

EFFECT OF LAMINATION ANGLE AND THICKNESS ON ANALYSIS OF COMPOSITE PLATE UNDER THERMO MECHANICAL LOADING

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Abstract: Analytical formulation and solution of stress analysis of composite plate subjected to thermo mechanical loading for various ply orientation and thickness of lamina are studied. The main aim of the paper is to investigate how mechanical and thermo mechanical loading would affect the stress ratio and stress distribution of composite plate. The plate is composed of layers of glass-epoxy composite and the orientation of the layers is assumed to be antisymmetric about the neutral axis of the laminate. The plate is subjected to a combined mechanical loading of tensile force and moment along x direction. The thermo mechanical stress is calculated for different ply orientation and thickness ratio, subjected to a temperature change and mechanical loading. The effect of number of lamina and varying thickness of laminate on the stress ratio and stress distribution is studied. The results in this paper are obtained by use of MATLAB Programming and by Finite element software ANSYS 14. Results obtained from both the methods are compared. Such type of loading finds wide application in aircraft flying at high altitude, marine application, medical devices etc.

KEYWORDS: Ply orientation, classical lamination theory, thermo mechanical, laminate.

1. Introduction

Composite material is a structure that is composed of two or more material combined at a macroscopic level and are not soluble to each other, in such a way that one phase is called reinforcing phase and the other phase in which it is embedded is called matrix. Reinforcing phase may be in the form of fiber, flakes, particles etc. Laminated composite find wide application in aerospace, marine, medical, automobile etc. Industries owing to their advantages of high strength, fatigue life, stiffness, temperature-dependent behavior, corrosion resistance, thermal insulation, wear resistance, thermal conductivity, attractiveness, acoustical insulation and low weight. The increase in the strength, stiffness and reduction in weight of engineering structure can be achieved by using composite material. Further reduction can be achieved by using optimum ply orientation, thickness or stacking sequence. Most of the research is focused on design of composite structure for optimum ply orientation and stacking sequence.

L.-W.Chen & L.-Y.Chen[1] studied the effect of thermal stress in composite plate by using finite element method. Onur Sayman [2] studied an elastic-plastic type of thermal stress analysis which was carried on metal-matrix composite beams of fiber reinforced of steel and metal-matrix of aluminum. R. Özcan [3] explained through his paper about residual stress analysis of a thermoplastic laminated composite plate for an in-plane loading. Frank R. Jones, Fangming Zhao [4] investigated the effects of thermal stresses developed on the stress transformation between resin and short fiber using the method of photo elasticity. Wu and

Tauchert [4,5] developed a model to obtain thermal stresses and deformations in anti-symmetric angle ply and cross-ply laminates and symmetric laminates.

Noor and Burton [6] obtained solutions for the bending, buckling and vibration of anti-symmetrically laminated rectangular plates, periodic in the in-plane directions. Savithri and Varadan [7] studied plates under uniformly distributed and concentrated loads. P. Vidal et al. [8] presented the thermo-mechanical analysis of bi-dimensional laminated beams by using a variables separation method. Thermal and mechanical responses are assessed by comparing with other approaches available in literature, exact solutions and 2D finite element computation. Zineb et al. [9] study the influence of varying thickness glass epoxy composite plate under the pure bending moment. Joshi & Biggers [10] determine the optimal thickness distribution over the plate, for which the thickness distributions that maximize the buckling load are determined. Javadi et al. [11] formulate a three dimensional solutions for contact area in laminated composite pinned joints with symmetric and non-symmetric stacking sequences. Kormaníková, E. [12] studied an optimal design of laminate circular cylindrical. Choudhary S, et al. [13] formulate a simple higher order theory for dynamic analysis of composite plate. Most of the research paper focuses on the optimum design of composite plate by optimizing thickness for bending, buckling, vibration or thermal load using finite element method or variable separation method. Ply orientation is a very important parameter which effects the stress distribution in composite plate. Research paper on the study of the effect of ply orientation on stress analysis of composite plate under thermo mechanical load is rare.

Thermo mechanical stress can be calculated by classical laminate theory, the first order shear deformation theory, higher order theories and finite element method. In this paper classical laminate theory is used to compute the stresses in a four layered anti symmetric composite plate under different ply orientation and loading conditions. Thermo mechanical stress is determined by superposing the stress generated for the mechanical and thermal loading condition. The study is carried out for angle ply orientation varied from 0 degree to 90 degree. The effect of no. of lamina and varying thickness ratio on the stress ratio and stress distribution of composite plate is studied. This paper determines the stress ratio for different orientation of the lamina to check which stacking sequence would give the lowest ratio. The stacking sequence of the laminate with the lowest stress ratio is likely of an optimized type.

2. Analysis of composite plate

A laminate is a collection of lamina arranged in a specified manner. Adjacent lamina may be of the same or different materials and their fiber orientations with respect to a reference axis (by convention, the x-axis) may be arbitrary. Classical lamination theory (CLT) is applicable to orthotropic continuous fiber laminated composites only. the classical lamination theory is only valid for thin laminates (span 'a' and 'b' > 10×thickness 't') with small displacement 'w' in the transverse direction ($w \ll t$) [IIT lecture notes]

2.1. Assumption of Classical lamination theory (CLT)

The assumptions of classical lamination theory are stated as [14]:

1. Each layer of the laminate is quasi-homogeneous and orthotropic.
2. The laminate is thin compared to the lateral dimensions and is loaded in its plane.
3. State of stress is plane stress.
4. All displacements are small compared to the laminate thickness.
5. Displacements are continuous throughout the laminate.

6. Straight lines normal to the middle surface remain straight and normal to that surface after deformation.

7. Transverse normal strain ε_z is negligible compared to the in-plane strains ε_x and ε_y .

8. Strain-displacement and stress-strain relations are linear.

2.2 Stress analysis of lamina

A lamina is a thin layer of a composite material that is generally of a thickness on the order of 0.005 in. (0.125 mm). A laminate is constructed by stacking a number of such lamina in the direction of the lamina thickness. The constitutive equations for a general linear elastic solid relate the stress and strain tensors through the expression:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \quad \varepsilon_{ij} = S_{ijkl}\sigma_{kl} \quad (i, j, k, l = 1, 2, 3) \quad (1)$$

Where C_{ijkl} = stiffness components, S_{ijkl} = compliance components.

Fiber reinforced composite materials are considered to have orthotropic elasticity because these materials possess three mutually perpendicular planes of the elastic symmetry. For a unidirectional reinforced lamina in the 1-2 plane as shown in Figure, a plane stress state is defined by: $\sigma_3 = 0, \tau_{23} = 0, \tau_{31} = 0$ $\sigma_1 \neq 0, \sigma_2 \neq 0, \tau_{12} \neq 0$

Moreover the stress- strains relations in 1-2 planes:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{11} \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (2)$$

where $Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$, $Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$, $Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}$, $Q_{66} = G_{12}$, $\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$

Strain- stress relation is obtained as:

$$[\varepsilon] = [Q]^{-1}[\sigma] \quad (3)$$

Stress strain relation in $x - y$ coordinate system:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad \text{where } [\bar{Q}] = [T]^{-1}[Q][R][T][R]^{-1} \quad (4)$$

$$T = \begin{bmatrix} C^2 & S^2 & 2CS \\ S^2 & C^2 & -2CS \\ -SC & SC & C^2 - S^2 \end{bmatrix} \text{ and } R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

where $C = \cos \theta$, $S = \sin \theta$.

