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A PARADIGM FOR THE ANALYSIS OF PRELOADED BOLTED JOINTS

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Abstract: The purpose of this paper is to present a paradigm, or guide, to the analysis of preloaded bolted joints made using multiple bolts. Classical analysis methods are applied to the interaction of the joint elements subjected to combinations of both in-plane and out-of-plane loads and moments. The distribution of the external loads and moments within the preloaded joint is determined in relationship to individual bolts. An analysis of loads and stresses in individual bolts and dowels along with flange bending and thread shear in tapped or threaded holes is developed. The article brings together a number of concepts and links them into a practical design analysis process that is applicable for many cases of preloaded bolted joints and are adequate to demonstrate the structural integrity of each element of the joint. Interpretation of results, within the context of design standards, is provided. In some cases finite element methods may be more appropriate, and the methods discussed can be used in the validation process.

KEYWORDS: bolted joint, preloaded bolt, bolt preload, bolt tension, multiple bolt, multi bolt

1 Introduction

Threaded connections, in particular bolted joints, are a common engineering feature found in most manufactured equipment and in many structures.

In preloaded (or pretensioned) joints the bolts are first tightened sufficient to establish closure of the joint with alignment of the mating components, and then further tightened to produce the required bolt preload and (more importantly) a compressive load at the faying surface. The faying surface of a joint member is the prepared surface (i.e. machined or ground) that is in contact with the faying surface of another member of the joint.

The important benefits of preloaded joints are in producing a stiff joint, without slippage. The bolts have a significant mean stress but experience a low working stress range. This low stress range is a major factor in why preloaded joints have good fatigue performance.

In many cases the method of analysis presented here will be adequate to demonstrate the structural integrity of the joint and compliance with design standards. When equipment is being designed to meet specific standards, any safety factors, partial safety factors and design factors or allowable design stresses and loads required by the standard should be incorporated into the analysis and should take precedence over any equivalent factors suggested here.

2 Nomenclature

A_s	Tensile area of each bolt thread
AS_n	Shear area of the internal thread
$D_{s.min}$	Minimum major diameter of the external thread
$E_{n.max}$	Maximum effective (pitch) diameter of the internal thread

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$F_{B_{(n)}}$	Minimum bolt preload required to prevent slip at bolt 'n'
$F_{b.max_{(n)}}$	Maximum bolt load on bolt 'n'
$F_{br_{(n)}}$	Bolt-related external load for bolt 'n'
F_{dp}	Design preload
$F_{dwl_{(r)}}$	Resultant shear load for dowel 'r'
$F_{f_{(n)}}$	Faying surface contact force local to bolt 'n'
F_p	Preload in each bolt
$F_{p.max}$	Maximum possible preload in any bolt
$F_{p.min}$	Minimum possible preload in any bolt
$F_{s.br_{(n)}}$	Resultant bolt-related shear load for bolt 'n'
F_x	External In-plane force acting in x-direction
$F_{x.br_{(n)}}$	Bolt-related shear load in x-direction for bolt 'n'
$F_{x.dwl_{(r)}}$	Dowel shear load in x-direction for dowel 'r'
F_y	External In-plane force acting in y-direction
$F_{y.br_{(n)}}$	Bolt-related shear load in y-direction for bolt 'n'
$F_{y.dwl_{(r)}}$	Dowel shear load in y-direction for dowel 'r'
F_z	External axial load in direction of 'z' axis
$I_{xx.j}$	Second moment of area of joint about 'x' axis
L_e	Bolt edge distance (distance from bolt centre to flange edge)
L_{eng}	Length of thread engagement
$L_{eng.min}$	Minimum length of thread engagement required
M_x	External moment acting about 'x' axis
M_x'	Resultant moment
M_y	External moment acting about 'y' axis
M_z	External torsional moment acting on joint
N_b	Number of bolts in joint
N_d	Number of dowels in joint
$P_{f_{(n)}}$	Contact pressure at faying surface local to bolt 'n'
p_{thrd}	Thread pitch
QS_n	Yield, or proof, shear strength of the internal thread material
UTS_s	Ultimate tensile strength of the bolt or external thread
$x_{(n)}$	Coordinate of bolt 'n'
$x_{(r)}$	Coordinate of dowel 'r'
Y_s	Tensile yield/proof stress of internal thread material
$y_{(n)}$	Coordinate of bolt 'n'

