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HYDROMAGNETIC THERMAL INSTABILITY OF COMPRESSIBLE WALTERS' (MODEL B') ROTATING FLUID PERMEATED WITH SUSPENDED PARTICLES IN POROUS MEDIUM

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Abstract: In this paper, the thermal instability of compressible Walters' (Model B') rotating fluid permeated with suspended particles (fine dust) in porous medium in hydromagnetics is considered. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the rotation, magnetic field, suspended particles and viscoelasticity introduce oscillatory modes. For stationary convection, Walters' (Model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid and it is observed that rotation has stabilizing effect, suspended particles are found to have destabilizing effect on the system, whereas the medium permeability has stabilizing or destabilizing effect on the system under certain conditions. The magnetic field has destabilizing effect in the absence of rotation, whereas in the presence of rotation, magnetic field has stabilizing or destabilizing effect under certain conditions.

NOMENCLATURE

q – velocity of fluid,
q_d – velocity of suspended particles,
p – pressure,
$\mathbf{g}$ – gravitational acceleration vector,
g – gravitational acceleration,
k_1 – medium permeability,
T – temperature,
t – time coordinate,
c_f – heat capacity of fluid,
c_{pt} – heat capacity of particles,
mN – mass of the particle per unit volume,
k – wave number of disturbance,
k_x, k_y – wave numbers in x and y directions,
p_1 – thermal Prandtl number,
P_l – dimensionless medium permeability

GREEK SYMBOLS

ε – medium porosity,
ρ – fluid density,
μ – fluid viscosity,
μ' – fluid viscoelasticity,
υ – kinematic viscosity,
υ' – kinematic viscoelasticity,
η – particle radius,
κ – thermal diffusivity,
α – thermal coefficient of expansion,
β – adverse temperature gradient,
θ – perturbation in temperature,
n – growth rate of the disturbance,
δ – perturbation in respective physical quantity,
ζ – Z-component of vorticity
$\mathbf{\Omega}$ – rotation vector having components $(0, 0, \Omega)$
μ_e – magnetic permeability

1. INTRODUCTION

A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Chandra [2] observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Bénard-type cellular convection with the fluid descending at a cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion "Columnar instability". He added an aerosol to mark the flow pattern.

Bhatia and Steiner [3] have studied the thermal instability of a Maxwellian visco-elastic fluid in the presence of magnetic field while effect of rotation on thermal instability of a visco-elastic fluid has been studied by Sharma [4].

The medium has been considered to be non-porous in all the above studies. Lapwood [5] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [6] whereas Scanlon and Segel [7] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

Sharma and Sunil [8] have studied the thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walters' (Model B') elastico-viscous fluid having relevance in chemical technology and industry. Walters' [9] reported that the mixture of polymethyl methacrylate and pyridine at 25 °C containing 30.5 g of polymer per litre with density 0.98 g per litre behaves very nearly as the Walters (Model B') elastico-viscous fluid. Walters' (Model B') elastico-viscous fluid forms the basis for the manufacture of many important polymers and useful products.

When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Veronis [10] simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, and the motions of infinitesimal amplitude are considered.

A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law which states that the usual viscous term in the equations of motion of Walters' (Model B') fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\right]\boldsymbol{q}$, where μ and μ' are the viscosity and viscoelasticity of the incompressible Walters' (Model B') fluid, k_1 is the medium permeability and $\boldsymbol{q}$ is the Darcian (filter) velocity of the fluid.

Sharma and Rana [11] have studied thermal instability of incompressible Walters' (Model B') elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium. Sharma and Rana [12] have also studied the thermosolutal instability of incompressible Walters' (Model B') rotating fluid in the presence of magnetic field and variable gravity field in porous medium.

The Bénard problem (the onset of convection in a horizontal layer uniformly heated from below) for incompressible Rivlin–Ericksen rotating fluid permeated with suspended particles and variable gravity field in porous medium was studied by Rana and Kumar [13]. Recently, Rana and Kango [14] have studied the effect of rotation on thermal instability of compressible Walters' (Model B') fluid in porous medium. In the present paper, the study is extended to the compressible Walters' (Model B') rotating fluid permeated with suspended particles in the presence of magnetic field in porous medium.

2. MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Consider an infinite horizontal layer of an electrically conducting compressible Walters' (Model B') rotating fluid of depth d in a porous medium bounded by the planes $z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by a uniform rotation $\boldsymbol{\Omega}(0, 0, \Omega)$ and uniform vertical magnetic field $\boldsymbol{H}(0, 0, H)$, (Fig. 1). This layer is heated from below such that a steady adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

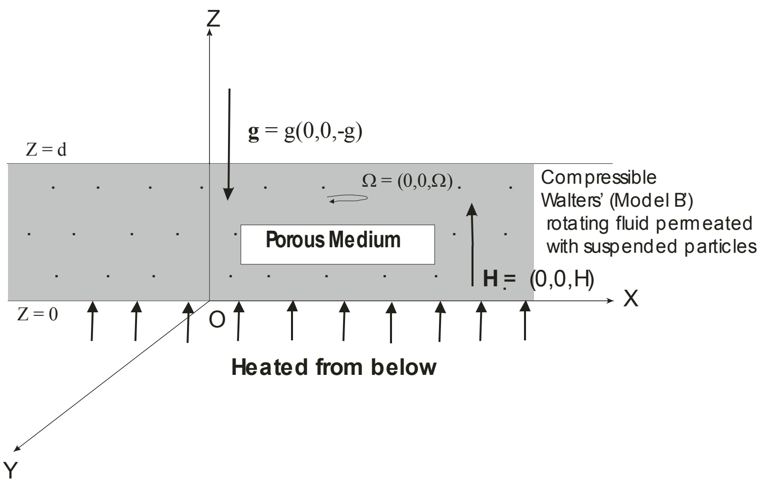

Fig. 1. Schematic sketch of physical situation

The hydromagnetic equations in porous medium (Chandrasekhar [1], Walters' [9]) relevant to the problem are

$$\frac{1}{\varepsilon}\left[\frac{\partial q}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\boldsymbol{q}\right]=1\frac{1}{\rho_m}\nabla p+g\left(1+\frac{\partial\rho}{\rho_m}\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)q$$
$$+\frac{2}{\eta}(q\times\Omega)+\frac{K'N}{\rho_m\varepsilon}(\boldsymbol{q}_d-\boldsymbol{q})+\frac{\mu_e}{4\pi\rho_m}(\nabla\times\boldsymbol{H})\times\boldsymbol{H} \tag{1}$$

$$\nabla.\boldsymbol{q}=0 \tag{2}$$

$$\frac{\partial T}{\partial t}+(\boldsymbol{q}.\nabla)T+\frac{mNC_{pt}}{\rho_m c_f}\left[\varepsilon\frac{\partial}{\partial t}+\boldsymbol{q}_d.\nabla\right]T=\kappa\nabla^2 T\ , \tag{3}$$

$$\nabla.\boldsymbol{H}=0\,, \tag{4}$$

$$\varepsilon\frac{\partial\boldsymbol{H}}{\partial t}=\nabla\times(\boldsymbol{q}\times\boldsymbol{H})+\varepsilon\eta\nabla^2\boldsymbol{H}\ . \tag{5}$$

Here, $\boldsymbol{q_d}(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of the particles respectively, c_f and c_{pt}, denote the heat capacity of pure fluid, heat capacity of the particles, respectively, and c'_f, c'_{pt} heat capacities analogous to solute. $K' = 6\pi\eta\rho\upsilon$, where η is the particle radius, is the Stokes drag coefficient, $\boldsymbol{q_d} = (l, r, s)$ and $\bar{x} = (x, y, z)$. κ and κ' denote the thermal diffusivity and solute diffusivity, respectively.

Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign. The buoyancy force on the particles is neglected. Interparticle reactions are not considered, since we assume that the distances between the particles are quite large compared with their diameters.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial q_d}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}_d.\nabla)\boldsymbol{q}_d\right] = K'N(\boldsymbol{q} - \boldsymbol{q}_d)\,, \tag{6}$$

$$\varepsilon\frac{\partial N}{\partial t} + \frac{1}{\varepsilon}\nabla.(N\boldsymbol{q}_d) = 0\,. \tag{7}$$