2.3 Stress analysis in laminate

2.3.1 Determination of laminate stiffness [A B D] matrix

Consider a laminate made of n plies shown in Figure 4.6. Each ply has a thickness of t_k . Then the thickness of the laminate h is $h = \sum_{k=1}^n t_k$

Then, the location of the mid plane is $h/2$ from the top or the bottom surface of the laminate. The z -coordinate of each ply k surface (top and bottom) is given by:

$$h_{k-1} = -\frac{h}{2} + \sum_1^{k-1} t \quad (\text{top surface}), h_{k-1} = -\frac{h}{2} + \sum_1^k t \quad (\text{bottom surface}) \quad (5)$$

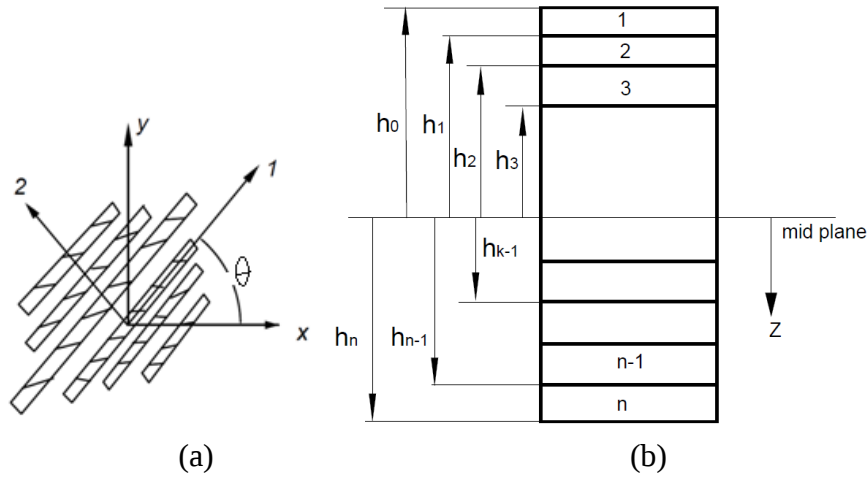


Fig. 1 (a) Rotation of Principle material axis from reference 'x-y' axis, (b) location of plies in laminate [15]

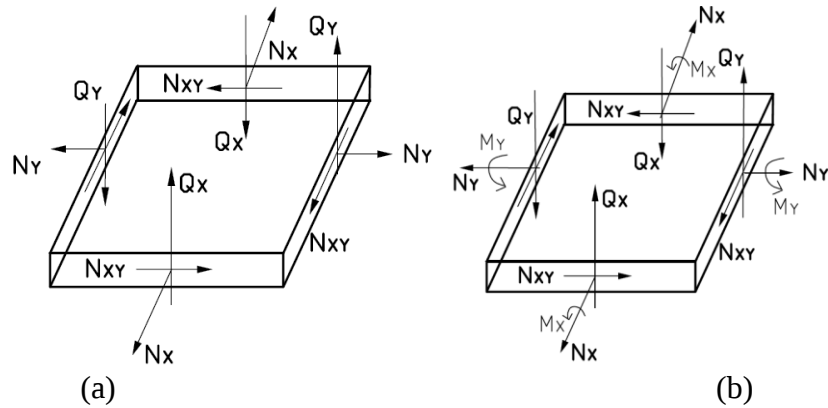


Fig. 2 Composite plate subjected to (a) Normal force/ length, (b) Bending moment / length [15]

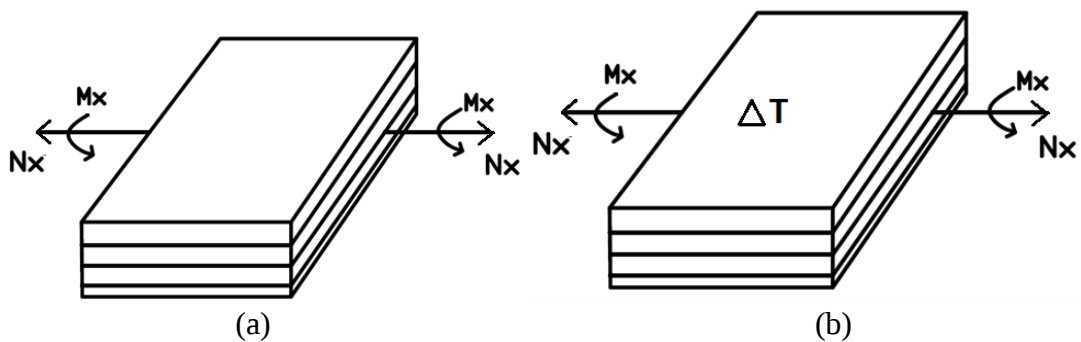


Fig. 3 Composite plate subjected to (a) combined mechanical load (normal force and bending moment), (b) Thermo mechanical load

Force per unit length (N) and the bending moment per unit length(M) of a laminate are given as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (7)$$

N_x, N_y , = normal force per unit length, N_{xy} = shear force per unit length,

M_x, M_y , = Bending moment per unit length, M_{xy} = Twisting moments per unit length

ε^0 = midplane strain of laminate in x-y coordinate and k =laminate curvature.

Combining the above two matrix as:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ k \end{bmatrix} \quad (8)$$

$$A_{ij}(\text{extensional stiffness matrix}) = \sum_{k=1}^n [\overline{Q}_{ij}]_k (h_k - h_{k-1}), \quad i = 1,2,6, \quad j = 1,2,6$$

$$B_{ij}(\text{extension - bending coupling matrix}) = \frac{1}{2} \sum_{k=1}^n [\overline{Q}_{ij}]_k (h_k^2 - h_{k-1}^2), \quad i = 1,2,6, \quad j = 1,2,6$$

$$D_{ij}(\text{bending stiffness matrix}) = \frac{1}{3} \sum_{k=1}^n [\overline{Q}_{ij}]_k (h_k^3 - h_{k-1}^3), \quad i = 1,2,6, \quad j = 1,2,6$$

2.3.2 Determination of stress of laminate

The mid plane strain and curvature can be determined from ABD matrix as:

$$\begin{bmatrix} \varepsilon^0 \\ k \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N \\ M \end{bmatrix} \quad (9)$$

Stresses of laminate in the k^{th} lamina as:

$$[\sigma]_k = [\bar{Q}]_k [\varepsilon^0] + h[\bar{Q}]_k [k] \quad (10)$$

Strain of laminate in the k^{th} lamina as:

$$[\varepsilon]_k = [\varepsilon^0] + h[k] \quad (11)$$

2.4 Thermo mechanical stresses

Thermal stresses are developed due to cooling down or heating from processing temperatures to operating temperatures different from processing temperatures. Each ply in a laminate gets stressed by the deformation differences of adjacent lamina.

The mechanical strains developed by thermal loads are:

$$[\varepsilon^M] = [\varepsilon] - [\varepsilon^T] \quad \text{where} \quad [\varepsilon] = [\varepsilon^0] + z[k] \quad (12)$$

The thermal stresses are given by:

$$[\sigma^T] = [\bar{Q}][\varepsilon^M] \quad (13)$$

Mid plane strain and curvature is calculated by:

$$\begin{bmatrix} N^T \\ M^T \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ k \end{bmatrix} \quad (14)$$

where

$$\begin{aligned} [N^T] &= \Delta T \sum_{k=1}^n [\overline{Q}_{ij}]_k [\alpha]_k (h_k - h_{k-1}), \\ [M^T] &= \frac{1}{2} \Delta T \sum_{k=1}^n [\overline{Q}_{ij}]_k [\alpha]_k (h_k^2 - h_{k-1}^2) \end{aligned} \quad (15)$$

Thus, if both mechanical and thermal loads are applied, then superpose the mechanical loads to the fictitious thermal loads to find the ply-by-ply stresses and strains in the laminate or separately apply the mechanical and thermal loads and then add the resulting stresses and strains from the solution of the two problems.

