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VARIANTS OF THE TRAVELING SALESMAN PROBLEM

PATTERSON Mike
Dillard College of Business, Midwestern State University, USA

FRIESEN Daniel
University of North Texas, Dallas Campus, USA

Abstract:
This paper includes an introduction to the concept of spreadsheet optimization and modeling as it specifically applies to combinatorial problems. One of the best known of the classic combinatorial problems is the "Traveling Salesman Problem" (TSP). The classic Traveling Salesman Problem has the objective of minimizing some value, usually distance, while defining a sequence of locations where each is visited once. An additional requirement is that the tour ends in the same location where the tour started. Variants of the classic Traveling Salesman Problem are developed including the Bottleneck TSP and the Variation Bottleneck TSP.

Key words: *Optimization, Spreadsheet, Traveling Salesman, Combinatorial*

1. Introduction

This paper includes a general introduction to the concept of spreadsheet optimization and modeling as it specifically applies to combinatorial problems. One of the best known of the classic combinatorial problems is the "Traveling Salesman Problem" (TSP). In the definitive The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization (1985) Lawler et. al. introduce the Traveling Salesman Problem (TSP) as "a salesman, starting from his home city, is to visit exactly once each city on a given list and then return home, it is plausible for him to select the order in which he visits the cities so that the total of the distances traveled in his tour is as small as possible". Cummings (June 2000) traces the roots of the problem to Karl Menger, who defined the problem, which he referred to as the "Messenger Problem" published "Das botenproblem" in Ergenbnisse eines Mathematischen Kolloquiums in 1932. In tracing the historical roots of the TSP Lawler et. al. (1985) credit Merrill Flood as being responsible for publicizing the conceptual basis for the TSP in the operations community. Robinson's paper "On the Hamiltonian Game" (1949) is frequently cited as a seminal work. Lawler et. al. (1985) cite "Solution of a large-scale traveling-salesman problem by Dantzig, Fulkerson and Johnson (1954) as a critical event in the development of the TSP and combinatorial optimization in general. Dantzig, one of the giants in operations research, is frequently referred to as the “Father of Linear Programming” (Albers and Reid, 1986). Among the more popular combinatorial problems which have been subject to optimization include permutations, sequencing and scheduling, the minimal spanning tree, and the traveling salesman problem. Tuza (2001) provides a particularly comprehensive set of challenging and unsolved combinatorial problems.

2. General Model Description

The classic Traveling Salesman Problem has the objective of minimizing some value, usually distance, while defining a sequence of locations where each is visited once. An additional requirement is that the tour ends in the same location where the tour started. The spreadsheet model utilized in this paper is patterned after a model developed by Patterson and Harmel (2003). The initial base problem is defined in Table 1, cells D4:R14. There are fourteen locations to be visited on the tour. The upper portion of Table 1 displays the direct distance between each of the locations. The lower portion of the spreadsheet (Cells D20:E34) display the distance between the cities when they are in the initial (i.e. alphabetical/random) sequence. The sum of the initial tour distance (5508) is also displayed.

Excel Solver was utilized as the software for development of this model. Solver is an add-in software tool in Excel for modeling general purpose linear and non-linear optimization problems. It was developed by Frontline Systems. Solver provides an option known as the “alldifferent” constraint, which is particularly useful in combinatorial problems, such as the traveling salesman problem. (Frontline Systems, n.d.)

Table 2 displays the lower portion of the spreadsheet with the formula view. Figure 1 displays the Solver parameters for the classical traveling salesman problem. The objective is to minimize the total distance traveled. The constraints include that each city must be visited once and that the tour must end in the same city which it began. The alldifferent constraint efficiently defines these constraints. The standard evolutionary engine is required in order to utilize the alldifferent option required to satisfy the constraints. Figure 2 displays a visual of the problem. Table 3 displays the Solver solution to the classic Traveling Salesman Problem. The visual solution is shown in Figure 3. As indicated the total distance traveled for the suggested trip is 2,549 miles.

