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# WAVE COUPLING ANALYSIS IN THE ELASTIC WAVEGUIDES OF QUARTER WAVELENGTH NOISE ABSORBERS

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**Abstract:** The analytical model of a fluid filled cylindrical shell is used to analyse the wave propagation in the quarter wavelength absorbers. It has been found, that due to the strong fluid-structure interaction, the considerable wave speed reduction can be reached using a compliant shell. Thus via parametric studies, the dimensions of the absorbers are assessed to reach more efficient and compact design. Furthermore, the use of the compliant materials allows treating the fluid to be incompressible, which helps to simplify the calculations.

**KEYWORDS:** Wave propagation, Quarter wavelength dampener, Shell theory, Wave speed, Heavy fluid loading, Fluid structure interaction

## 1 Introduction

In hydraulic systems, the noise generated by the source (a pump, piston, etc.) can be transmitted via fluid medium (fluid-borne noise), via a pipe/other structural elements (structure-borne noise) or a combination of the two (fluid-structure noise). In many situations this noise is unwanted and should be minimized or fully avoided [1]. Considering the fluid-borne noise, it can be attenuated either by means of a machine's re-design (as e.g. in [2]) or using the active or passive way of noise control. Focusing at the passive alternatives, one can apply compliant hoses, expansion chambers, tuning coils, Helmholtz resonators, or other silencer configurations [3].

Recently, there have been published articles dedicated to the modelling and analysis of various Helmholtz resonator arrangements [4], even with ability to act as energy harvesters [5], or an article dealing with applications of hydraulic double expansion chamber [6] which aimed to attenuate the excessive noise in systems with heavy fluid loading. Furthermore, the interaction between the fluid medium and structural element has been analyzed in number of papers, e.g. in [7, 8].

In this article, we are focused at the other way to reduce the noise, implementing the tuning cables, also known as 'quarter wavelength' resonators [3]. There is a number of arrangements of the resonators used (e.g. in-line, side), but the principle of operation is the same for all of them. It is based on destructive interference of the incoming and reflected waves. As the name suggests, they are designed to have the length one quarter (or a multiple) of the wavelength of the wave to be attenuated. To find the value of the wavelength, one needs to know the wave speed (which is in general dependent on the fluid-structure interaction) and the frequency at which the noise is generated. A mathematical model of a cylindrical in-line quarter wavelength resonator based on transfer matrix analysis or numerical FEM and some parametric studies has been published in [9 - 11].

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This article deals with a strong two-way coupling (fluid-structure as well as structure-fluid) analysed for compliant elastic waveguides. The analytical model, based on the shell theory, is applied to a flexible thin-shell quarter wavelength fluid filled attenuator. A parametric study is performed and the influence of the material and geometric parameters on the phase speed of the first wave propagating in the waveguide is assessed.

## 2 Mathematical model

Let us assume the quarter-wavelength absorber to be attached to a pipe sideways and being filled with heavy fluid, as shown in Fig. 1. It can be modelled as a thin homogeneous cylindrical shell with the inner radius *R* and the thickness *h*. For the rigid part, the Kirchhoff-type shell theory is used, while the fluid is modelled using Navier-Stokes equations. In order to follow the thin shell assumption the ratio between the shell's wall thickness and pipe's mean radius is to be $h/R < 0.25$. Furthermore, there is done an assumption that the inner radius $R_i$ is large enough to ignore all the viscous as well as non-linear effects in the fluid.

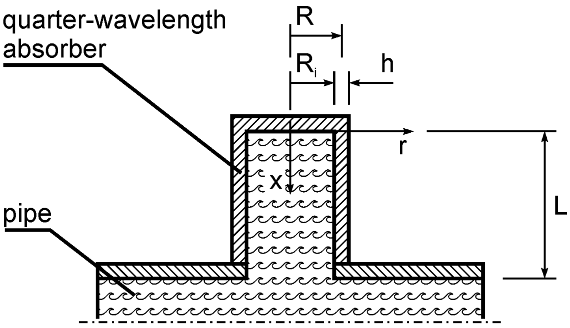



Fig. 1 sketch of the quarter wavelength absorber assumed

The motion of the shell is described with the coordinates $u_m$ and $v_m$, while for the fluid dynamics the potential $\varphi_m$ is used. The governing equations of the shell then become [12]:

$$-\frac{d^2\tilde{u}_m}{d\tilde{x}^2}+\frac{1-\nu}{2}m^2\tilde{u}_m-\frac{1+\nu}{2}m\frac{d\tilde{v}_m}{d\tilde{x}}-\nu\frac{d\tilde{w}_m}{d\tilde{x}}-\Omega^2\tilde{u}_m=0 \tag{1}$$