Stress ratio can be calculated as:

$$\text{Stress ratio (SR)} = \frac{\text{Maximum stress}}{\text{Minimum stress}} \quad (16)$$

3. Problem formulation

A composite plate made up of four layers glass epoxy material subjected to combined loading condition (shown in fig) is considered for study. The layers are arranged anti symmetrically $[+\alpha \ -\alpha \ +\alpha \ -\alpha]$. Stresses developed in the composite plate for different ply orientation (0° - 90°) are studied for loading condition: combined load of normal force and bending moment in x direction. ($N_x = 1000 \frac{N}{m}$, $M_x = 400 \text{ N} - m$). The effect of lamination angle on the stress distribution of composite plate is analyzed for different length to thickness ratio (a/h) of composite plate. Stress ratio is computed for a/h ratio of 5, 10 and 20. Classical lamination theory is used to model the stress analysis of composite plate. From this analysis the optimum lamination angle corresponding to minimum stress ratio can be obtained. The properties of glass epoxy composite material are shown in the table. The entire ply is of equal thickness.

Table 1: Properties of Glass- epoxy composite

Fiber volume fraction	Longitudinal modulus of elasticity (E_1)	Transverse modulus of elasticity (E_2)	Shear Modulus (G_{12})	Poisson's ratio (ν_{12})
60%	38.6 GPa	8.27 GPa	4.14 GPa	0.3

A. Steps to analyze composite plate

- Determine the reduced stiffness matrix $[Q]$ for each ply (eq. 2) using the properties of composite material (table-1)
- Determine the transformed reduced stiffness matrix $[\bar{Q}]$ for each ply (eq. 2) using $[Q]$ and angle of orientation.
- Determine the coordinate of the top and bottom surface of the laminate (eq.5)
- Determine the laminate stiffness ABD matrix (eq. 8) using $[\bar{Q}]$ and coordinate of the laminate.
- Determine the mid plane strain and curvature by substituting the value of ABD matrix and applied force and moment (eq. 9).
- Determine the global stress and strain in each ply (eq.10, eq. 11) using mid plane strain and curvature and location of each ply.

4. Result and Discussion

The stress analysis in composite plate subjected to thermo mechanical load for different layer orientation, thickness and number of lamina is computed by using Classical lamination theory method (CLT). CLT is solved by developing MATLAB program and by using ANSYS software. The lamina is arranged anti symmetrically in the laminate. The lamination angle is

varied from 0° to 90°. Since magnitude of stress at 0° and 90° lamination angle are almost same. The influence of lamination angle and thickness on the stress distribution in composite plate subjected to thermal load only and combined normal force and bending moment in x direction is also studied. Minimum stress ratio for mechanical load and thermo mechanical load is obtained at lamination angle [+30] and [+60] respectively (table 2 & 3). Whereas for thermal load stress ratio remain constant for different lamination angle. Therefore the optimum lamination angle for combined mechanical load is [+30] and for thermo mechanical load is [+60]. Table 2 and 3 shows the comparison of stress ratio (SR) obtained from Classical Lamination Theory (CLT) with FEA (ANSYS) respectively. Both the result shows good agreement with each other with minor percentage error (3% -5%).

Fig 4, 5 and 6 shows the effect of lamination angle on stress distribution in composite plate subjected to mechanical load, thermal load and thermo mechanical load respectively. Optimum lamination angle for mechanical load is [+30], [+45] and [+35] for stress along x axis, y axis and shear stress respectively. Optimum lamination angle for thermal load and thermo mechanical load is [+30], [+60] and [+45] for stress along x axis, y axis and shear stress respectively. And all the stresses are zero for 0° and 90° lamination angle. Fig 7, 8 and 9 shows the variation of stress with laminate thickness (z/h) for lamination angle [+0]as, [+90]as, [+30]as, [+60]as and [+45]as for mechanical load, thermal load and thermo mechanical load respectively.

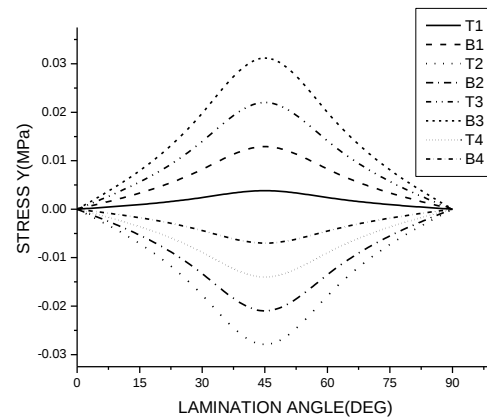
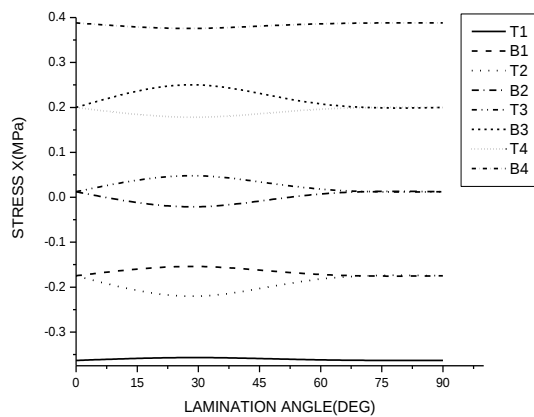
Fig10 shows that stress ratio decreases with the increase of a/h ratio for different lamination angle and stress ratio for mechanical load is greater than that of thermo mechanical load. Stress ratio (SR) for thermal load is constant. Fig 11 shows that optimum stress ratio is obtained at lamination angle of [+30] for mechanical load and [+60] for thermo mechanical load for different a/h ratio. Fig 12 shows the variation of stress ratio with a/h ratio for different lamination angle for mechanical and thermo mechanical load. Fig 13 shows that stress ratio decreases with the increase of the number of lamina.

Table 2 Comparison of stress ratio (SR) obtained from Classical Lamination Theory with FEA (ANSYS) for mechanical load.

MECHANICAL LOAD											
STRESS RATIO		CLT					ANSYS				
LAMINA	a/h	STACKING SEQUENCE					STACKING SEQUENCE				
		[0]s	[30]s	[45]s	[60]s	[90]s	[0]s	[30]s	[45]s	[60]s	[90]s
4	5	1.069	1.056	1.061	1.067	1.069	1.12245	1.108485	1.113735	1.120245	1.12245
	10	1.034	1.027	1.030	1.033	1.034	1.085595	1.07877	1.081395	1.084545	1.085595
	20	1.017	1.014	1.015	1.016	1.017	1.06764	1.06428	1.06554	1.067115	1.06764
8	5	1.1429	1.1293	1.1345	1.1407	1.1429	1.200045	1.185765	1.191225	1.197735	1.200045
	10	1.1053	1.0626	1.0651	1.068	1.1053	1.160565	1.11573	1.118355	1.1214	1.160565
	20	1.0339	1.0308	1.032	1.0334	1.0339	1.085595	1.08234	1.0836	1.08507	1.085595
12	5	1.2222	1.2079	1.2134	1.22	1.2222	1.28331	1.268295	1.27407	1.281	1.28331
	10	1.1053	1.0988	1.1013	1.1043	1.1053	1.160565	1.15374	1.156365	1.159515	1.160565
	20	1.0513	1.0482	1.0494	1.0508	1.0513	1.103865	1.10061	1.10187	1.10334	1.103865

Table 3 Comparison of stress ratio (SR) obtained from Classical Lamination Theory with FEA (ANSYS) for thermo mechanical load.