$y_{(r)}$	Coordinate of dowel 'r'
y'	Transposed coordinate
μ_f	Friction coefficient at faying surface
$\sigma_{b.f}$	Flange bending stress
τ_n	Internal thread shear stress
θ	Angle of resultant moment
θ_{thrd}	Thread flank angle (included angle between thread flanks)

3 Preloaded Bolt Joint Theory

Ideally, preloading the joint's bolts induces a uniform (or near uniform) compressive stress at the faying surface. When external loads are applied to the joint any resulting tensile stress components act to reduce this compressive stress. As long as the faying surface retains some compressive stress the joint will continue to perform as if it were a continuous member. It is only when a resulting tensile stress component attempts to exceed the pre-compression at the faying surface that separation of the joint occurs. At this point the joint can be deemed to have failed, even though none of the constituent parts of the joint have failed.

A detailed analysis of preloaded bolted joint load distributions has been discussed by Welch (2018) [1]. A less detailed method of design analysis was developed for use in a static analysis to demonstrate the structural integrity of the joint. This method of design analysis assumes each preloaded bolt influences an approximately circular region of the faying surface that surrounds it. From this, it also assumes that the regions influenced by the bolts can be considered as springs.

Figure 1 illustrates the bolt-related loads on one half of the joint resulting from the combined loading of the external axial load and external out-of-plane moment.

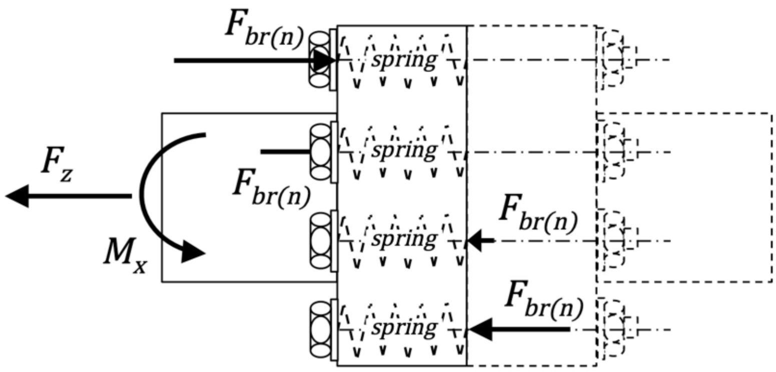

Fig. 1 Bolt Related Loads.

An external moment applied to the joint results in loads on each of the effective springs. These loads are proportional to the distance of the bolt/spring from the neutral axes of the bolt group. These bolt-related loads are given by the following equation.

$$F_{br_{(n)}} = \frac{F_z}{N_b} + \frac{M_x \cdot y_{(n)}}{\sum_n y_{(n)}^2} \tag{1}$$

where the neutral axes, or centroid, of the bolt group are defined by $\frac{1}{N_b} \cdot \sum_n x_{(n)} = 0$ and $\frac{1}{N_b} \cdot \sum_n y_{(n)} = 0$

Equation (1) uses an implied second moment of area of $I_{xx.j} = A_s \cdot \sum_n y_{(n)}^2$ which is less than the actual second moment of area of the joint. The bolt-related loads are not the loads on the bolts; they are the external load components on each bolt assembly, comprising a bolt and its surrounding region of flange.

It is common for the out-of-plane moment on a joint to be described by a pair of moments, M_x and M_y, acting about the principle axes of the joint, or another convenient pair of perpendicular axes. These moments and their resultant, M'_x, are illustrated in Figure 2.

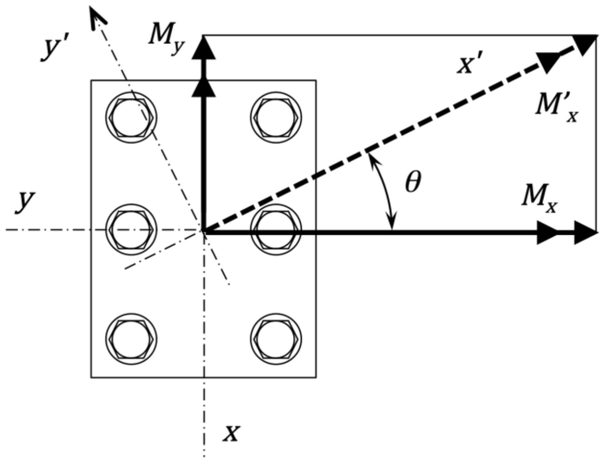

Fig. 2 Orientation of resultant moment.