Table 1: Direct Distance Traveling Salesman Problem

1\A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
2			To	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3		City		Abilene	Amarillo	Austin	Corpus Christi	Dallas	El Paso	Houston	Laredo	Lubbock	McAllen	Odessa	San Antonio	Waco	Wichita Falls
4	From	1	Abilene	0	266	213	387	180	439	348	373	162	480	304	244	183	141
5		2	Amarillo	266	0	478	636	361	418	596	609	119	728	255	493	423	225
6		3	Austin	213	478	0	192	192	573	162	232	368	300	334	79	102	283
7		4	Corpus Christi	387	636	192	0	377	691	207	141	526	152	432	143	287	474
8		5	Dallas	180	361	192	377	0	617	238	424	322	491	347	271	91	136
9		6	El Paso	439	418	573	691	617	0	730	602	344	745	274	548	610	552
10		7	Houston	348	596	162	207	238	730	0	311	510	345	494	197	180	371
11		8	Laredo	373	609	232	141	424	602	311	0	498	143	422	154	334	490
12		9	Lubbock	162	119	368	526	322	344	510	498	0	618	137	382	345	208
13		10	McAllen	480	728	300	152	491	745	345	143	618	0	565	236	401	572
14		11	Odessa	304	255	334	432	347	274	494	422	137	565	0	336	340	293
15		12	San Antonio	244	493	79	143	271	548	197	154	382	236	336	0	181	336
16		13	Waco	183	423	102	287	91	610	180	334	345	401	340	181	0	198
17		14	Wichita Falls	141	225	283	474	136	552	371	490	208	572	293	336	198	0
18																	
19			From\To														
20		1	Abilene	Miles													
21		2	Amarillo	266													
22		3	Austin	478													
23		4	Corpus Christi	192													
24		5	Dallas	377													
25		6	El Paso	617													
26		7	Houston	730													
27		8	Laredo	311													
28		9	Lubbock	498													
29		10	McAllen	618													
30		11	Odessa	565													
31		12	San Antonio	336													
32		13	Waco	181													
33		14	Wichita Falls	198													
34		1	Abilene	141													
35			Total	5508													

Table 2: Formula View Initial Sequence Traveling Salesman Problem

1\A	B	C	D	E	F
19			From\To		
20		1	=VLOOKUP(C20,citytab,2)		
21		2	=VLOOKUP(C21,citytab,2)	=INDEX(mileage,C20,C21)	=ABS(E21-E37)
22		3	=VLOOKUP(C22,citytab,2)	=INDEX(mileage,C21,C22)	=ABS(E22-E37)
23		4	=VLOOKUP(C23,citytab,2)	=INDEX(mileage,C22,C23)	=ABS(E23-E37)
24		5	=VLOOKUP(C24,citytab,2)	=INDEX(mileage,C23,C24)	=ABS(E24-E37)
25		6	=VLOOKUP(C25,citytab,2)	=INDEX(mileage,C24,C25)	=ABS(E25-E37)
26		7	=VLOOKUP(C26,citytab,2)	=INDEX(mileage,C25,C26)	=ABS(E26-E37)
27		8	=VLOOKUP(C27,citytab,2)	=INDEX(mileage,C26,C27)	=ABS(E27-E37)
28		9	=VLOOKUP(C28,citytab,2)	=INDEX(mileage,C27,C28)	=ABS(E28-E37)
29		10	=VLOOKUP(C29,citytab,2)	=INDEX(mileage,C28,C29)	=ABS(E29-E37)
30		11	=VLOOKUP(C30,citytab,2)	=INDEX(mileage,C29,C30)	=ABS(E30-E37)
31		12	=VLOOKUP(C31,citytab,2)	=INDEX(mileage,C30,C31)	=ABS(E31-E37)
32		13	=VLOOKUP(C32,citytab,2)	=INDEX(mileage,C31,C32)	=ABS(E32-E37)
33		14	=VLOOKUP(C33,citytab,2)	=INDEX(mileage,C32,C33)	=ABS(E33-E37)
34		=C20	=D20	=INDEX(mileage,C33,C34)	=ABS(E34-E37)
35			Total	=SUM(E21:E34)	=SUM(F21:F34)