THERMO MECHANICAL LOAD											
STRESS RATIO		CLT					ANSYS				
LAMINA	a/h	STACKING SEQUENCE					STACKING SEQUENCE				
		[0]s	[30]s	[45]s	[60]s	[90]s	[0]s	[30]s	[45]s	[60]s	[90]s
4	5	1.069	0.969	0.959	0.9029	1.069	1.12245	1.01745	1.00695	0.948045	1.12245
	10	1.0339	0.8948	0.8668	0.753	1.0339	1.085595	0.93954	0.91014	0.79065	1.085595
	20	1.0168	0.7748	0.7284	0.6722	1.0168	1.06764	0.81354	0.76482	0.70581	1.06764
8	5	1.1429	0.9982	0.9962	0.9872	1.1429	1.200045	1.04811	1.04601	1.03656	1.200045
	10	1.069	0.9816	0.9734	0.9382	1.069	1.12245	1.03068	1.02207	0.98511	1.12245
	20	1.0339	0.9358	0.9162	0.8574	1.0339	1.085595	0.98259	0.96201	0.90027	1.085595
12	5	1.2222	1.0031	1.0026	1.0035	1.2222	1.28331	1.053255	1.05273	1.053675	1.28331
	10	1.1053	0.9987	0.9956	0.9845	1.1053	1.160565	1.048635	1.04538	1.033725	1.160565
	20	1.0513	0.9776	0.9675	0.9354	1.0513	1.103865	1.02648	1.015875	0.98217	1.103865



List of symbols used

σ	Stress	G	Shear modulus
ϵ	Strain	ν	Poisson's ratio
θ	Lamination angle(deg)	T1	Top of layer 1
γ	Shear strain at plane	B1	Bottom of layer1
τ	Shear stress at plane	T2	Top of layer 2
E	Young's Modulus	B2	Bottom of layer 2
h	Total laminate thickness	T3	Top of layer 3
z	thickness	B3	Bottom of layer3
a	Plate length	T4	Top of layer 4

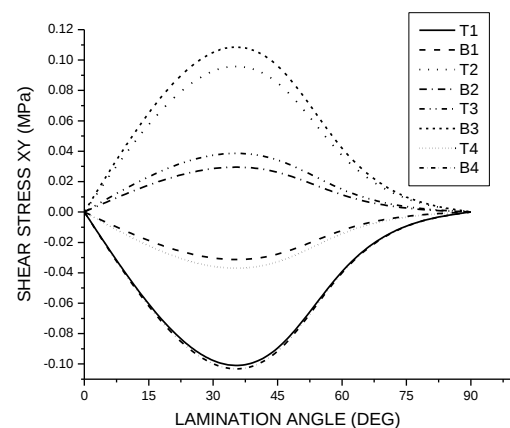


Fig. 4 Composite plate subjected to combined mechanical load ($N_x = 1000 \text{ N} - m$, $M_x = 400 \text{ N} - m$ and $a/h = 5$) a) stress in x direction b) stress in y direction c) Shear stress XY for different lamination angle.

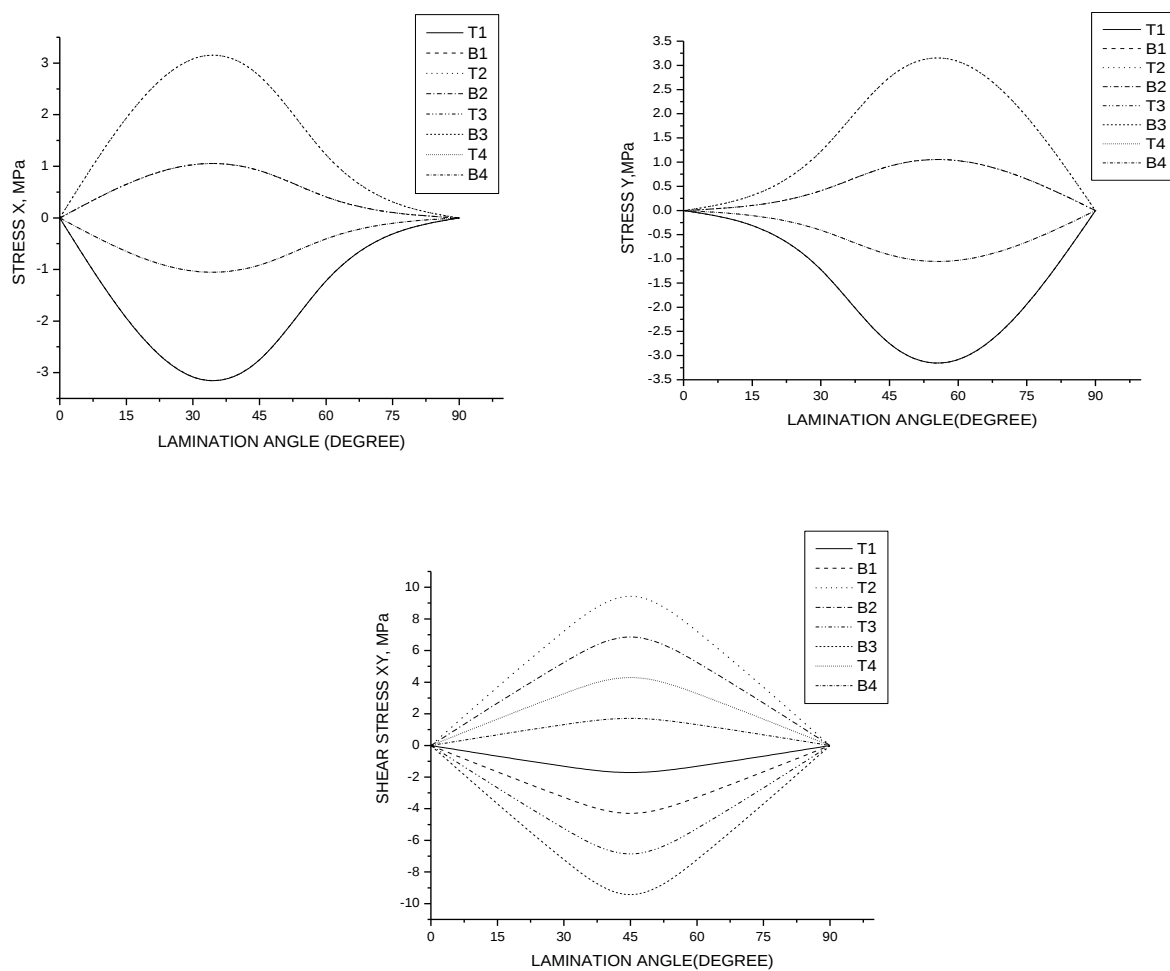


Fig. 5 Composite plate subjected to thermal load ($\Delta T = 75^{\circ}\text{C}$) a) stress in x direction b) stress in y direction c) Shear stress XY for different lamination angle.