$$\begin{aligned}&\frac{1+\nu}{2}m\frac{d\tilde{u}_m}{d\tilde{x}}-\frac{1-\nu}{2}\frac{d^2\tilde{v}_m}{d\tilde{x}^2}+m^2\tilde{v}_m-\frac{\tilde{h}^2}{6}(1+\nu)\frac{d^2\tilde{v}_m}{d\tilde{x}^2}+\frac{\tilde{h}^2}{12}m^2\tilde{v}_m+\cdots\\&m\tilde{w}_m+\frac{\tilde{h}^2}{12}m^3\tilde{w}_m-\frac{\tilde{h}^2}{12}(2-\nu)m\frac{d^2\tilde{w}_m}{d\tilde{x}^2}-\Omega^2\tilde{v}_m=0\end{aligned} \tag{2}$$

$$\begin{aligned}&\nu\frac{d\tilde{u}_m}{d\tilde{x}}+m\tilde{v}_m+\frac{\tilde{h}^2}{12}m^3\tilde{v}_m-\frac{\tilde{h}^2}{12}(2-\nu)m\frac{d^2\tilde{v}_m}{d\tilde{x}^2}+\tilde{w}_m+\frac{\tilde{h}^2}{12}\frac{d^4\tilde{w}_m}{d\tilde{x}^4}-\cdots\\&\frac{\tilde{h}^2}{6}m^2\frac{d^2\tilde{w}_m}{d\tilde{x}^2}+\frac{\tilde{h}^2}{12}m^4\tilde{w}_m-\Omega^2\tilde{w}_m-i\frac{\Omega\tilde{\rho}}{\tilde{h}}\tilde{\varphi}_m=0\end{aligned} \tag{3}$$

with the circumferential wave number *m*, Poisson's ratio *ν*, wave speed in the fluid *c* and excitation frequency *ω*. The equations are already expressed in the non-dimensional form, with the coordinates scaled with respect to the radius R: $\tilde{r}=r/R$, $\tilde{x}=x/R$, $\tilde{v}=v/R$, and $\tilde{w}=w/R$, and $\tilde{\varphi}=\varphi/R$ and using the non-dimensional parameters: $\Omega^2=[\rho(1-\nu^2)\omega^2R^2]/E$, $\tilde{\rho}=\rho_{fl}/\rho$, $\tilde{\gamma}=c/c_{fl}$, $k=k_{dim}\,R$, $\kappa^2=k^2+\tilde{\gamma}^2\Omega^2$, $\mathrm{c}=\sqrt{\mathrm{E}/[\rho(1-\nu^2)]}$.

The fluid's dynamics is describing by:

$$\frac{d^2\tilde{\varphi}_m}{d\tilde{r}^2}+\frac{1}{\tilde{r}}\frac{d\tilde{\varphi}_m}{d\tilde{r}}-\frac{m^2}{\tilde{r}^2}\tilde{\varphi}_m+\frac{d^2\tilde{\varphi}_m}{d\tilde{x}^2}+\tilde{\gamma}^2\Omega^2\tilde{\varphi}_m=0 \tag{4}$$

At the fluid-structure interface, following continuity condition needs to be satisfied:

$$\left.\frac{\partial \tilde{\varphi}_m}{\partial r}\right|_{\tilde{r}=1} = i\Omega\tilde{w}_m \tag{5}$$

The ‘fluid-to-structure’ coupling is formulated by Eq. (5). In conventional duct acoustics, absolutely rigid walls of the shell are considered, so that the right hand side of Eq. (5) is equal to zero.

On the other hand, the ‘structure-to-fluid’ coupling is formulated by the last term in Eq. (3). For ‘light fluid loading’ (e.g. air-filled steel pipe), it is negligibly small. In this article, we are interested in ‘heavy fluid loading’ (e.g. water in a pipe). Thus in what follows, we take into account both the ‘fluid-to-structure’ and ‘structure-to-fluid’ coupling.