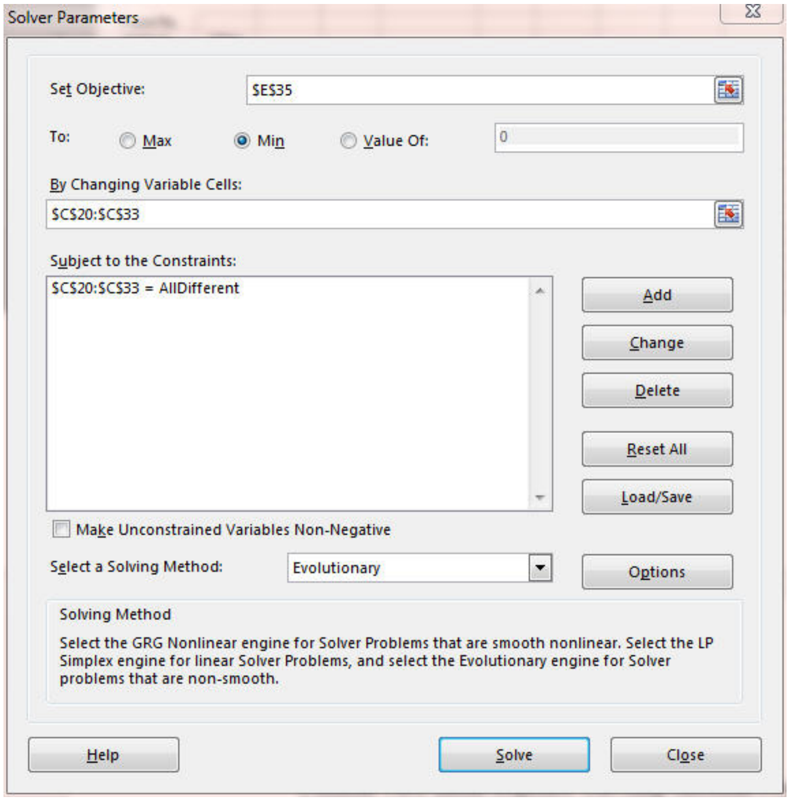

Figure 1: Solver Parameters Traveling Salesman Problem

Figure 2: Texas Map of Traveling Salesman Problem

As stated previously, the primary purpose of this study is to present two alternative models to the TSP. The first variation to the classic Traveling Salesman Problem is the Bottleneck Traveling Salesman Problem (BTSP). Conceptually, the BTSP is quite similar to the TSP. The constraints are identical in that each location must be visited once. In other words, each location must be entered once and exited once. Also, the trip must end in the same city where it began. The only real formulation difference is that rather than minimizing the total distance of the trip, the objective is to minimize the distance of the single longest intercity trip (Lawler, et. al., 1985). With this one change to the objective function, the classic TSP is converted to a "minimax" problem where the objective is to minimize the maximum single distance between two cities. The second alteration to the TSP is to modify the objective function to minimize the degree of variation around the mean intercity trip difference. In other words, to the extent it is possible, we desire to make each intercity trip as close as possible to the mean distance. Once again the constraints are identical to the TSP. The difference is in the statement of the objective function. In our formulation we chose to minimize the standard deviation of the distances for the selected tour sequence. Table 4 displays the complete initial spreadsheet for the all three new models. Additional statistical calculations displayed include the intercity maximum value, the mean intercity distance and the standard deviation for the intercity distances. Table 5 displays the formula view for relevant cells. In addition, column F displays the deviation from the mean for each intercity trip.

Table 3: Solution to Traveling Salesman Problem

1\A	B	C	D	E
19			From\To	
20		1	Abilene	Miles
21		3	Austin	213
22		12	San Antonio	79
23		8	Laredo	154
24		10	McAllen	143
25		4	Corpus Christi	152
26		7	Houston	207
27		13	Waco	180
28		5	Dallas	91
29		14	Wichita Falls	136
30		2	Amarillo	225
31		9	Lubbock	119
32		11	Odessa	137
33		6	El Paso	274
34		1	Abilene	439
35			Total	2549