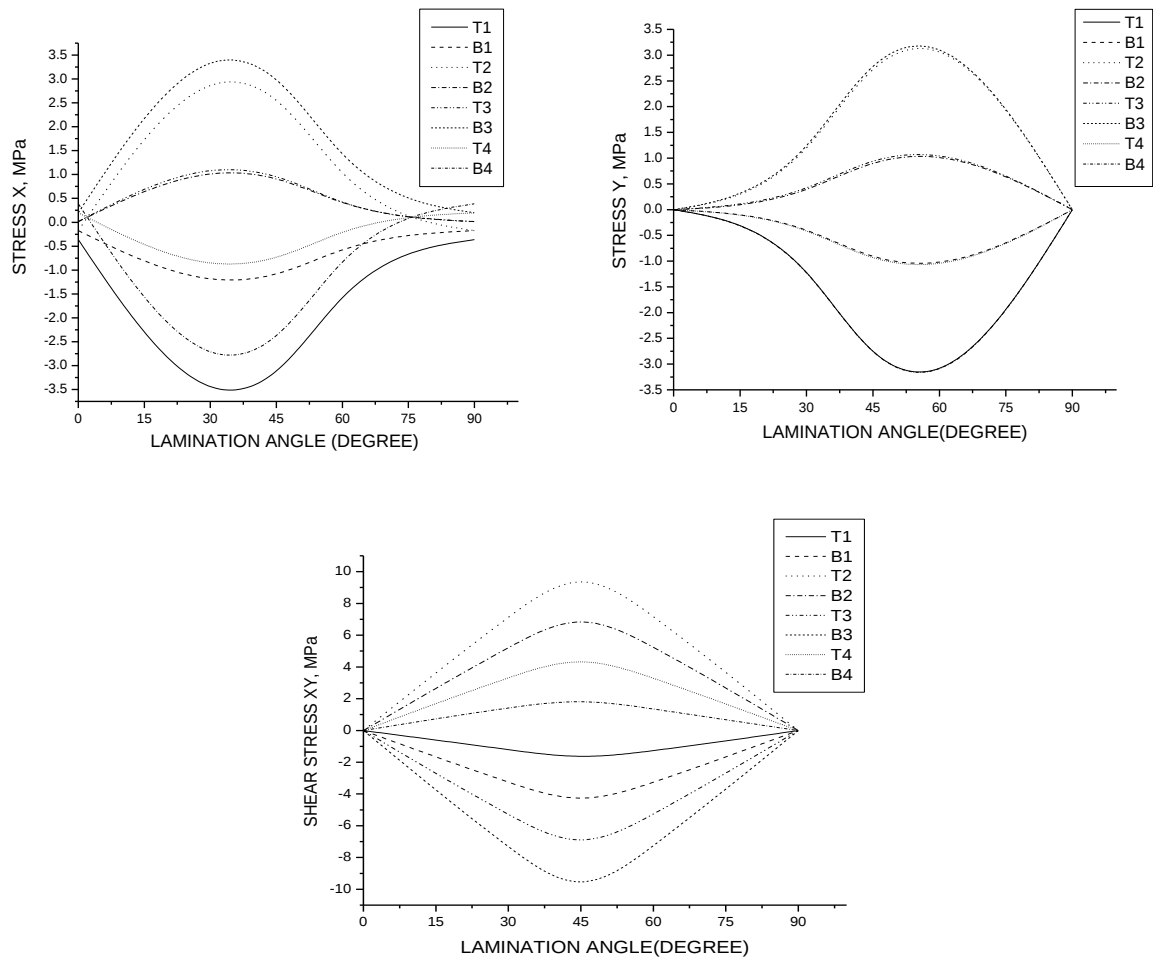


Fig. 6 Composite plate subjected to Thermo mechanical load ($N_x = 1000 \text{ N} - m$, $M_x = 400 \text{ N} - m$, $\Delta T = 75^\circ\text{C}$ and $a/h = 5$) a) stress in x direction b) stress in y direction c) Shear stress XY for different lamination angle.

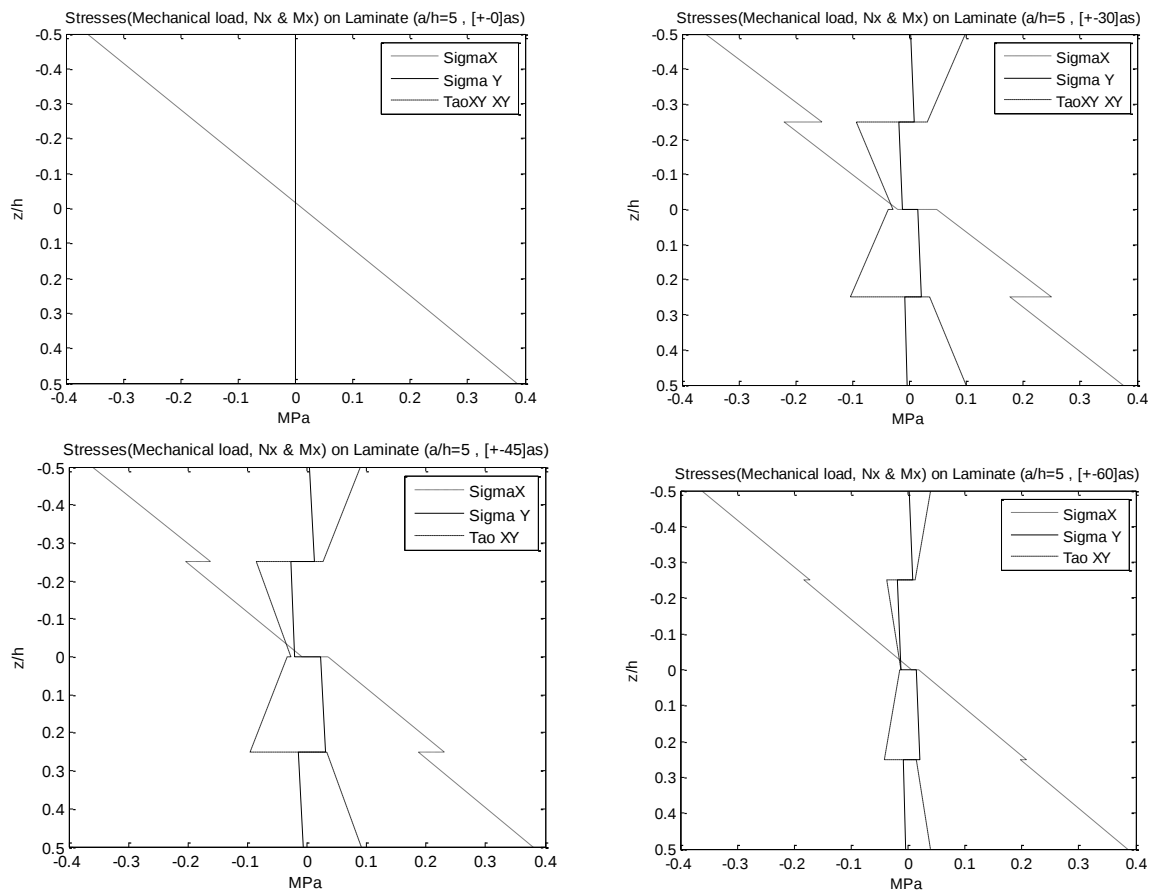


Fig. 7 Variation of stress with laminate thickness (z/h) for lamination angle (a) $[+0]_{as}$ & $[+90]_{as}$ (b) $[+30]_{as}$ (c) $[+45]_{as}$ and (d) $[+60]_{as}$ for combined mechanical load ($N_x = 1000 \text{ N} - m$, $M_x = 400 \text{ N} - m$ and $a/h = 5$)