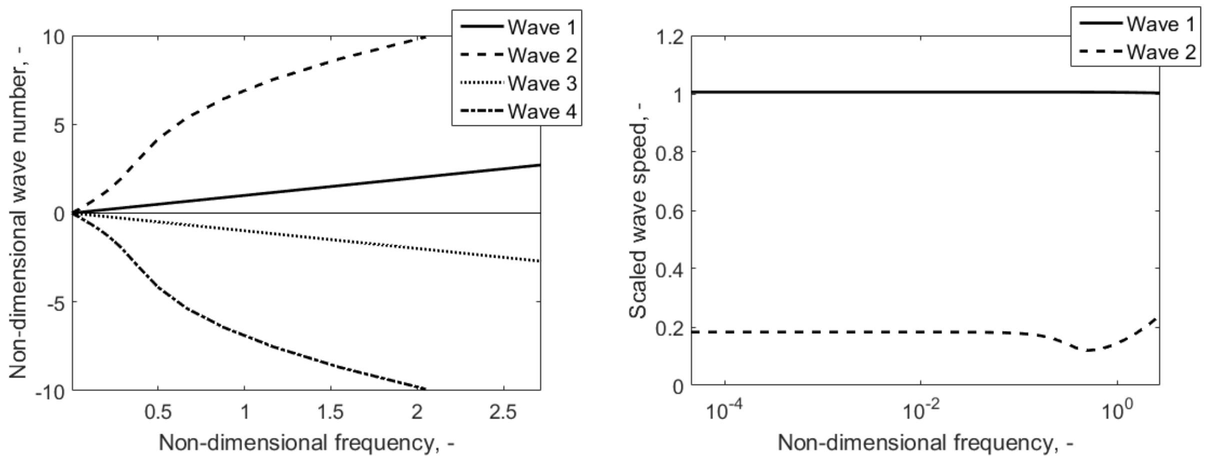
To design the quarter-wavelength absorber, wave numbers and corresponding wave speeds at excitation frequencies are required. Thus, one needs search for the dispersion equation and analysing the dispersion curve, the wave number $\tilde{k}$ for given frequency $\Omega$ is found and the phase speed is calculated using:

$$\tilde{c} = \frac{\Omega}{\tilde{k}} \tag{6}$$

We are interested in the waves propagating from zero frequency, thus also the analysis of the dispersion diagram will be limited only to the low frequency region (below the first cut-off frequency).

## 3 Results and discussion

Solving the dispersion equation for the rubber pipe (i.e. an elastic waveguide) filled with water and taking $m$=0, the diagram shown in Fig.2a is obtained. There are two waves propagating from zero frequency in both directions (i.e. in positive and negative planes of wave number $\tilde{k}$). The corresponding wave speeds (scaled with respect to the wave speed in the solid part) are plotted in Fig.2b. The speed of the first wave (Wave 1 in Fig.2b) is not dependent on frequency, and $\tilde{c}$ = 1. Therefore, this wave is non-dispersive, de-coupled from fluid medium and it propagates at the sound speed in a solid part. More interesting is however the second wave (Wave 2 in Fig.2b), which is non-dispersive ($\tilde{c}$=const.) for low frequencies and after some threshold ($\Omega$=0.5), the wave speed increases with the increased frequency. At the low frequencies, where the wave is non-dispersive, the wave speed is significantly lower than the sound speed in solid part of the waveguide, i.e. the wave is strongly coupled and influenced by the fluid medium.



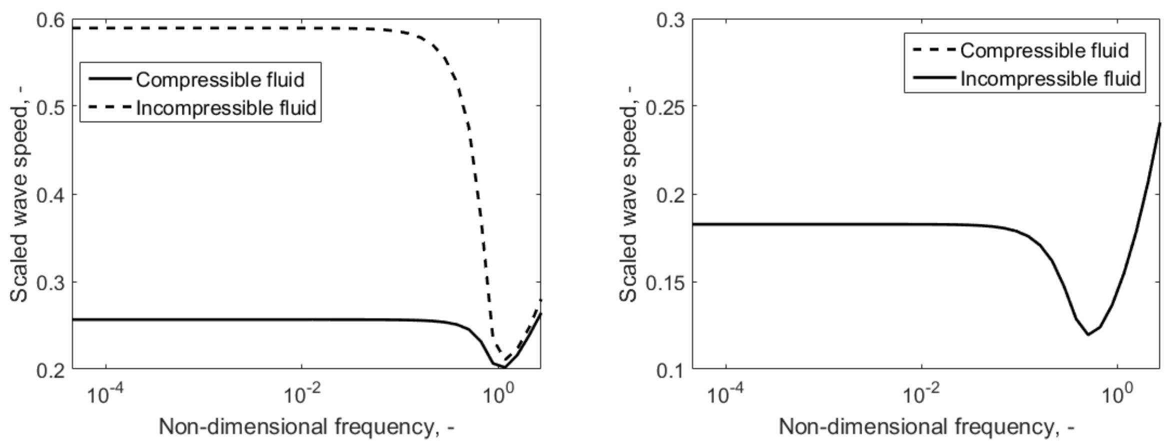

(a) (b)

Fig. 2 (a) Dispersion diagram for water filled rubber pipe with h/R = 1/10; (b) corresponding wave speeds for waves propagating in a positive direction

In a case of the quarter wavelength absorbers, as their length $L$ (Fig.1) should be ¼ of the wavelength of the wave to be absorbed (i.e. the destructive interference is reached), the lower wave speed means shorter the absorber (since for a fixed frequency the wavelength is $\lambda=c/f$). Thus this fluid-structure coupling can be effectively used in the absorber's design.

