



DOI 10.2478/sbe-2019-0016

SBE no. 14(1) 2019

VARIANTS OF THE TRAVELING SALESMAN PROBLEM

PATTERSON Mike

Dillard College of Business, Midwestern State University, USA

FRIESEN Daniel

University of North Texas, Dallas Campus, USA

Abstract:

This paper includes an introduction to the concept of spreadsheet optimization and modeling as it specifically applies to combinatorial problems. One of the best known of the classic combinatorial problems is the "Traveling Salesman Problem" (TSP). The classic Traveling Salesman Problem has the objective of minimizing some value, usually distance, while defining a sequence of locations where each is visited once. An additional requirement is that the tour ends in the same location where the tour started. Variants of the classic Traveling Salesman Problem are developed including the Bottleneck TSP and the Variation Bottleneck TSP.

Key words: *Optimization, Spreadsheet, Traveling Salesman, Combinatorial*

1. Introduction

This paper includes a general introduction to the concept of spreadsheet optimization and modeling as it specifically applies to combinatorial problems. One of the best known of the classic combinatorial problems is the "Traveling Salesman Problem" (TSP). In the definitive *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization* (1985) Lawler et. al. introduce the Traveling Salesman Problem (TSP) as "a salesman, starting from his home city, is to visit exactly once each city on a given list and then return home, it is plausible for him to select the order in which he visits the cities so that the total of the distances traveled in his tour is as small as possible". Cummings (June 2000) traces the roots of the problem to Karl Menger, who defined the problem, which he referred to as the "Messenger Problem" published "Das botenproblem" in *Ergebnisse eines Mathematischen Kolloquiums* in 1932. In tracing the historical roots of the TSP Lawler et. al. (1985) credit Merrill Flood as being responsible for publicizing the conceptual basis for the TSP in the operations community. Robinson's paper "On the Hamiltonian Game" (1949) is frequently cited as a seminal work. Lawler et. al. (1985) cite "Solution of a large-scale traveling-salesman problem by Dantzig, Fulkerson and Johnson (1954) as a

critical event in the development of the TSP and combinatorial optimization in general. Dantzig, one of the giants in operations research, is frequently referred to as the “Father of Linear Programming” (Albers and Reid, 1986). Among the more popular combinatorial problems which have been subject to optimization include permutations, sequencing and scheduling, the minimal spanning tree, and the traveling salesman problem. Tuza (2001) provides a particularly comprehensive set of challenging and unsolved combinatorial problems.

2. General Model Description

The classic Traveling Salesman Problem has the objective of minimizing some value, usually distance, while defining a sequence of locations where each is visited once. An additional requirement is that the tour ends in the same location where the tour started. The spreadsheet model utilized in this paper is patterned after a model developed by Patterson and Harmel (2003). The initial base problem is defined in Table 1, cells D4:R14. There are fourteen locations to be visited on the tour. The upper portion of Table 1 displays the direct distance between each of the locations. The lower portion of the spreadsheet (Cells D20:E34) display the distance between the cities when they are in the initial (i.e. alphabetical/random) sequence. The sum of the initial tour distance (5508) is also displayed.

Excel Solver was utilized as the software for development of this model. Solver is an add-in software tool in Excel for modeling general purpose linear and non-linear optimization problems. It was developed by Frontline Systems. Solver provides an option known as the “alldifferent” constraint, which is particularly useful in combinatorial problems, such as the traveling salesman problem. (Frontline Systems, n.d.)

Table 2 displays the lower portion of the spreadsheet with the formula view. Figure 1 displays the Solver parameters for the classical traveling salesman problem. The objective is to minimize the total distance traveled. The constraints include that each city must be visited once and that the tour must end in the same city which it began. The alldifferent constraint efficiently defines these constraints. The standard evolutionary engine is required in order to utilize the alldifferent option required to satisfy the constraints. Figure 2 displays a visual of the problem. Table 3 displays the Solver solution to the classic Traveling Salesman Problem. The visual solution is shown in Figure 3. As indicated the total distance traveled for the suggested trip is 2,549 miles.