The state variables pressure, density and temperature are expressed in the form (Spiegel and Veronis [10])

$$f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t)\,, \tag{8}$$

where f_m denotes for constant space distribution f, f_0 is the variation in the absence of motion, and $f'(x, y, z, t)$ is the fluctuation resulting from motion. The basic state of the system is

$$p = p(z), \rho = \rho(z), \quad T = T(z), \quad \boldsymbol{q} = (0, 0, 0), \quad \boldsymbol{q_d} = (0, 0, 0) \text{ and } N = N_0 \tag{9}$$

where

$$p(z) = p_m - \boldsymbol{g}\int_0^z(\rho_m + \rho_0)dz$$

$$\rho(z) = \rho_m[1 - \alpha_m(T - T_0) + +K_m(p - p_m)]$$

$$T = -\beta z + T_0,\ \alpha_m = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m = \alpha(\text{say})K_m = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m\,. \tag{10}$$

Here, p_m and ρ_m denote a constant space distribution of p and ρ, while T_0 and ρ_0 denote temperature and density of the fluid at the lower boundary.

Let $\boldsymbol{q}(u, v, w)$, $\boldsymbol{q_d}(l, r, s)$, θ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $\mathbf{q}(0, 0, 0)$, the perturbation in particle velocity $\mathbf{q_d}(0, 0, 0)$, temperature T, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation θ temperature is given by

$$\delta\rho = -\alpha\rho_m\theta\,. \tag{11}$$

Following the assumptions given by Spiegal and Veronis [10] and the results for compressible fluid, the flow equations are found to be the same as that of incompressible fluid except that the static temperature gradient β is replaced by the excess over the adiabatic $(\beta - g/c_p)$, c_p being specific heat of the fluid at constant pressure.

The linearized perturbation equations governing the motion of fluids are

$$\begin{aligned}\frac{1}{\varepsilon}\frac{\partial \boldsymbol{q}}{\partial t} &= -\frac{1}{\rho_m}\nabla\delta p - g\alpha\theta - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)q \\ &+ \frac{K'N}{\varepsilon}(\boldsymbol{q}_d - \boldsymbol{q}) + \frac{2}{\varepsilon}(\boldsymbol{q}\times\Omega) + \frac{\mu_e}{4\pi\rho_m}(\nabla\times\boldsymbol{h})\times\boldsymbol{H},\end{aligned} \tag{12}$$

$$\nabla.\boldsymbol{q} = 0\,, \tag{13}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\boldsymbol{q}_d = q\,, \tag{14}$$

$$(1+b\varepsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)(w+bs) + \kappa\nabla^2\theta\,, \tag{15}$$

$$\nabla.\boldsymbol{h} = 0\,, \tag{16}$$

$$\varepsilon\frac{\partial\boldsymbol{H}}{\partial t} = (H.\nabla)q + \varepsilon\eta\nabla^2 H\,, \tag{17}$$

where $b = \dfrac{mNC_{pt}}{\rho_m C_f}$ and w, s are the vertical fluid and particles velocity.

3. DISPERSION RELATION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, s, \theta, \zeta, h_z, \xi] = [W(z), S(z), \Theta(z), Z(z), K(z), X(z)]\exp(ik_x x + ik_y y + nt)\,, \quad (18)$$

where k_x and k_y are the wave numbers in the x and y directions, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (18) in (12)–(17), we get

$$\frac{n}{\varepsilon}\left[\frac{d^2}{dz^2} - k^2\right]W = -gk^2\alpha\theta - \frac{1}{k_1}(\upsilon - \upsilon' n)\left(\frac{d^2}{dz^2} - k^2\right)W$$
$$-\frac{mNn}{\varepsilon\rho_m\left(\frac{m}{K}n + 1\right)}\left(\frac{d^2}{dz^2} - k^2\right)W - \frac{2\Omega}{\varepsilon}\frac{dZ}{dz} + \frac{\mu_e}{4\pi\rho_m}\frac{d}{dz}\left(\frac{d^2}{dz^2} - k^2\right)K, \quad (19)$$

$$\frac{n}{\varepsilon}Z = -\frac{1}{k_1}(\upsilon - \upsilon' n) - \frac{mNn}{\varepsilon\rho_m\left(\frac{m}{K}n + 1\right)}Z + \frac{2\Omega}{\varepsilon}\frac{dW}{dz} + \frac{\mu_e H}{4\pi\rho_m}DX\,, \quad (20)$$

$$\varepsilon nX = H\frac{dZ}{dz} + \varepsilon n\left(\frac{d^2}{dz^2} - k^2\right)X\,, \quad (21)$$

$$\varepsilon nK = H\frac{dW}{dz} + \varepsilon n\left(\frac{d^2}{dz^2} - k^2\right)K\,, \quad (22)$$

$$(1 + b\varepsilon)n\theta = \left(\beta - \frac{g}{c_p}\right)(W + bS) + \kappa\left(\frac{d^2}{dz^2} - k^2\right)\theta\,. \quad (23)$$