As mentioned before, for light fluid loading of a shell, the fluid-structure interaction is negligible. On the other hand, considering the heavy fluid loading, the interaction needs to be assessed. Taking in account the steel shell (having the thickness ratio $h/R$ = 1/10) with a compressible fluid (i.e. for water taking $c_{\mathrm{fl}}$ = 1481 m/s) the wave speed varies with frequency as shown in Fig.3a. Then comparing it with the wave speeds obtained for an incompressible fluid ($c_{\mathrm{fl}} \rightarrow \infty$, i.e. $\tilde{\gamma} = 0$), one can see that the fluid's compressibility has a significant effect to the wave speed and the assumption of the incompressible fluid causes large overestimation in the wave speed. However, if we perform the same comparison for an elastic rubber pipe of the same geometry (Fig.3b), the wave speeds are identical. Thus the compressibility of the fluid has no effect on wave speeds and one can treat it as incompressible (which simplifies the calculations).

Furthermore, comparing the speeds for steel (at low frequencies $\tilde{c}\approx 0.26$) and rubber (at low frequencies $\tilde{c}\approx 0.18$), the hard steel shell and influence of the fluid compressibility are not providing such a speed reduction as soft shell with the same dimensions and filled with the same fluid can provide. Thus for more effective and compact absorber, softer and more compliant material is more suitable.

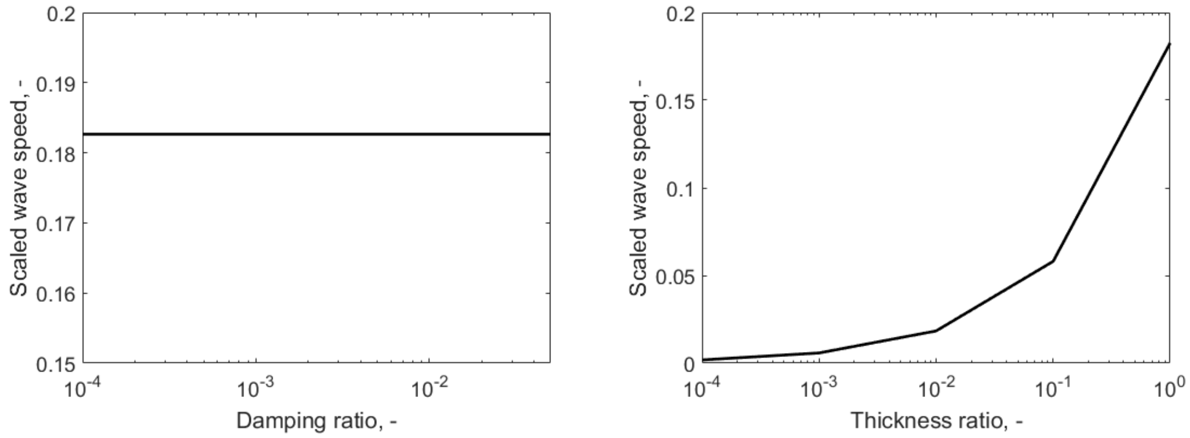


(a) (b)

Fig. 3 Comparison of the wave speeds as a function of frequency parameter when the fluid is treated to be compressible and incompressible: (a) for steel; (b) for rubber

The rubber and number of other flexible materials are also introducing some damping into the vibrating structure. To study the effect of damping on wave speed, a complex Young's modulus (with damping ratio $\zeta$) has been introduced. However, the comparison of the wave speeds for variable values of $\zeta$ reveals that even for 5% damping has no influence on the speed the wave propagates (Fig.4a). The light damping introduces small change in frequency, but that is not significant enough to be reflected in the wave speed variation. This resembles the mechanical systems with light damping where the eigenfrequency analysis shows a small change in a response for low damping. Yet one can conclude that damping is not a key parameter in absorber's design and it will not improve its direct performance.

The absorber as such can be manufactured as a tube with different wall thickness and diameter values. To study their influence on wave speed value, the thickness ratio $h/R$ is varied. Drawing our attention only to the frequencies with non-dispersive waves (i.e.