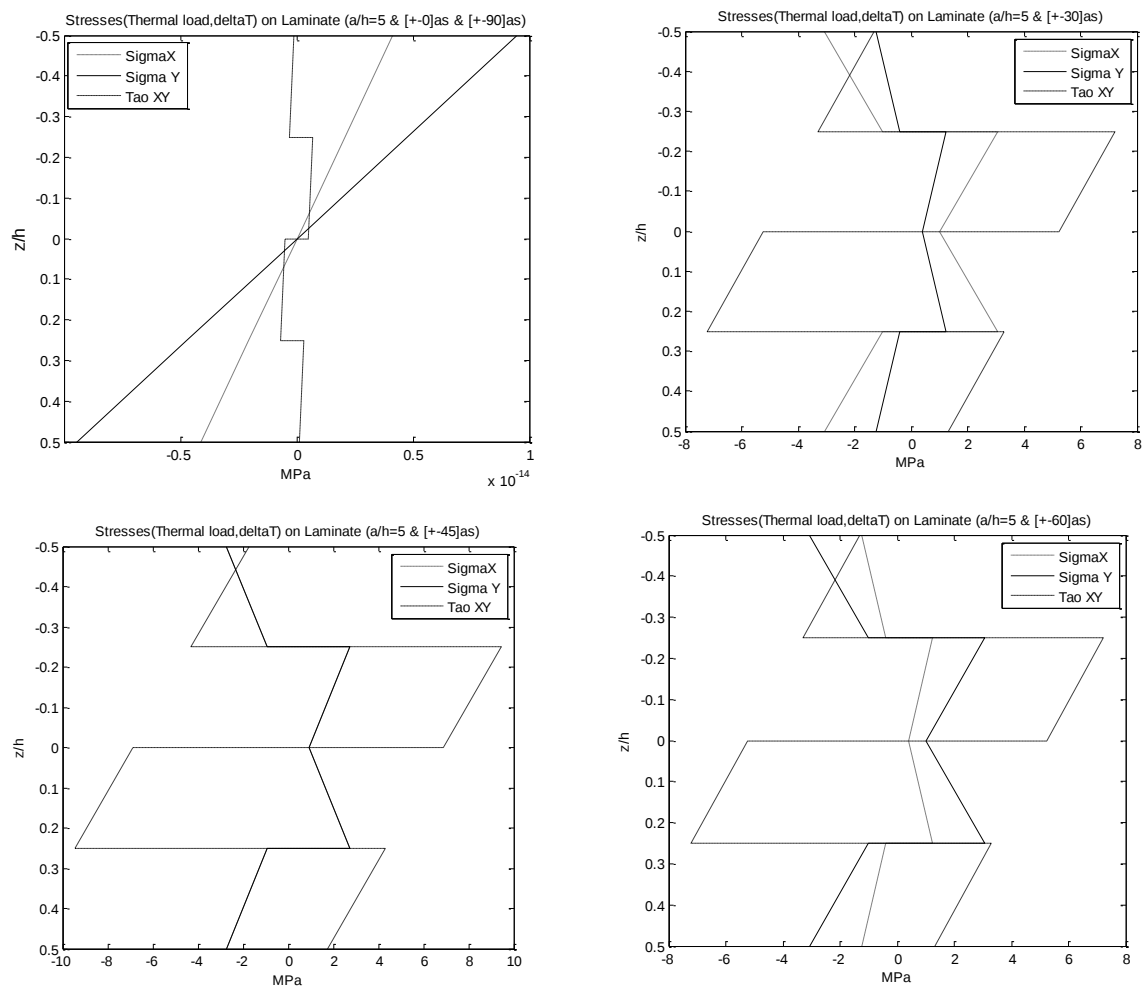


Fig. 8 Variation of stress with laminate thickness (z/h) for lamination angle (a) [+0] as & [+90] as (b) [+30] as (c) [+45] as and (d) [+60] as for thermal load ($\Delta T = 75$ and $a/h = 5$)

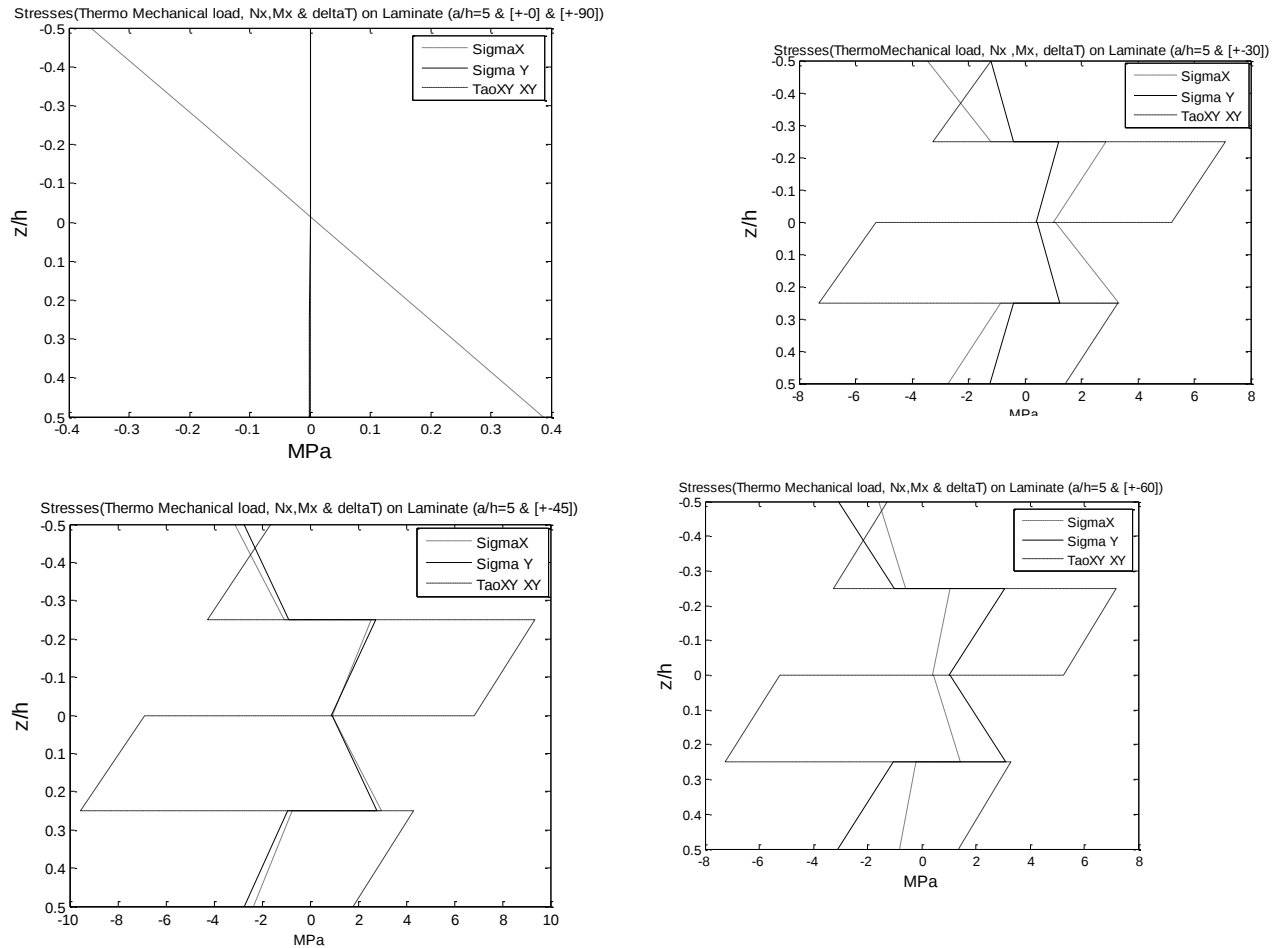


Fig. 9 Variation of stress with laminate thickness (z/h) for lamination angle (a) [+0] as & [+90] as (b) [+30] as (c) [+45] as and (d) [+60] as for thermo mechanical load ($N_x = 1000 \text{ N-m}$, $M_x = 400 \text{ N-m}$, $\Delta T = 75$ and $a/h = 5$)

