{\cos\theta\left(1 + \frac{6}{M_{tc}}\right) - \sqrt{3}\sin\theta}, \tag{11}$$

$$\frac{27 - 3M_{MN}^2}{3 - M_{MN}^2 + 8/9 M_{MN}^3 \sin\theta(3/4 - \sin^2\theta)} = \frac{27 - 3M_{tc}^2}{3 - M_{tc}^2 + \frac{2}{9} M_{tc}^3}. \tag{12}$$

State parameter at the image point

$$\psi_i = e - e_{c.i} \tag{13}$$

can be considered as the measure of the divergence of the current state from the critical state. Soil is in the critical state when $\psi_i = 0$.

Dilatancy D^p is connected to the state parameter ψ. It has been found in laboratory tests that extreme value of dilatancy is linearly proportional to the state parameter. In Nor Sand the relationship between the minimum dilatancy and the state parameter in the image state is given by

$$D^p_{\min} = \chi \psi_i \tag{14}$$

where χ is a model property determined in laboratory tests. When determined in triaxial compression it is denoted as χ_{tc}.

In order to limit dilation non-associated flow rule is applied in constitutive models. Conventional critical state models (e.g., Cam clay) derive the formulation of a yield surface from the assumption of normality (associated flow rule). Nor Sand controls dilatancy through the value of the mean stress at the image point p_i. Evolution of the yield surface (its expansion and contraction) for a dense sand is illustrated in Fig. 7. It is the general evolution of the yield surface during the pure shearing (p' = const.) in drained triaxial conditions. The initial configuration of the yield surface for dense sand is represented by the curve (a). Shear stress increases and the yield surface expands until stress ratio reaches the image state ($p'_i = p'$) at the configuration (b). During this stage volumetric strain decreases due to normality (void ratio decreases). Since the state parameter at the image point $\psi_i < 0$ expansion of the yield surface continues (p'_i increases), but now accompanied by increasing volume. Expansion of the yield surface stops when the internal cap is reached (c). Dilatancy continues until the image state and critical state coincide. The yield surface stops changing its configuration at (d).

The limit on the hardening (expansion of the yield surface) is given with respect to the current stress by

$$\left(\frac{p'_i}{p'}\right)_{\max} = \exp\left(-\frac{\chi_{tc}\psi_i}{M_{tc}}\right). \tag{15}$$

Equation (15) defines a planar cap within the yield surface presented in Fig. 7.

The hardening law is given by

$$\frac{dp'_i}{p'_i} = H\frac{M_i}{M_{tc}}\left(\frac{p'}{p'_i}\right)^2\left[\exp\left(-\frac{\chi_{tc}\psi_i}{M_{tc}}\right) - \frac{p'_i}{p'}\right]d\varepsilon_s \tag{16}$$

where H is a model parameter that has to be calibrated according to experimental data.

In Nor Sand well-known isotropic elastic relationships are assumed

$$\frac{K}{p'} = \frac{1+e}{\kappa}, \tag{17a}$$

$$I_r = \frac{G}{p'} = \frac{1+e}{\kappa}\frac{3(1-2\nu)}{2(1+\nu)}, \tag{17b}$$

where ν is Poisson's ratio and κ is the gradient of unloading/reloading line in e–ln p' space.

Typical values of parameters for sands are given after Jefferies and Been [15]:

$\Gamma = 0.9$–1.4,
$\lambda = 0.01$–0.07,
$M_{tc} = 1.2$–1.5,
$H = 50$–500,
$\chi_{tc} = 2.5$–4.5,
$I_r = 100$–600,
$\nu = 0.1$–0.3.