Since the resultant moment acts about a different axis to those used to define the joint, an alternative coordinate system, aligned to the resultant moment, needs to be considered. The angle between the transposed coordinate system and the joint coordinate system is given by:

$$\theta = arctan\left(M_y/M_x\right) \tag{2}$$

If M_x is negative then 180 degrees (π radians) needs to be added to the angle θ to ensure the direction of the resultant moment is in the correct 'quadrant'.

The transposed bolt coordinates are given by:

$$y'_{(n)} = y_{(n)} \cdot \cos(\theta) - x_{(n)} \cdot \sin(\theta) \tag{3}$$

where $x_{(n)}$ and $y_{(n)}$ are the coordinates of bolt 'n' defined with respect to the centroid of the bolt group.

The resultant moment, M'_x, is given by:

$$M'_x = M_x \cdot \cos(\theta) + M_y \cdot \sin(\theta) \tag{4}$$

When considering an out-of-plane moment that is not aligned with the joint x-axis the terms for y' and M'_x given by equations (3) and (4) should be used in equation (1).

Equations (1) to (4) follow the "right hand" rule. Loads are positive in the direction of the axes and positive moments act clockwise about the axes when viewed from the origin.

The external loads and moments produce only small load changes in the preloaded bolts. The major effect of the external loads is to induce significant changes in the faying surface contact pressures. Joint separation does not occur until the tensile bolt-related load exceeds the bolt preload. Hence, the tensile (positive) bolt related loads given by equation (1) should not exceed the minimum bolt preload.

$$F_{br_{(n)}} \leq F_{p.min} \tag{5a}$$

The maximum and minimum bolt preloads, $F_{p.max}$ and $F_{p.min}$, reflect the tolerance on the nominal preload, F_p. Table A8 of VDI 2230 Part 1, "Systematic calculation of high duty bolted joints with one cylindrical bolt" [2] suggests a tolerance of $\pm 17\%$ for angle controlled bolt tightening (turn of nut). The suggested tolerances for bolts tightened by a torque wrench are $\pm 23\%$ if the torque is determined by experiment or $\pm 33\%$ if the torque is determined by calculation based on friction.

Section 3.8 of British Standard BS 7608:1990, "*Code of practice for Fatigue design and assessment of steel structures*" [3] says that;

> 'If reliance is to be placed on this pre-load, it should be at least 1.5 times the design tension'

Hence, when designing against fatigue the following equation can be used in place of equation (5a):

$$F_{br_{(n)}} \leq F_{dp} \tag{5b}$$

where the design preload, F_{dp}, is given by:

$$F_{dp} = \frac{2}{3} \cdot F_p \tag{6}$$

The use of a design bolt preload takes account of a number of factors, which includes a tolerance on the bolt preload applied during assembly. Applying equations (5b) and (6) produces a "design against fatigue", but should not be regarded as a fatigue assessment.

Any safety factors, partial safety factors and design factors required by any standards being followed should be incorporated into equations (5a) and/or (5b).

It has been shown that external out-of-plane loads are carried mainly by changes in the faying surface contact pressure distribution and produce only a small change in bolt loads [1]. Section 3.8 of British Standard BS 7608:1990 [3] suggests that the working stress range of a bolt is up to a maximum of 20% of the applied external load. Hence, the maximum possible tensile load on a bolt can be estimated as:

$$F_{b.max_{(n)}} = F_{p.max} + 20\% \cdot F_{br_{(n)}} \tag{7}$$

This maximum load should not exceed the proof load for the bolt.

3.1 In-Plane Loads on the Joint

Bolted joints for mechanical engineering purposes are usually designed to support external in-plane loads and torsional moments by friction at the faying surface. In some joints dowels, or other positive method of location of the joint, can assist in supporting these loads.