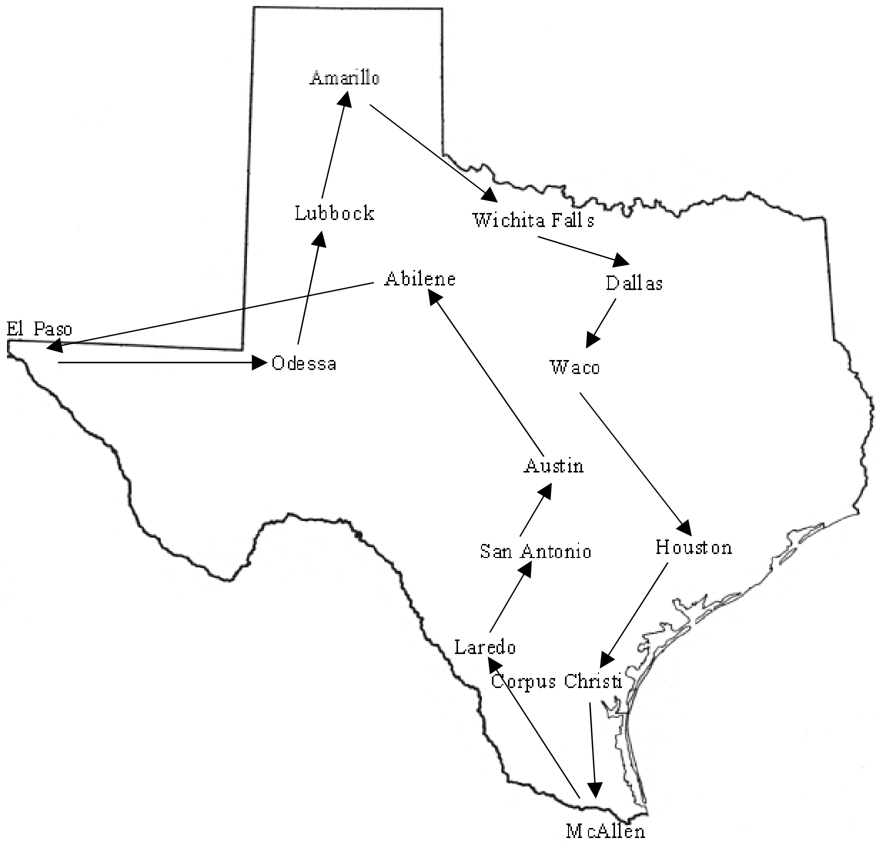

Figure 3: Texas Solution Map of Classic Traveling Salesman Problem

Table 4: Direct Distance Traveling Salesman Problem

1\A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
2			To	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3		City		Abilene	Amarillo	Austin	Corpus Christi	Dallas	El Paso	Houston	Laredo	Lubbock	McAllen	Odessa	San Antonio	Waco	Wichita Falls
4	From	1	Abilene	0	266	213	387	180	439	348	373	162	480	304	244	183	141
5		2	Amarillo	266	0	478	636	361	418	596	609	119	728	255	493	423	225
6		3	Austin	213	478	0	192	192	573	162	232	368	300	334	79	102	283
7		4	Corpus Christi	387	636	192	0	377	691	207	141	526	152	432	143	287	474
8		5	Dallas	180	361	192	377	0	617	238	424	322	491	347	271	91	136
9		6	El Paso	439	418	573	691	617	0	730	602	344	745	274	548	610	552
10		7	Houston	348	596	162	207	238	730	0	311	510	345	494	197	180	371
11		8	Laredo	373	609	232	141	424	602	311	0	498	143	422	154	334	490
12		9	Lubbock	162	119	368	526	322	344	510	498	0	618	137	382	345	208
13		10	McAllen	480	728	300	152	491	745	345	143	618	0	565	236	401	572
14		11	Odessa	304	255	334	432	347	274	494	422	137	565	0	336	340	293
15		12	San Antonio	244	493	79	143	271	548	197	154	382	236	336	0	181	336
16		13	Waco	183	423	102	287	91	610	180	334	345	401	340	181	0	198
17		14	Wichita Falls	141	225	283	474	136	552	371	490	208	572	293	336	198	0
18																	
19			From\To														
20		1	Abilene	Miles	Dev.												
21		2	Amarillo	266	127.429												
22		3	Austin	478	84.5714												
23		4	Corpus Christi	192	201.429												
24		5	Dallas	377	16.4286												
25		6	El Paso	617	223.571												
26		7	Houston	730	336.571												
27		8	Laredo	311	82.4286												
28		9	Lubbock	498	104.571												
29		10	McAllen	618	224.571												
30		11	Odessa	565	171.571												
31		12	San Antonio	336	57.4286												
32		13	Waco	181	212.429												
33		14	Wichita Falls	198	195.429												
34		1	Abilene	141	252.429												
35			Total	5508	2290.86												
36			Intercity Max.	730													
37			Mean	393.43													
38			Std. Dev.	191.24													