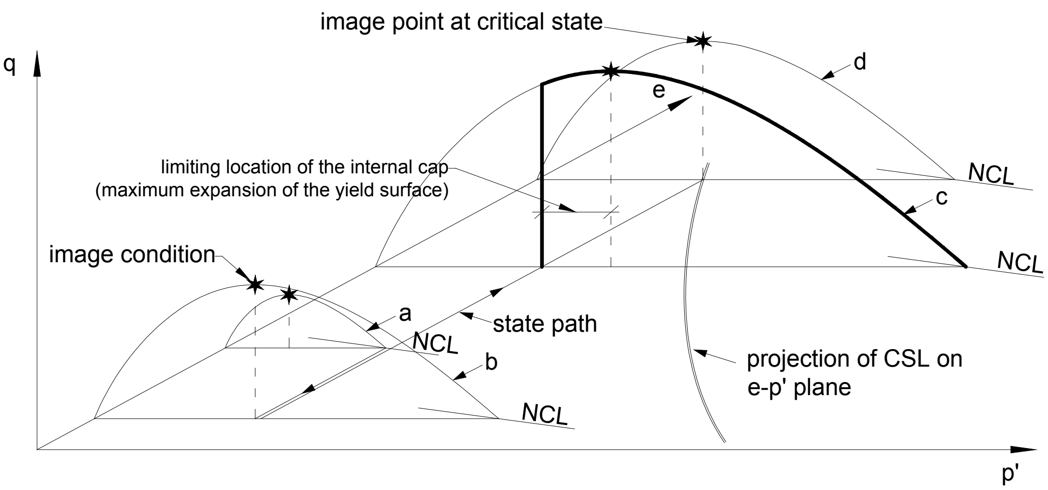

Fig. 7. Evolution of the yield surface

6. SIMULATIONS OF UNDRAINED TRIAXIAL COMPRESSION OF SAND

To show the ability of Nor Sand to simulate the static liquefaction a set of parameters has been assumed and the element tests performed. Parameters determining CSL in e–ln p space are: $\Gamma = 1.2$, $\lambda = 0.01$.

Critical friction ratio in triaxial compression is $M_{tc} = 1.2$ which corresponds to effective critical friction angle $\phi_c' = 30°$. Plastic hardening modulus for loading $H = 200$. Coefficient relating maximum dilatancy to the state parameter $\chi_{tc} = 3.5$ which is a typical average value for sands according to Jefferies and Been [15]. Elastic shear rigidity $I_r = 300$ and Poisson's ratio $\nu = 0.3$.

Simulations of triaxial compression have been carried out with the assumption that the tests start from the initial effective mean stress $p_0' = 100$ kPa (pressure of water in a triaxial apparatus cell). A set of various sand densities has been considered. Loose sand, initially in the contractive state, is represented by three initial values of the state parameter $\psi_0 = 0.068$; 0.035; 0.01. These values correspond to the values of void ratio $e_0 = 1.222$; 1.189; 1.164. Dense sand, initially in dilative state, is represented by the initial values of the state parameter $\psi_0 = -0.01$; -0.02 which correspond to $e_0 = 1.144$; 1.134.

Results of the numerical simulations of undrained triaxial compression are presented in Fig. 8. Figure 8a shows stress paths for sands with different initial parameters. The model predicts unstable behaviour for all assumed initial states. Three simulations have exhibited partial liquefaction whereas for two full static liquefaction has been achieved.

Figure 8b presents stress-strain curves corresponding to stress paths. One curve ($\psi_0 = 0.068$) exhibits complete loss of strength. Another curve ($\psi_0 = 0.035$) ends up at the residual state ($q_{\text{res}} = 4$ kPa). Both curves go through the peak deviatoric stresses followed by rapid strain softening. Three other curves which represent limited liquefaction show the increase of strength (hardening) after the temporary drop of shearing resistance.