Again, assume that the preload in each bolt influences the region of the faying surface that surrounds the bolt. Also assume each region influenced by a bolt can be considered as a spring. Then the bolt-related loads resulting from the combined loading of the external in-plane loads and torsional moments are given by the following equations.

$$F_{x.br_{(n)}} = \frac{F_x}{N_b} - \frac{M_z \cdot y_{(n)}}{\sum_n \left(x_{(n)}^2 + y_{(n)}^2\right)} \tag{8}$$

$$F_{y.br_{(n)}} = \frac{F_y}{N_b} + \frac{M_z \cdot x_{(n)}}{\sum_n \left(x_{(n)}^2 + y_{(n)}^2\right)} \tag{9}$$

$$F_{s.br_{(n)}} = \sqrt{F_{x.br_{(n)}}^{\ 2} + F_{y.br_{(n)}}^{\ 2}} \tag{10}$$

Equation (8) gives the bolt-related shear load component acting in the *x*-direction. Similarly, equation (9) gives the bolt-related shear load component acting in the *y*-direction. Equation (10) gives the resultant bolt-related shear load at each bolt. These shear loads are carried across the faying surface by friction.

The minimum bolt preload required to both maintain closure of the joint and prevent slip is given by:

$$F_{B_{(n)}} = F_{br_{(n)}} + \frac{F_{s.br_{(n)}}}{\mu_f} \tag{11}$$

where μ_f is the friction coefficient at the faying surface.

If the joint does not incorporate dowels or other means of positive location then equation (11) should use a dynamic friction coefficient for μ_f. Dynamic friction will be less than static friction. Hence, any potential movement or slip, resulting from impact or dynamic loads overcoming static friction, will be arrested.

If dowels are incorporated into the joint face, they act to prevent movement or slip that would otherwise occur if the load overcame static friction. Hence, the minimum preload can be determined using a static friction coefficient for μ_f.

Joint slip, particularly on load reversals, could induce self-loosening of the bolts. Slip at macroscopic levels could produce fretting at the faying surface, which in turn could lead to a loss of preload and bolt loosening. Hence, the tensile (positive) bolt related loads given by equation (11) should not exceed the bolt preload, as expressed by the following equation.

$$F_{B_{(n)}} \leq F_p \tag{12}$$

Equations (11) and (12) can be combined to give the alternative equation:

$$F_{s.br_{(n)}} = \left(F_p - F_{br_{(n)}}\right) \cdot \mu_f \tag{13}$$

Equations (12) and (13) use the nominal bolt preload F_p. Some standards may instead require the design bolt preload, or the minimum preload, to be used for calculating the contact pressure.

Again, any safety factors, partial safety factors and design factors required by any standards being followed should be incorporated into the static analysis.

3.2 Bolt shear and Bending

Because the load analysis is "disconnected" from the geometry it is not possible to separate out the in-plane loads on the bolts and dowels, or other positive means of location. Generally, for typical flange thicknesses of 1.5 to 2 times the bolt diameter, bolt shear and bolt bending will not be an issue provided equations (10) and (11) or equation (12) show that the bolt preload can support the bolt-related shear load along with the bolt-related tensile load.

A detailed theoretical study of bolt bending [4] shows that in-plane loads and moments produce both bending stresses and additional tensile stresses in the bolts. Section 3.2.4 of VDI 2230 Part 1 [2] says;

> 'In highly preloaded bolted joints there is generally no risk of self-loosening by rotation. In the case of bolts with low bending resistance, additional locking may be necessary in order to avoid an inadmissible loss of preload. Locking means to prevent loosening by rotation ensure that at least 80% of the assembly preload remains as residual preload.'

Long bolts can be considered as having low bending resistance. Hence, bolts with a grip length of say 4 or more times the nominal bolt diameter are likely experience relaxation of preload. A preload of 80% of that which would be produced by the bolt assembly, or make up, torque should be used in equations (5a), (5b), (6) and equations (12) or (13) when calculating load limits.