Table 5: Formula View Initial Sequence Three Traveling Salesman Problem

1\A	B	C	D	E	F
19			From\To		
20		1	=VLOOKUP(C20,citytab,2)		
21		2	=VLOOKUP(C21,citytab,2)	=INDEX(mileage,C20,C21)	=ABS(E21-E37)
22		3	=VLOOKUP(C22,citytab,2)	=INDEX(mileage,C21,C22)	=ABS(E22-E37)
23		4	=VLOOKUP(C23,citytab,2)	=INDEX(mileage,C22,C23)	=ABS(E23-E37)
24		5	=VLOOKUP(C24,citytab,2)	=INDEX(mileage,C23,C24)	=ABS(E24-E37)
25		6	=VLOOKUP(C25,citytab,2)	=INDEX(mileage,C24,C25)	=ABS(E25-E37)
26		7	=VLOOKUP(C26,citytab,2)	=INDEX(mileage,C25,C26)	=ABS(E26-E37)
27		8	=VLOOKUP(C27,citytab,2)	=INDEX(mileage,C26,C27)	=ABS(E27-E37)
28		9	=VLOOKUP(C28,citytab,2)	=INDEX(mileage,C27,C28)	=ABS(E28-E37)
29		10	=VLOOKUP(C29,citytab,2)	=INDEX(mileage,C28,C29)	=ABS(E29-E37)
30		11	=VLOOKUP(C30,citytab,2)	=INDEX(mileage,C29,C30)	=ABS(E30-E37)
31		12	=VLOOKUP(C31,citytab,2)	=INDEX(mileage,C30,C31)	=ABS(E31-E37)
32		13	=VLOOKUP(C32,citytab,2)	=INDEX(mileage,C31,C32)	=ABS(E32-E37)
33		14	=VLOOKUP(C33,citytab,2)	=INDEX(mileage,C32,C33)	=ABS(E33-E37)
34		=C20	=D20	=INDEX(mileage,C33,C34)	=ABS(E34-E37)
35			Total	=SUM(E21:E34)	=SUM(F21:F34)
36			Intercity Max.	=MAX(E21:E34)	
37			Mean	=AVERAGE(E21:E34)	
38			Std. Dev.	=STDEV(E21:E34)	