Table 1: Direct Distance Traveling Salesman Problem

1\A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73
74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145
146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163
164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181
182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199
200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217
218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235
236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253
254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271
272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289
290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307
308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325
326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343
344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361
362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379
380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397
398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415
416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433
434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451
452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469
470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487
488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505
506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523
524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541
542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559
560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577
578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595
596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613
614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631
632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649
650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667
668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685
686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703
704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721
722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739
740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757
758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775
776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793
794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811
812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829
830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847
848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865
866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883
884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901
902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919
920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937
938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955
956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973
974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991
992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009
1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027
1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045
1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063
1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081
1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099
1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117
1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135
1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149	1150	1151	1152	1153
1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171
1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182	1183	1184	1185	1186	1187	1188	1189
1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207
1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221	1222	1223	1224	1225
1226	1227	1228	1229	1230	1231	1232	1233	1234	1235	1236	1237	1238	1239	1240	1241	1242	1243
1244	1245	1246	1247	1248	1249	1250	1251	1252	1253	1254	1255	1256	1257	1258	1259	1260	1261
1262	1263	1264	1265	1266	1267	1268	1269	1270	1271	1272	1273	1274	1275	1276	1277	1278	1279
1280	1281	1282	1283	1284	1285	1286	1287	1288	1289	1290	1291	1292	1293	1294	1295	1296	1297
1298	1299	1300	1301	1302	1303	1304	1305	1306	1307	1308	1309	1310	1311	1312	1313	1314	1315
1316	1317	1318	1319	1320	1321	1322	1323	1324	1325	1326	1327	1328	1329	1330	1331	1332	1333
1334	1335	1336	1337	1338	1339	1340	1341	1342	1343	1344	1345	1346	1347	1348	1349	1350	1351
1352	1353	1354	1355	1356	1357	1358	1359	1360	1361	1362	1363	1364	1365	1366	1367	1368	1369
1370	1371	1372	1373	1374	1375	1376	1377	1378	1379	1380	1381	1382	1383	1384	1385	1386	1387
1388	1389	1390	1391	1392	1393	1394	1395	1396	1397	1398	1399	1400	1401	1402	1403	1404	1405
1406	1407	1408	1409	1410	1411	1412	1413	1414	1415	1416	1417	1418	1419	1420	1421	1422	