Equations (19)–(23) in non dimensional form become

$$\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right](D^2 - a^2)W + \frac{ga^2d^2\alpha\theta}{\upsilon} + \frac{2\Omega d^3}{\varepsilon\upsilon}DZ$$
$$-\frac{\mu_e Hd}{4\pi\upsilon\rho_m}(D^2 - a^2)DK = 0, \quad (24)$$

$$\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right] = \left(\frac{2\Omega d^3}{\varepsilon\upsilon}\right)DW + \frac{\mu_e Hd}{4\pi\upsilon\rho_m}DX\,, \quad (25)$$

$$[D^2 - a^2 - p_1\sigma]X = -\left(\frac{Hd}{\varepsilon\eta}\right)DZ\,, \quad (26)$$

$$[D^2 - a^2 - p_2\sigma]K = -\left(\frac{Hd}{\varepsilon\eta}\right)DW\,, \tag{27}$$

$$[D^2 - a^2 - Ep_1\sigma]\theta = -\frac{gd^2}{\kappa c_p}(G-1)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W\,, \tag{28}$$

where we have put $a = kd$, $\sigma = \frac{nd^2}{\upsilon}$, $\tau = \frac{m}{K'}$, $\tau_1 = \frac{\tau\upsilon}{d^2}$, $M = \frac{mN}{\rho_0}$, $E = 1 + b\varepsilon$, $B = b + 1$, $F = \frac{\upsilon'}{d^2}$, $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $p_1 = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number, $p_2 = \frac{\upsilon}{\eta}$ is the magnetic Prandtl number, and $D^* = d\frac{d}{dz}$ and the superscript * is suppressed.

Applying the operator $(D^2 - a^2 - p_2\sigma)$ to equation (25) to eliminate X between equations (25) and (26), we get

$$\left\{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2-p_2\sigma)+\frac{Q}{\varepsilon}D^2\right\}W = \frac{2\Omega d^2}{\upsilon}(D^2-a^2-p_2\sigma)DW. \tag{29}$$

Eliminating K, Θ and Z between equations (24)–(29), we obtain

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2)(D^2-a^2-E_1p_1\sigma)(D^2-a^2-p_2\sigma)W$$
$$-Ra^2\left(\frac{G-1}{G}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)(D^2-a^2-p_2\sigma)W+\frac{Q}{\varepsilon}(D^2-a^2)(D^2-a^2-E_1p_1\sigma)W \tag{30}$$
$$\left[\frac{\frac{T_A}{\varepsilon^2}(D^2-a^2-E_1p_1\sigma)(D^2-a^2-p_2\sigma)}{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2-E_1p_1\sigma)+\frac{Q}{\varepsilon}D^2}\right]D^2W = 0,$$

where $R = \frac{g\alpha\beta d^4}{\upsilon\kappa}$ is the thermal Rayleigh number, $Q = \frac{\mu_e H^2 d^2}{4\pi\upsilon\rho_m\eta}$ is the Chandrasekhar number and $T_A = \left(\frac{2\Omega d^2}{\upsilon}\right)^2$ is the Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar [1])

$$W = D^2W = DZ = \Theta = 0 \quad \text{at} \quad z = 0 \ \text{and} \ 1 \tag{31}$$

and the components of h are continuous. Since the components of the magnetic field are continuous and the tangential components are zero outside the fluid, we have

$$DK = 0 \tag{32}$$

on the boundaries. Using the boundary conditions (31) and (32), we can show that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$, and hence, the proper solution of equation (30) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad W_0 \quad \text{is a constant.} \tag{33}$$