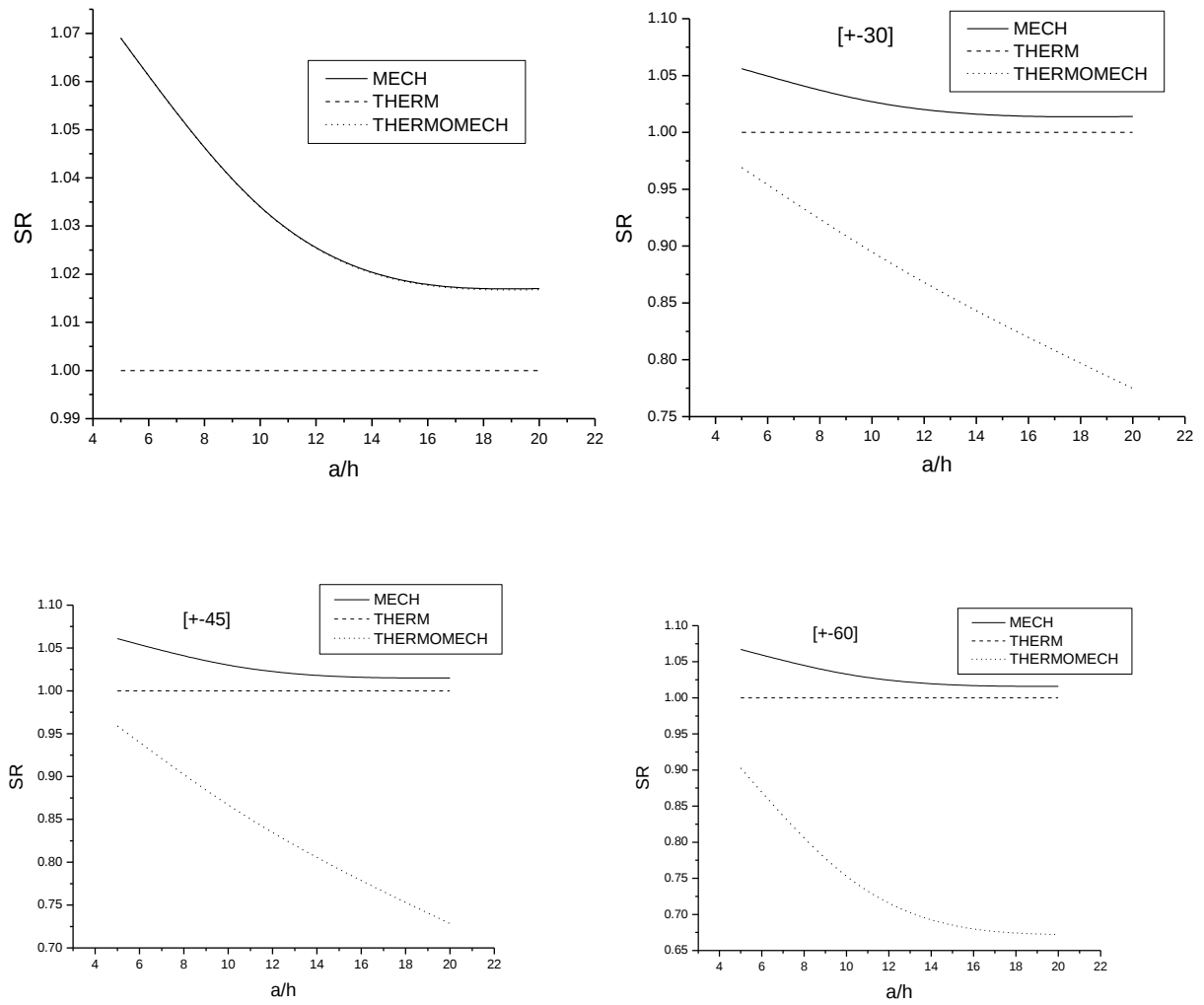


Fig. 10 Comparison of the variation of stress ratio (SR) with thickness ratio (a/h) for mechanical, thermal and thermo mechanical load for different lamination angle.

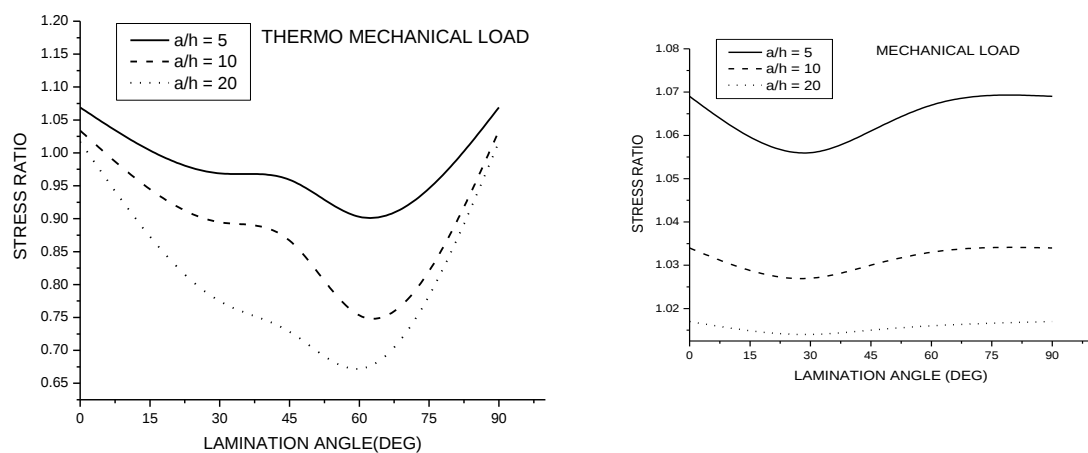


Fig. 11: variation of stress ratio with lamination angle for different a/h ratio for mechanical and thermo mechanical load.