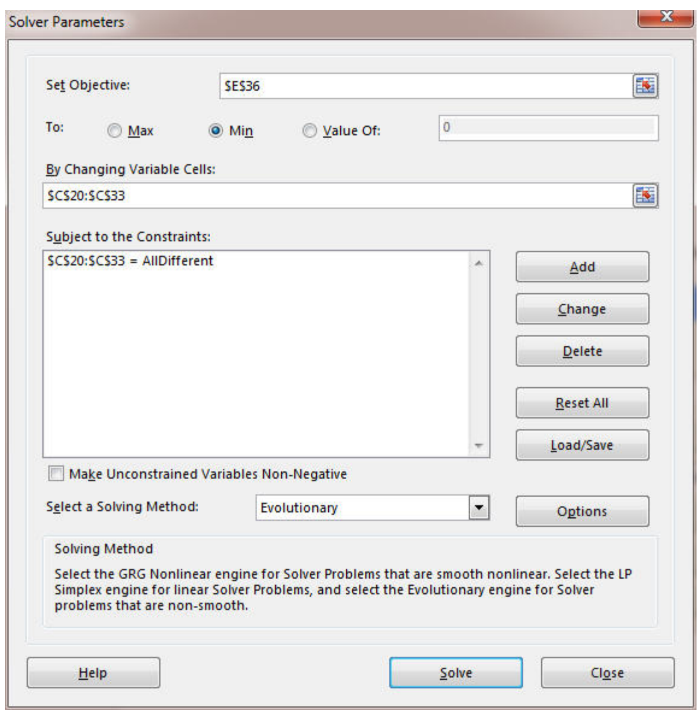

Figure 4: Solver Parameters Bottleneck Traveling Salesman Problem

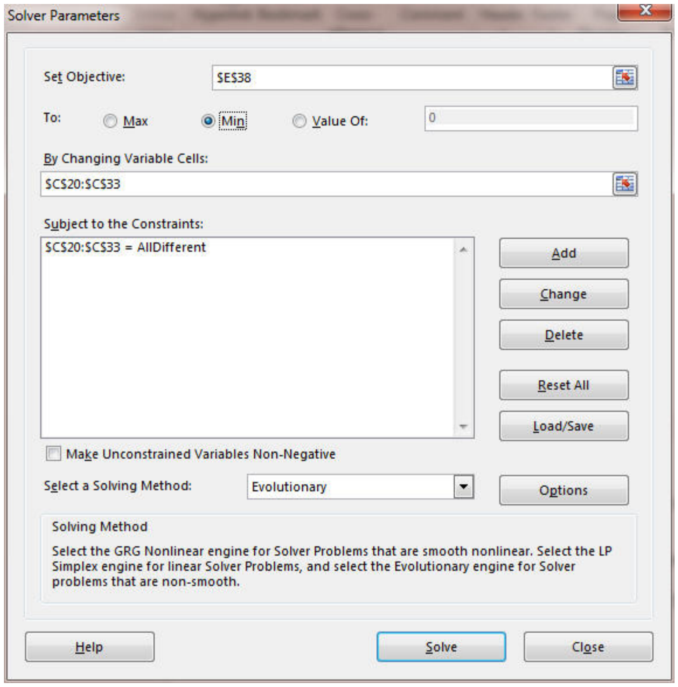

Figure 5: Solver Parameters Variation Traveling Salesman Problem

Table 7 displays a summary of the suggested tours. The first suggested tour is the baseline and could be considered as a random trip since it is in alphabetical order. For this baseline tour, the total distance is 5,508 miles, the bottleneck trip is 730 and the standard deviation is 191.2 miles. In comparison the classic TSP solution reduces the total trip distance by almost 3,000 miles to 2,549. The Bottleneck TSP solution results in the longest single intercity trip of 344 miles. Multiple runs indicate multiple optimal solutions with the same result of 344 miles which is the distance from Lubbock to El Paso. The Variation TSP, which minimizes the standard deviation, results in a tour with a standard deviation of 26.69. Figure 5 displays the visual map of the suggested Bottleneck tour. Figure 6 displays the suggested Variation Bottleneck tour, which attempts to provide the most uniform intercity distance between cities.

Table 7: Summary of Suggested Tours Traveling Salesman Problem

	Alpabetical				Classic		
	From\To				From\To		
1	Abilene	Miles	Dev.	7	Houston	Miles	Dev.
2	Amarillo	266	127.43	4	Corpus Christi	207	24.93
3	Austin	478	84.571	10	McAllen	152	30.07
4	Corpus Christi	192	201.43	8	Laredo	143	39.07
5	Dallas	377	16.429	12	San Antonio	154	28.07
6	El Paso	617	223.57	3	Austin	79	103.1
7	Houston	730	336.57	1	Abilene	213	30.93
8	Laredo	311	82.429	6	El Paso	439	256.9
9	Lubbock	498	104.57	11	Odessa	274	91.93
10	McAllen	618	224.57	9	Lubbock	137	45.07
11	Odessa	565	171.57	2	Amarillo	119	63.07
12	San Antonio	336	57.429	14	Wichita Falls	225	42.93
13	Waco	181	212.43	5	Dallas	136	46.07
14	Wichita Falls	198	195.43	13	Waco	91	91.07
1	Abilene	141	252.43	7	Houston	180	2.071
	Total	5508			Total	2549	
	Intercity	730			Intercity	439	
	Mean	393.4			Mean	182.1	
	Std. Dev.	191.2			Std. Dev.	91.21	
	Bottleneck				Variation		
	From\To				From\To		
11	Odessa			12	San Antonio		
6	El Paso	274	43.714	14	Wichita Falls	336	27.21
9	Lubbock	344	113.71	7	Houston	371	7.786
14	Wichita Falls	208	22.286	10	McAllen	345	18.21
2	Amarillo	225	5.2857	13	Waco	401	37.79
1	Abilene	266	35.714	8	Laredo	334	29.21
5	Dallas	180	50.286	1	Abilene	373	9.786
13	Waco	91	139.29	4	Corpus Christi	387	23.79
4	Corpus Christi	287	56.714	5	Dallas	377	13.79
7	Houston	207	23.286	2	Amarillo	361	2.214
12	San Antonio	197	33.286	6	El Paso	418	54.79
10	McAllen	236	5.7143	9	Lubbock	344	19.21
8	Laredo	143	87.286	3	Austin	368	4.786
3	Austin	232	1.7143	11	Odessa	334	29.21
11	Odessa	334	103.71	12	San Antonio	336	27.21
	Total	3224			Total	5085	
	Intercity	344			Intercity	418	
	Mean	230.3			Mean	363.2	
	Std. Dev.	68.98			Std. Dev.	26.69	