Substituting equation (33) in (30), we obtain the dispersion relation

$$\begin{aligned} R_1 x = \left(\frac{G}{G-1}\right)\Bigg\{ & \left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}\right](1+x)(1+x+E_1p_1i\sigma_1) \\ & \left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)+\frac{Q_1}{\varepsilon}\frac{(1+x)(1+x+E_1p_1i\sigma_1)}{1+x+p_2i\sigma_1}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) \\ & +\frac{\frac{T_{A_1}}{\varepsilon^2}(1+x+E_1p_1i\sigma_1)}{\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)\Bigg\}, \end{aligned} \tag{34}$$

where $R_1 = \dfrac{R}{\pi^4}$, $T_{A_1} = \dfrac{T_A}{\pi^4}$, $x = \dfrac{a^2}{\pi^2}$, $i\sigma_1 = \dfrac{\sigma}{\pi^2}$, $P = \pi^2 P_l$, $Q_1 = \dfrac{Q}{\pi^4}$.

Equation (34) is the required dispersion relation accounting for the effect of suspended particles, magnetic field, medium permeability, variable gravity field, rotation on thermal convection in Walters' (Model B') elastico-viscous fluid in porous medium.

4. OSCILLATORY MODES

Here we examine the possibility of oscillatory modes, if any, in Walters' (Model B') elastico-viscous fluid due to the presence of suspended particles, rotation, magnetic field, viscoelasticity and variable gravity field. Multiplying equation

(24) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (25)–(28) with the help of boundary conditions (31) and (32), we obtain

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]I_1-\frac{\mu_e\varepsilon\eta}{4\pi\upsilon\rho_m}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_2+p_2\sigma^* I_3)$$
$$-\frac{\alpha a^2 gk}{\upsilon\beta}\left(\frac{G}{G-1}\right)\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_4+E_1p_1\sigma^* I_5)+d^2\left[\frac{\sigma^*}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma^*}{P_l}\right]I_6\,, \quad (35)$$
$$+\frac{\mu_e\varepsilon\eta d^2}{4\pi\upsilon\rho_m}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_7+p_2\sigma^* I_8)=0,$$

where

$$I_1=\int_0^1(|DW|^2+a^2|W|^2)dz\,,$$

$$I_2=\int_0^1(|D^2K|^2+a^4|K|^2+2a|DK|^2)dz\,,$$

$$I_3=\int_0^1(|DK|^2+a^2|K|^2)dz\,,$$

$$I_4=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)dz\,,$$

$$I_5=\int_0^1|\Theta|^2\,dz\,,$$

$$I_6=\int_0^1|Z|^2\,dz\,,$$

$$I_7=\int_0^1(|DX|^2+a^2|X|^2)dz$$

$$I_8=\int_0^1|X|^2\,dz\,.$$

The integral part I_1–I_8 are all positive definite. Putting $\sigma = i\sigma_i$ in equation (35), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left\{\left[\frac{1}{\varepsilon}\left(1+\frac{M}{1+\tau_1^2\sigma_1^2}\right)-\frac{F}{P_l}\right](I_1-d^2I_4)\right.$$
$$-\frac{\mu_e\varepsilon\eta}{4\pi\upsilon\rho_m}\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_2+\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}p_2I_3\right]$$
$$+\frac{\alpha a^2gk}{\upsilon\beta}\left(\frac{G}{G-1}\right)\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_4+\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}E_1p_1I_5\right]. \tag{36}$$
$$\left.+\frac{\mu_e\varepsilon\eta d^2}{4\pi\upsilon\rho_m}\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_6+\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}p_2I_8\right]\right\}=0$$

Equation (36) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of rotation, magnetic field, suspended particles, viscoelasticity.

5. THE STATIONARY CONVECTION AND DISCUSSIONS

For stationary convection putting $\sigma = 0$ in equation (34) reduces it to

$$R_1=\frac{1+x}{xB}\left(\frac{G}{G-1}\right)\left[\frac{1+x}{P}+\frac{Q_1}{\varepsilon}+\frac{T_{A_1}(1+x)P}{\{\varepsilon(1+x)+Q_1P\}\varepsilon}\right], \tag{37}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B, P, Q_1 and Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ.

Let the non-dimensional number G accounting for compressibility effect be kept as fixed, then we get

$$\overline{R_c}=\left(\frac{G}{G-1}\right)R_c, \tag{38}$$

where $\overline{R_c}$ and R_c denote the critical number in the presence and absence of compressibility, respectively. Thus, the effect of compressibility is to postpone the instability on the onset of thermal instability. The cases $G = 1$ and $G < 1$ correspond to infinite and negative values of Rayleigh numbers due to compressibility, which is not relevant to the present study.