Generation of pore-water pressure is shown in Fig. 8c. In cases showing limited liquefaction pore-water pressure rises and then drops more or less rapidly heading towards negative values (suction). In undrained tests volume is constant so the void ratio is also constant (Fig. 8d). Changes of the effective mean stress are coupled with changes of pore-water pres-

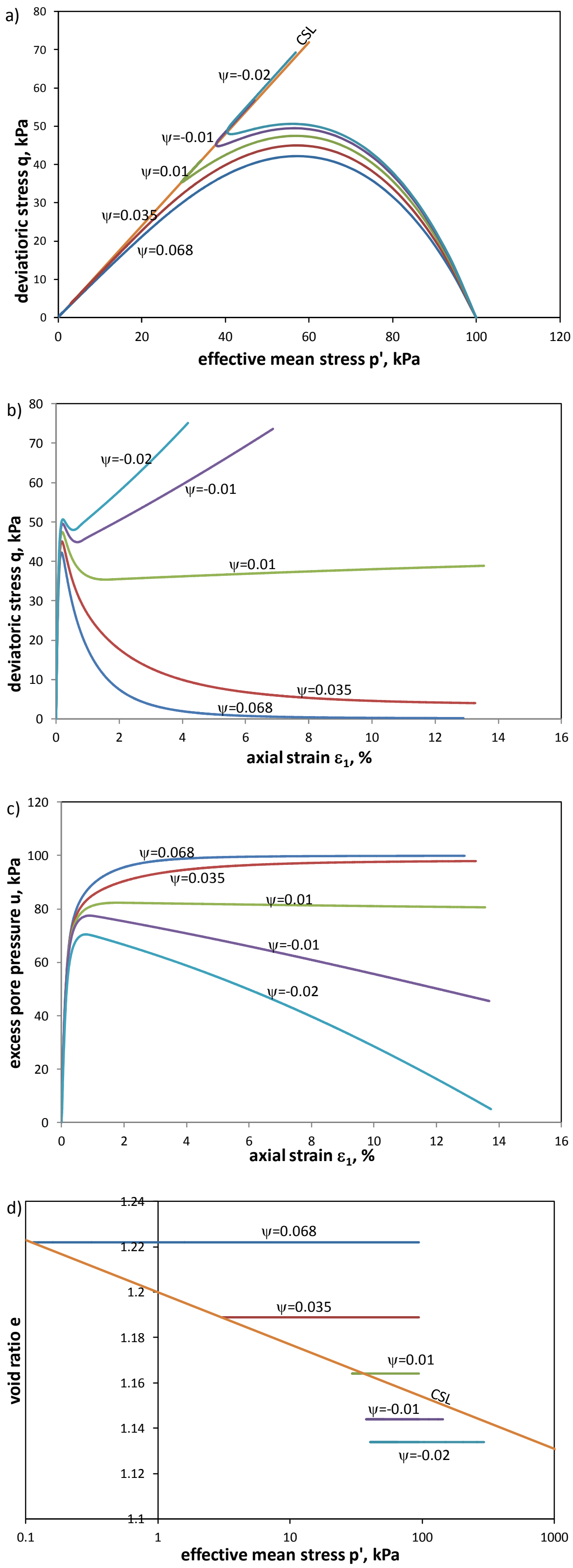

Fig. 8. Results of numerical simulations of undrained triaxial compression

sure. Whenever pore-water pressure increases the effective mean stress drops and vice versa. All state paths in Fig. 8d tend to the critical state line.

7. CONCLUSIONS

The reproduction of sand behaviour in undrained conditions can be performed within the framework of critical state soil mechanics. Thanks to decoupling the image stress from the critical state hardening of Nor Sand goes far more than for primary critical state models like Cam clay. Parameter M_i is a function of the state parameter ψ and as such it is introduced into the flow rule. Limited hardening is introduced into the concept of the yield surface by introducing the internal cap which violates the traditional approach to the yield surface concept. Nevertheless, Nor Sand predicts realistic dilatancy with the associated flow rule.

Nor Sand model is relatively simple with only seven non-dimensional parameters. It is an isotropic model based on the associated flow rule, capable of predicting unstable behaviour of sands in undrained loading, i.e., limited and full static liquefaction.

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