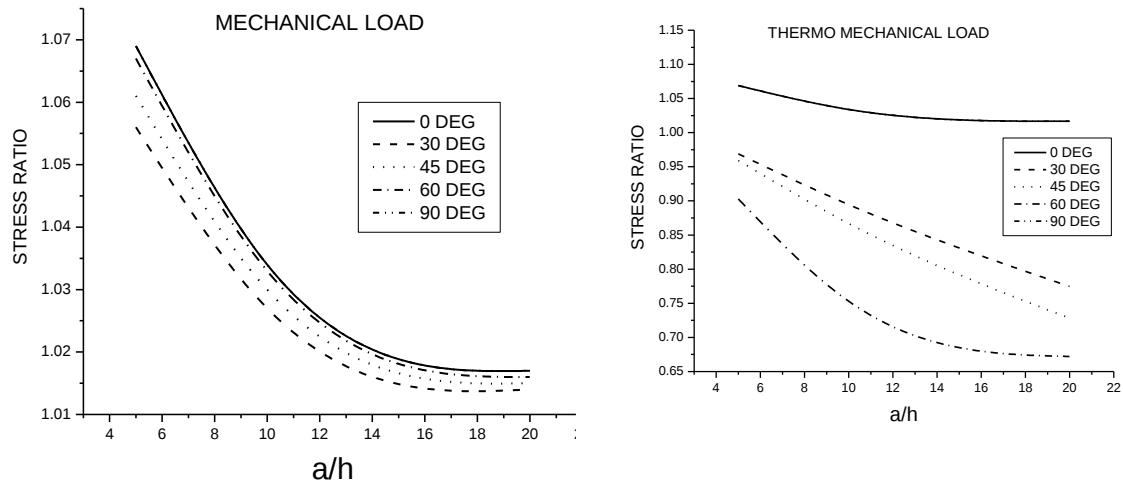
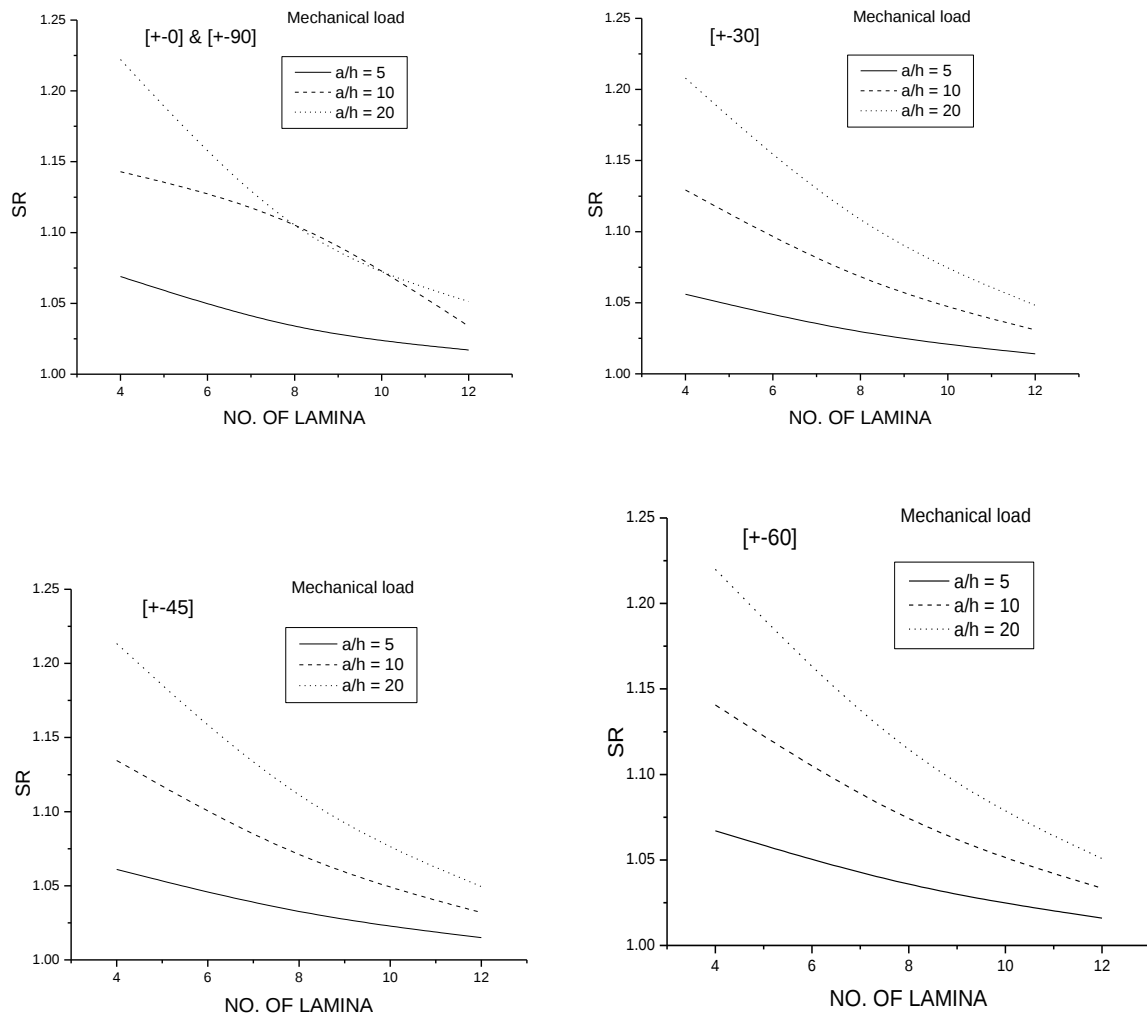


Fig. 12: variation of stress ratio with a/h ratio for different lamination angle for mechanical and thermo mechanical load.



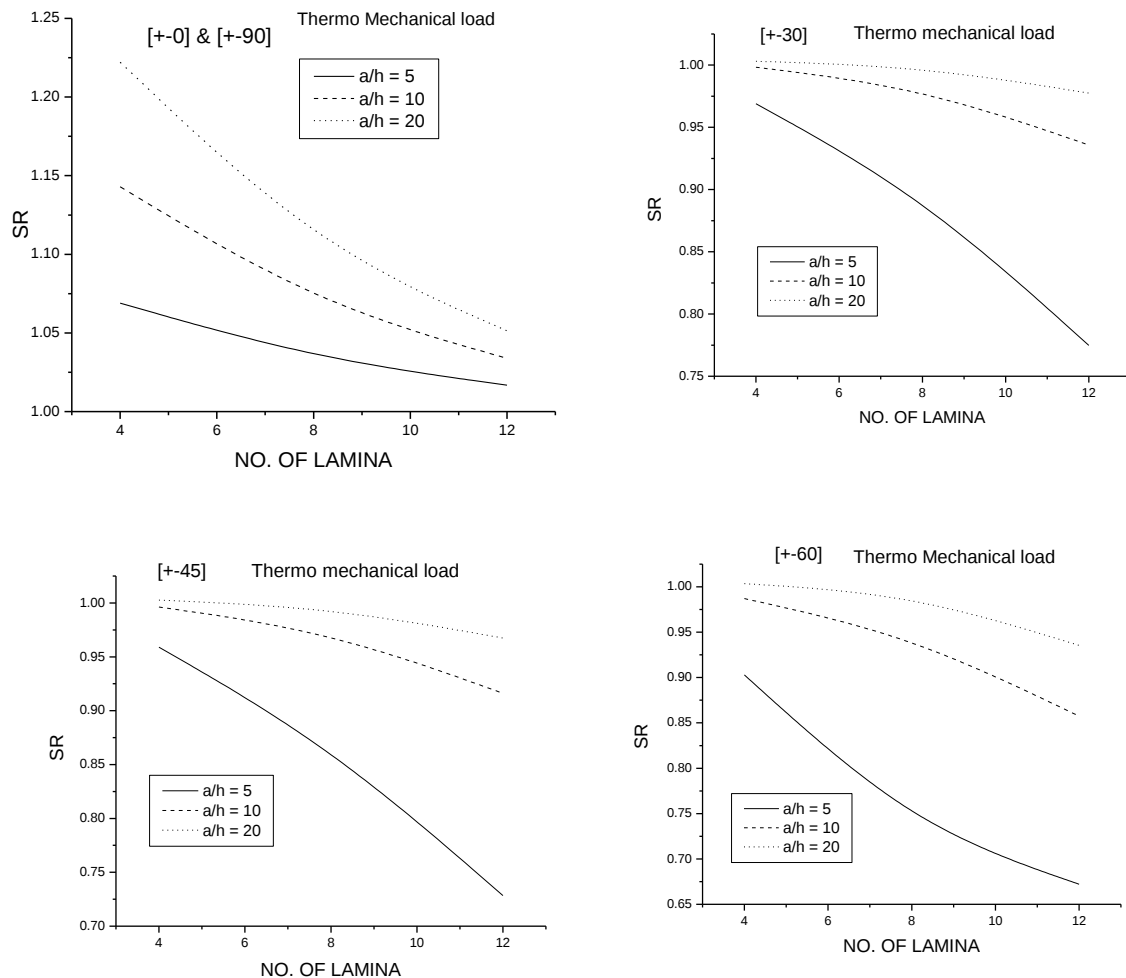


Fig. 13 variation of stress ratio with no. of lamina for different lamination angle for mechanical and thermo mechanical load.

5. Conclusion

Composite plate subjected to thermo mechanical load find wide application in aerospace, military, medical, automobile sector etc. This paper present a model to analyze a composite plate subjected to different loading conditions. Classical laminate theory is used to solve the model. It is concluded from the analysis that the optimum lamination angle for combined mechanical load is $[+30]$ and for thermo mechanical load is $[+60]$ at which the stress ratio is minimum. The stress ratio (SR) decreases with the increase in a/h ratio and number of lamina. The problem is solved by using both CLT method and FEA. The result of both the method shows good agreement with each other with minor percentage error (3% -5%). Finite element tool (ANSYS) may be a more appropriate method to find the approximate solution to the stress and strain analysis of complex composite plate problem where it is not possible to solve the mathematical model.

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