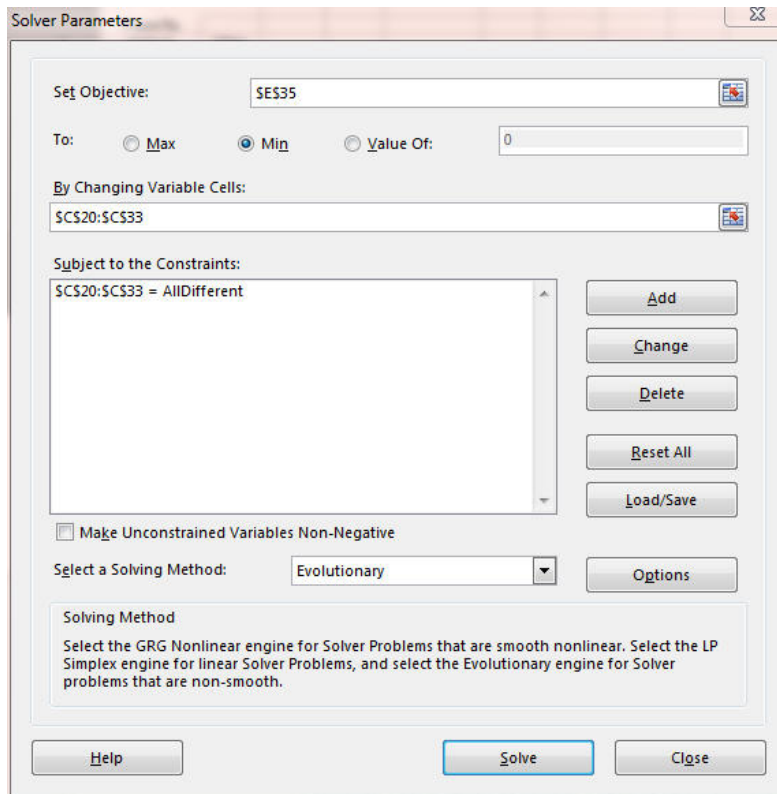


Figure 1: Solver Parameters Traveling Salesman Problem



Figure 2: Texas Map of Traveling Salesman Problem

As stated previously, the primary purpose of this study is to present two alternative models to the TSP. The first variation to the classic Traveling Salesman Problem is the Bottleneck Traveling Salesman Problem (BTSP). Conceptually, the BTSP is quite similar to the TSP. The constraints are identical in that each location must be visited once. In other words, each location must be entered once and exited once. Also, the trip must end in the same city where it began. The only real formulation difference is that rather than minimizing the total distance of the trip, the objective is to minimize the distance of the single longest intercity trip (Lawler, et. al., 1985). With this one change to the objective function, the classic TSP is converted to a “minimax” problem where the objective is to minimize the maximum single distance between two cities. The second alteration to the TSP is to modify the objective function to minimize the degree of variation around the mean intercity trip difference. In other words, to the extent it is possible, we desire to make each intercity trip as close as possible to the mean distance. Once again the constraints are identical to the TSP. The difference is in the statement of the objective function. In our formulation we chose to minimize the standard deviation of the distances for the selected tour sequence. Table 4 displays the complete initial spreadsheet for the all three new models. Additional statistical calculations displayed include the intercity maximum value, the mean intercity distance and the standard deviation for the intercity distances. Table 5 displays the formula view for relevant cells. In addition, column F displays the deviation from the mean for each intercity trip.

Table 3: Solution to Traveling Salesman Problem

1\A	B	C	D	E
19			From\To	
20		1	Abilene	Miles
21		3	Austin	213
22		12	San Antonio	79
23		8	Laredo	154
24		10	McAllen	143
25		4	Corpus Christi	152
26		7	Houston	207
27		13	Waco	180
28		5	Dallas	91
29		14	Wichita Falls	136
30		2	Amarillo	225
31		9	Lubbock	119
32		11	Odessa	137
33		6	El Paso	274
34		1	Abilene	439
35			Total	2549