4 Dowels

The dowels assist in carrying the out-of-plane loads by "pegging" the joint, preventing slip and the associated reduction in friction. The stiffness of each dowel in shear will be less than the stiffness of the region of the flange that is under the influence of a bolt. Hence a conservative estimate of the in-plane load carried by each dowel can be made using the following equation:

$$F_{x.dwl_{(r)}} = \frac{F_x}{N_b + N_d} - \frac{M_z \cdot y_{(r)}}{\sum_n\left(x_{(n)}^2 + y_{(n)}^2\right) + \sum_r\left(x_{(r)}^2 + y_{(r)}^2\right)} \tag{14}$$

$$F_{y.dwl_{(r)}} = \frac{F_y}{N_b + N_d} + \frac{M_z \cdot x_{(r)}}{\sum_n\left(x_{(n)}^2 + y_{(n)}^2\right) + \sum_r\left(x_{(r)}^2 + y_{(r)}^2\right)} \tag{15}$$

$$F_{dwl_{(r)}} = \sqrt{F_{x.dwl}{}_{(r)}^2 + F_{y.dwl}{}_{(r)}^2} \tag{16}$$

Equations (14) and (15) uses coordinates about the centroid of the combined bolt and dowel locations, and may be different to that used for equations (8) and (9).

The centroid used in equations (14) and (15) is at the position where: $\frac{\sum_n x_{(n)} + \sum_r x_{(r)}}{N_b + N_d} = 0$ and $\frac{\sum_n y_{(n)} + \sum_r y_{(r)}}{N_b + N_d} = 0$

Often equations (14) to (16) are not used to size dowels. Instead dowels are sized to support all of the shear loads, without considering friction at the faying surface. This is not a condition that would occur in normal operation but it does provide the joint with the ability to maintain some structural integrity in the event of bolts loosening in service.

5 Flange Bending

The joint flange(s) will experience bending due to pressure at the faying surface acting at the edge of the flange, as illustrated in Figure 3.

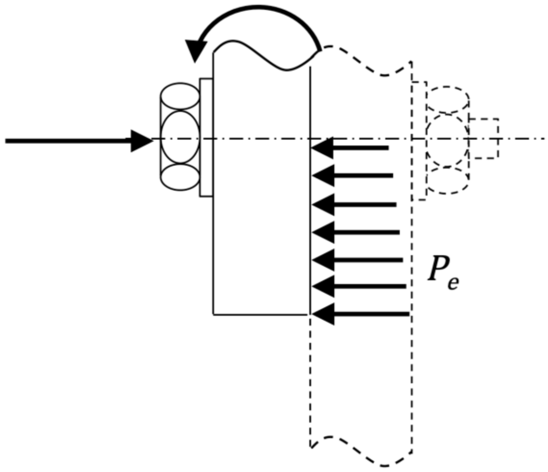

Fig. 3 Pressure Load on Flange Face.

The compressive (negative) bolt related loads given by equation (1) act to increase the contact pressure on the region of faying surface surrounding the bolt.

The contact forces acting at the faying surface local to each bolt are given by:

$$F_{f_{(n)}} = F_{br_{(n)}} - F_{p.max} \tag{17}$$

The negative sign in equation (17) indicates that the tensile preload in the bolts produce a compressive stress at the faying surface. Assuming that the circular region of faying surface influenced by the bolts closest to the edge of the flange extends to the edge, then the contact pressure is given by:

$$P_{f_{(n)}} = \frac{F_{f_{(n)}}}{\pi \cdot L_e^2} \tag{18}$$

The flange bending stress is calculated using the most compressive value of contact pressure, $P_{f_{(n)}}$, as the flange edge pressure. The bending stress in the flange is given by:

$$\sigma_{b.f} = \frac{-3 \cdot L_e^2 \cdot P_e}{t_f^2} \tag{19}$$

where the contact pressure at the edge of the faying surface, P_e, is taken to be the most compressive (negative) value of contact pressure $P_{f_{(n)}}$.

6 Thread Shear

In the preceding sections it was assumed that the bolted connections were made using appropriate grade nuts. However, it is not unusual for bolts to be tightened into threaded, or tapped, holes within one of the mating components. In these cases the thread shear stresses will need to be calculated.

It is usual for bolted connections to be designed so that the bolt will fail in tension at the threaded portion of the shank before failure by thread shearing. This type of failure gives an early indication that failure has occurred during assembly, due to over-tightening, or due to overloading during service.