(a) (b)

Fig. 4 wave speed (non-scaled) as a function of (non-scaled frequency): (a) variable damping ratio; (b) variable shell thickness ratio

sufficiently low frequencies), the Fig.4b shows, that as the thickness ratio is reduced, the speed (scaled with respect to speed of sound in the shell) is lower. It means that for the compliant absorber, thanks to the fluid structure interaction, the speed of the compound wave is lower than the one in either shell ($\tilde{c}$=1) or in the fluid ($\tilde{c}$>>1).

Unlike the acoustic waves propagating in the absorbers with light (compressible) fluid and rigid shell, the speed of wave propagating absorbers with heavy fluid loading (which can be considered as incompressible) can be significantly lower than the ones in the shell or in fluid itself. And the membrane like shells with small thickness are capable to reduce the wave speeds and so dimensions of the quarter wavelength absorbers. However, with the model used, the results obtained for very small thicknesses, $h/R\to0$ the speed $\tilde{c}\to0$, which is not physical. This is caused by a lack of the pre-stress which needs to be applied to the membrane shell. Therefore for the thin membrane shells with negligible bending stiffness, the model needs to be revisited and the pre-stress added. This task is left as a future work.

## CONCLUSION

The fluid structure interaction in the quarter wavelength absorbers was analysed by means of the model of the shell exposed to heavy fluid loading. It has been found that unlike for the light fluid loading, considering the compliant materials, fluid's compressibility can be neglected, which simplifies the calculations. Also there was identified a significant reduction in the wave speed obtained for compliant waveguides, which has a practical impact on the absorber's dimensions, since the absorber's length is dependent on the wave speed. Furthermore, it was observed, that model used is not valid for small thicknesses and further assessment of the thresholds should be done.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## REFERENCES


[1] S. Ziaran. Znizovanie kmitania a hluku v priemysle, Slovenska Technicka Univerzita v Bratislave, **2006**. ISBN 8022723665.

[2] R. Klas, F. Pochyly. The modification of the mixed flow pump with respect to its stability. *Journal of Mechanical Engineering – Strojnícky časopis* **2011** (62), No. 2, 93 – 103.

[3] G. E. Totten, V. J. De Negri. Handbook of Hydraulic Fluid Technology, $2^{nd}$ Edition, CRC Press, **2017**. ISBN 9781138077348.

[4] T. Ichiyanagi, T. Kuribayashi, T. Nishiumi. Investigations on Wave Propagation of the Pressure Pulsation in a Helmholtz-Type Hydraulic Silencer With a Hemispherical Vessel, ASME. *J. Vib. Acoust.*, **2016** (139), No. 1, 011003-1 – 11003-8, DOI: 10.1115/1.4034618.

[5] E. Skow, Z. Koontz, K. Cunefare, A. Erturk. Hydraulic pressure energy harvester enhanced by Helmholtz resonator, In Proc. SPIE 9431, Active and Passive Smart Structures and Integrated Systems, **2015**, id. 943104, pp. 11. DOI: 10.1117/12.2084343.

[6] F. Yang, B. Deng, Filtering performance and optimization of double-chamber compound hydraulic attenuators, Proc IMechE Part C: *J Mechanical Engineering Science* **2018** (232), No. 18, 3250 – 3262. DOI: 10.1177/0954406217733293.

[7] S.V. Sorokin. Non-linear oscillations of a baffled elastic plate in heavy fluid loading conditions. *Journal of Sound and Vibration* **2000** (232), No. 3, 619 – 643.

[8] E. Wiszt, E. Wisztova. Comparison of acoustical and mechanical couplings of vibrating mechanical oscillators. *Journal of Mechanical Engineering – Strojnícky časopis* **1998** (49), No. 1, 40 – 51.

[9] M. Hastings, C. Chen. Analysis of Tuning Cables for Reduction of Fluidborne Noise in Automotive Power Steering Hydraulic Lines, SAE Technical Paper 931295, **1993**. DOI: 10.4271/931295.

[10] K. A. Marek, N. E. Earnhart, K. A. Cunefare. Model and analysis of a cylindrical in-line hydraulic suppressor with a solid compressible liner. *Journal of Sound and Vibration*, **2014** (333), No. 24, 6312 – 6331. DOI: 10.1016/j.jsv.2014.07.006

[11] J. Yu, S. Popescu. Measurements of Transmission and Attenuation Characteristics of Fluidborne Noise in Fluid Hoses with/without Tuning Cables, SAE Technical Paper 2001-01-1610, **2001**. DOI: 10.4271/2001-01-1610

[12] A.W. Leisa. Vibrations of shells, Acoustical Society of America, New York, **1993**. ISBN 1563962934