To study the effects of suspended particles, rotation and medium permeability, we examine the behavior of $\frac{dR_1}{dB}$, $\frac{dR_1}{dT_{A_1}}$, $\frac{dR_1}{dQ_1}$, and $\frac{dR_1}{dP}$ analytically.

Equation (37) yields

$$\frac{dR_1}{dB} = -\frac{1+x}{xB^2}\left(\frac{G}{G-1}\right)\left[\frac{1+x}{P} + \frac{Q_1}{\varepsilon} + \frac{T_{A_1}(1+x)P}{\{\varepsilon(1+x)+Q_1P\}\varepsilon}\right], \tag{39}$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system. This stabilizing effect is in agreement with the earlier work of Scanlon and Segel [7] and Rana and Kumar [13].

From equation (37), we get

$$\frac{dR_1}{dT_{A_1}} = \frac{1+x}{xB}\left(\frac{G}{G-1}\right)\frac{(1+x)P}{\{\varepsilon(1+x)+Q_1P\}\varepsilon}, \tag{40}$$

which shows that rotation has stabilizing effect on the system. This stabilizing effect is an agreement of the earlier work of Rana and Kumar [13] and Rana and Kango [14].

From equation (37), we get

$$\frac{dR_1}{dQ_1} = \frac{1+x}{xB}\left(\frac{G}{G-1}\right)\left[\frac{1}{\varepsilon} - \frac{T_{A_1}(1+x)P^2}{\{\varepsilon(1+x)+Q_1P\}^2\varepsilon}\right] \tag{41}$$

which implies that magnetic field stabilizes the system, if

$$\{\varepsilon(1+x)+Q_1P\}^2 > T_{A_1}(1+x)P^2$$

and destabilizes the system, if

$$\{\varepsilon(1+x)+Q_1P\}^2 < T_{A_1}(1+x)P^2$$

In the absence of rotation, magnetic field has stabilizing effect on the system.

It is evident from equation (37) that

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xB}\left(\frac{G}{G-1}\right)\left[\frac{1}{P^2} - \frac{T_{A_1}(1+x)}{\{\varepsilon(1+x)+Q_1P\}^2}\right] \tag{42}$$

From equation (42), we observe that medium permeability has destabilizing effect when $\{\varepsilon(1+x)+Q_1P\}^2 < T_{A_1}(1+x)P^2$. and stabilizing effect when $\{\varepsilon(1+x)+Q_1P\}^2 > T_{A_1}(1+x)P^2$. This stabilizing effect is an agreement of the earlier work of Sharma and Rana [12], Rana and Kumar [13] and Rana and Kango [14]. In the absence of rotation, $\dfrac{dR_1}{dP}$ is always negative implying thereby the destabilizing effect of medium permeability.

6. CONCLUSIONS

The thermal instability of Walters' (Model B') elastico-viscous rotating fluid permitted with suspended particles in the presence of magnetic field in porous medium has been investigated. The main conclusions are as follows:

(i) For stationary convection, Walters' (Model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid.

(ii) It is clear from equation (30) that the effect of compressibility is to postpone the onset of thermal instability.

(iii) The expressions for $\dfrac{dR_1}{dB}$, $\dfrac{dR_1}{dT_{A_1}}$, $\dfrac{dR_1}{dQ_1}$, and $\dfrac{dR_1}{dP}$ are examined analytically and it has been found that the rotation has stabilizing effect and suspended particles are found to have destabilizing effect on the system whereas the medium permeability has a stabilizing/destabilizing effect on the system for $\{\varepsilon(1 + x) + Q_1P\}^2 < T_{A_1}(1 + x)P^2/\{\varepsilon(1 + x) + Q_1P\}^2 > T_{A_1}(1 + x)P^2$. The magnetic field has stabilizing/destabilizing effect on the system for $\{\varepsilon(1 + x) + Q_1P\}^2 > T_{A_1}(1 + x)P^2/\{\varepsilon(1 + x) + Q_1P\}^2 < T_{A_1}(1 + x)P^2$.

(iv) The presence of rotation, suspended particles, magnetic field and viscoelasticity introduced oscillatory modes.

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