If the internal thread material has a tensile strength the same as or greater than the bolt, then the thread engagement needs to be at least that of a standard nut (plus an allowance for the chamfered lead of the bolt if required). However, if the internal thread is formed within a material having a lower ultimate strength than the bolt then the shear stress in the internal thread and the minimum length of thread engagement required has to be considered.

The shear area of the internal thread at the minimum major diameter of the external thread is given by:

$$AS_n = \frac{\pi \cdot L_{eng} \cdot D_{s.min}}{p_{thrd}} \left(\frac{p_{thrd}}{2} + tan\left(\frac{\theta_{thrd}}{2}\right) \cdot (D_{s.min} - E_{n.max}) \right) \tag{20}$$

Equation (20) is based on that given in Appendix A of British Standard BS 3580:1964 "*Guide to the design considerations on: The strength of screw threads*" [5]. However, the equation presented here has been put into a format that can be applied to any thread form commonly used for bolts.

It has been shown that the first full internal thread supports a large proportion of the load and is subject to yielding [6]. When determining the length of thread engagement, L_{eng}, it can be assumed that one half of each of the end threads in the thread engagement does not contribute to carrying load in shear. Hence, the active length of thread engagement is the nominal length of engagement less one thread pitch. If the end of the bolt is within the tapped hole then the length of thread engagement should account for the bolt's chamfered lead.

British Standard BS 3692:2001 "*metric precision hexagon bolts, screws and nuts – specification*" [7] calls for a tolerance of 6H/6g on nuts and bolts. This is the usual tolerance for precision high strength fasteners, where the use of a standard nut results in short lengths of thread engagement, typically 0.8d (where d is the nominal thread size). Internal threads in materials with lower shear strength than nuts require a longer thread engagement and tolerances of 6H/6g can lead to interference between the internal and external threads. It is common practice to change the tolerance of the internal thread to accommodate the pitch and flank angle errors of long thread engagements. Often the true tolerance on the internal thread is not recorded within the design data, but should be reflected in the analysis.

The internal thread shear stress is given by:

$$\tau_n = \frac{F_b}{AS_n} \tag{21}$$

where F_b is the maximum bolt load, which can be taken to be the bolt preload F_p.

The minimum length of thread engagement required is given by:

$$L_{eng.min} = \frac{p_{thrd} \cdot A_s \cdot UTS_s}{\pi \cdot D_{s.min} \left(\frac{p_{thrd}}{2} + tan\left(\frac{\theta_{thrd}}{2}\right) \cdot (D_{s.min} - E_{n.max}) \right) \cdot QS_n} \tag{22}$$

Again, this equation is based on those given in British Standard BS 3580:1964 Appendix A [5]. The effect of the internal thread material having a lower ultimate strength than the bolt has been incorporated into equation 27.

Equation (22) uses the yield/proof based shear strength instead of an ultimate shear strength, based on half the ultimate strength, as suggested in BS 3580:1964 [5]. Using the yield/proof yield strength protects the (more expensive) internally threaded component from permanent deformation of its thread. For steel $QS_n = Y_s/\sqrt{3}$ where Y_s is the yield/proof stress.

CONCLUSION

In many cases the method of analysis presented here will be adequate to demonstrate the structural integrity of the joint and compliance with design standards.

Preloading the bolts of the joint induces a compressive stress at the faying surface. The bolt preload keeps the joint closed when external loads are applied to the joint, producing tensile stress components that act to reduce this compressive stress. As long as the faying surface retains some compressive stress the joint will continue to perform as if it were a continuous member.

External out-of-plane moments and tensile loads are supported mainly by a reduction in the contact pressure at the faying surface, with only a small proportion of the external loads acting to increase the bolt tensile stress. This results in a low working stress range, which is a major factor in why preloaded joints have good fatigue performance and high stiffness.

External in-plane loads and torsional moments on the joint are supported mainly by friction at the faying surface. In some joints dowels, or other positive method of location of the joint, can assist in supporting the shear loads by preventing movement or slip at the joint face.

The joint flange(s) experience bending due to the pressure at the faying surface acting at the edge of the flange.

The approach presented here cannot be applied to all bolted joint configurations. In some cases a detailed analysis has to be considered. In particular, joints with single bolts, or a single line of bolts, are usually special cases and have to be considered from first principles.

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