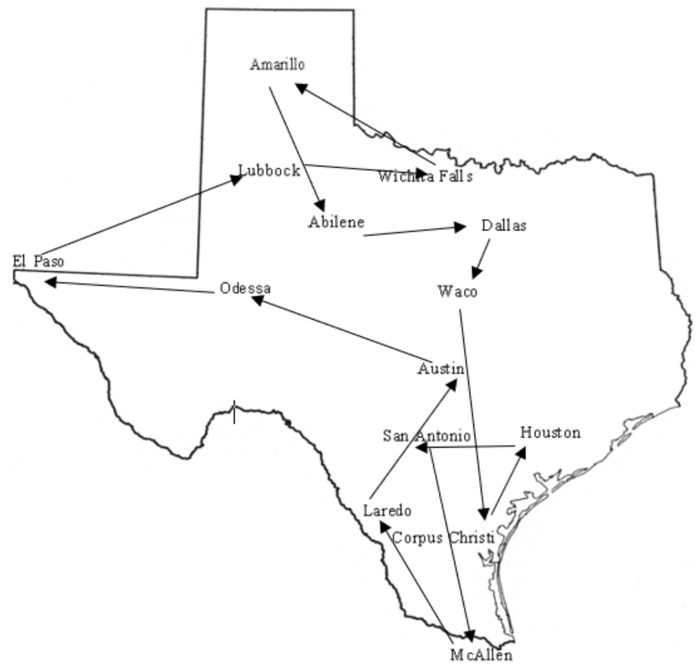

Figure 5: Texas Solution Map of Bottleneck Traveling Salesman Problem

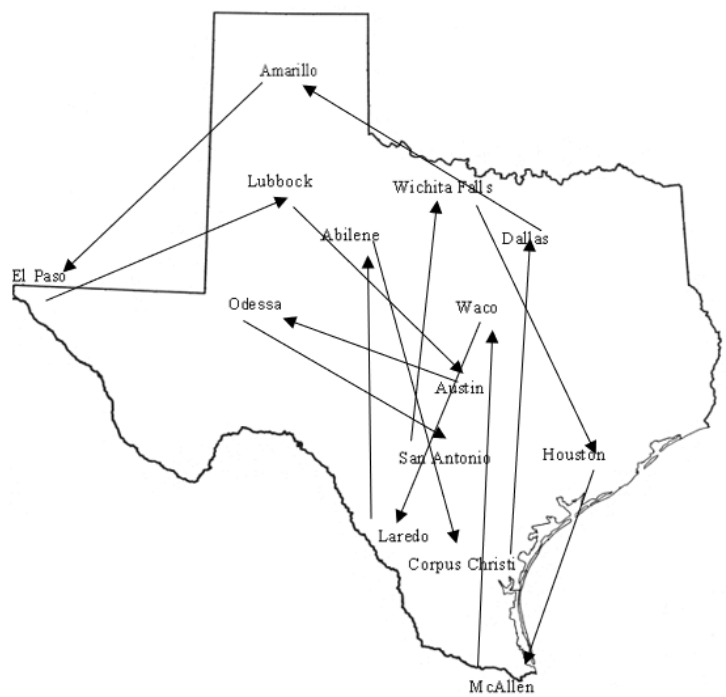

Figure 6: Texas Solution Map of Variation Traveling Salesman Problem

The presence of multiple optimal solutions to the BTSP led the authors to further explore the model formulation. Using the minimize objective function for the maximum intercity distance found in a tour efficiently located the bottleneck to be the trip between El Paso and Lubbock. With the current formulation, any tour which included this trip and satisfied the constraints would be considered optimal. The authors determined that the trip between El Paso and Lubbock could be determined to be mandatory by the inclusion of a constraint which required that the maximum intercity distance be required to be equal to the distance between these two cities, i.e. 344 miles. With this additional constraint, the objective function was then modified to minimize the total tour distance. Thus it accomplishes two objectives. First, the tour must include the route from Lubbock to El Paso. Secondly, the model identifies the shortest total tour distance, which includes the bottleneck route. Figure 7 displays the Solver parameters.

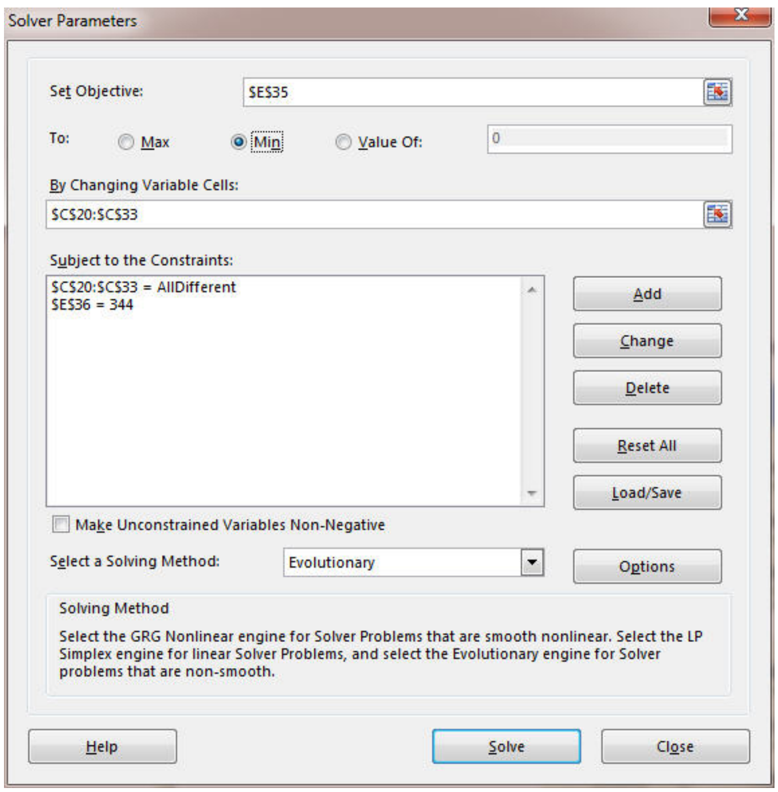

Figure 7: Solver Parameters Second Bottleneck Variation Traveling Salesman Problem

Table 8 displays the Solver solution to the newly formulated BTSP. This solution does include the bottleneck trip from Lubbock to El Paso. Thus, the bottleneck trip is reduced by 609 miles (3224-2615) in comparison to the original BTSP model formulation. The original classic TSP solution resulted in a total tour distance of 2,549 miles. The new formulation increases the total distance to 2,615, a difference of 66 miles. Figure 8 displays a map of the suggested tour.

Table 8: Summary of Suggested Tour Modified Bottlenecks Traveling Salesman Problem

	From\To		
14	Wichita Falls		
5	Dallas	136	50.79
13	Waco	91	95.79
7	Houston	180	6.786
4	Corpus Christi	207	20.21
10	McAllen	152	34.79
8	Laredo	143	43.79
12	San Antonio	154	32.79
3	Austin	79	107.8
1	Abilene	213	26.21
9	Lubbock	162	24.79
6	El Paso	344	157.2
11	Odessa	274	87.21
2	Amarillo	255	68.21
14	Wichita Falls	225	38.21
	Total	2615	794.6
	Intercity	344	
	Mean	186.786	
	Std. Dev.	72.1997	

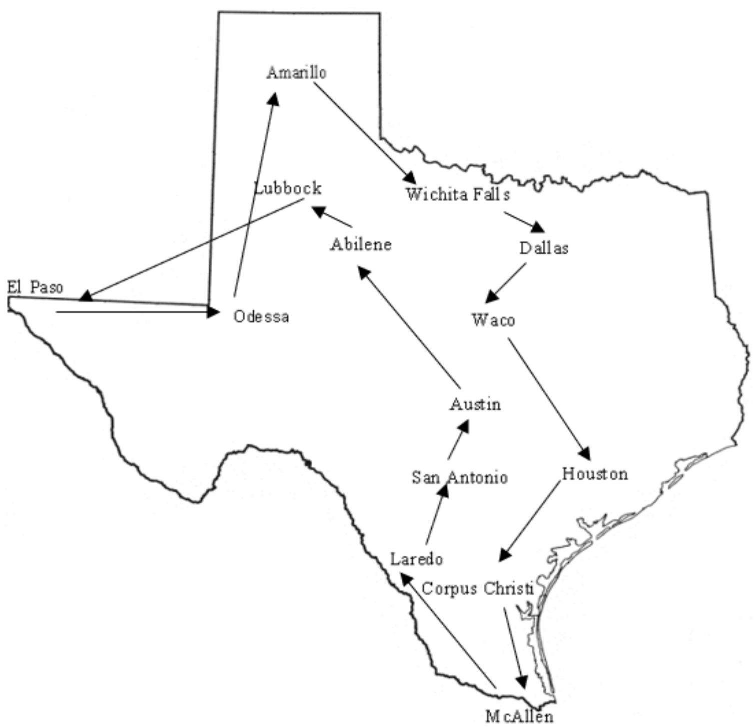

Figure 8: Texas Solution Map of Modified Bottleneck Traveling Salesman Problem

3. Summary and Conclusion

The purpose of the paper is to expand on the Traveling Salesman Problem and present a spreadsheet model which includes alternatives to the objective to minimizing the total distance of the tour. The additional objectives include minimizing the bottleneck distance and the degree of variation around the mean intercity trip difference. The Excel model formulation and solutions represent different approaches to the classic Traveling Salesman Problem.

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