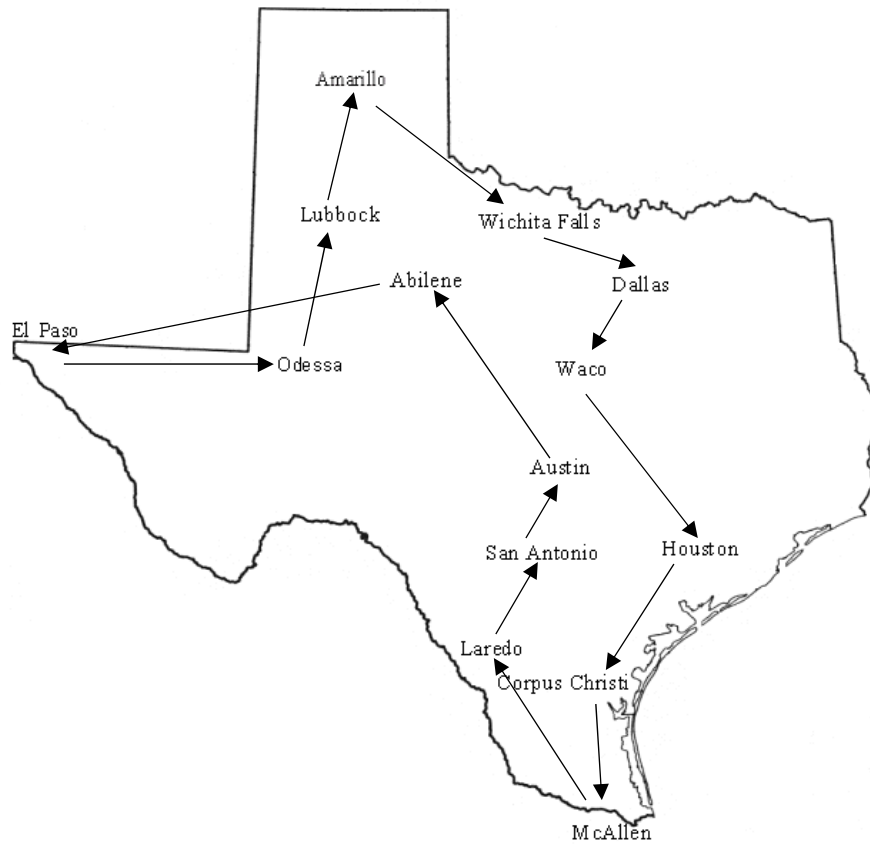


Figure 3: Texas Solution Map of Classic Traveling Salesman Problem

Table 4: Direct Distance Traveling Salesman Problem

1A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
2			To	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	City			Abilene	Amarillo	Austin	Corpus Christi	Dallas	El Paso	Houston	Laredo	Lubbock	McAllen	Odessa	San Antonio	Waco	Wichita Falls
4	From	1	Abilene	0	266	213	387	180	439	348	373	162	480	304	244	183	141
5	2	Amarillo	266	0	478	636	361	418	596	609	119	728	255	493	423	225	
6	3	Austin	213	478	0	192	192	573	162	232	368	300	334	79	102	283	
7	4	Corpus Christi	387	636	192	0	377	691	207	141	526	152	432	143	287	474	
8	5	Dallas	180	361	192	377	0	617	238	424	322	491	347	271	91	136	
9	6	El Paso	439	418	573	691	617	0	730	602	344	745	274	548	610	552	
10	7	Houston	348	596	162	207	238	730	0	311	510	345	494	197	180	371	
11	8	Laredo	373	609	232	141	424	602	311	0	498	143	422	154	334	490	
12	9	Lubbock	162	119	368	526	322	344	510	498	0	618	137	382	345	208	
13	10	McAllen	480	728	300	152	491	745	345	143	619	0	565	236	401	572	
14	11	Odessa	304	255	334	432	347	274	494	422	137	565	0	336	340	293	
15	12	San Antonio	244	493	79	143	271	548	197	154	382	236	336	0	181	336	
16	13	Waco	183	423	102	287	91	610	180	334	345	401	340	181	0	198	
17	14	Wichita Falls	141	225	283	474	136	552	371	490	208	572	293	336	198	0	
18																	
19			From/To														
20	1		Abilene	Miles	Dev.												
21	2		Amarillo	266	127.429												
22	3		Austin	478	84.5714												
23	4		Corpus Christi	192	201.429												
24	5		Dallas	377	16.4286												
25	6		El Paso	617	223.571												
26	7		Houston	730	336.571												
27	8		Laredo	311	82.4286												
28	9		Lubbock	498	104.571												
29	10		McAllen	618	224.571												
30	11		Odessa	565	171.571												
31	12		San Antonio	336	57.4286												
32	13		Waco	181	212.429												
33	14		Wichita Falls	198	195.429												
34	1		Abilene	141	252.429												
35			Total	5508	2290.86												
36			Intercity Max.	730													
37			Mean	393.43													
38			Std. Dev.	191.24													

Table 5: Formula View Initial Sequence Three Traveling Salesman Problem

1\A	B	C	D	E	F
19			From\To		
20	1	=VLOOKUP(C20,citytab,2)			
21	2	=VLOOKUP(C21,citytab,2)	=INDEX(mileage,C20,C21)	=ABS(E21-\$E\$37)	
22	3	=VLOOKUP(C22,citytab,2)	=INDEX(mileage,C21,C22)	=ABS(E22-\$E\$37)	
23	4	=VLOOKUP(C23,citytab,2)	=INDEX(mileage,C22,C23)	=ABS(E23-\$E\$37)	
24	5	=VLOOKUP(C24,citytab,2)	=INDEX(mileage,C23,C24)	=ABS(E24-\$E\$37)	
25	6	=VLOOKUP(C25,citytab,2)	=INDEX(mileage,C24,C25)	=ABS(E25-\$E\$37)	
26	7	=VLOOKUP(C26,citytab,2)	=INDEX(mileage,C25,C26)	=ABS(E26-\$E\$37)	
27	8	=VLOOKUP(C27,citytab,2)	=INDEX(mileage,C26,C27)	=ABS(E27-\$E\$37)	
28	9	=VLOOKUP(C28,citytab,2)	=INDEX(mileage,C27,C28)	=ABS(E28-\$E\$37)	
29	10	=VLOOKUP(C29,citytab,2)	=INDEX(mileage,C28,C29)	=ABS(E29-\$E\$37)	
30	11	=VLOOKUP(C30,citytab,2)	=INDEX(mileage,C29,C30)	=ABS(E30-\$E\$37)	
31	12	=VLOOKUP(C31,citytab,2)	=INDEX(mileage,C30,C31)	=ABS(E31-\$E\$37)	
32	13	=VLOOKUP(C32,citytab,2)	=INDEX(mileage,C31,C32)	=ABS(E32-\$E\$37)	
33	14	=VLOOKUP(C33,citytab,2)	=INDEX(mileage,C32,C33)	=ABS(E33-\$E\$37)	
34	=C20	=D20	=INDEX(mileage,C33,C34)	=ABS(E34-\$E\$37)	
35			Total	=SUM(E21:E34)	=SUM(F21:F34)
36			Intercity Max.	=MAX(E21:E34)	
37			Mean	=AVERAGE(E21:E34)	
38			Std. Dev.	=STDEV(E21:E34)	

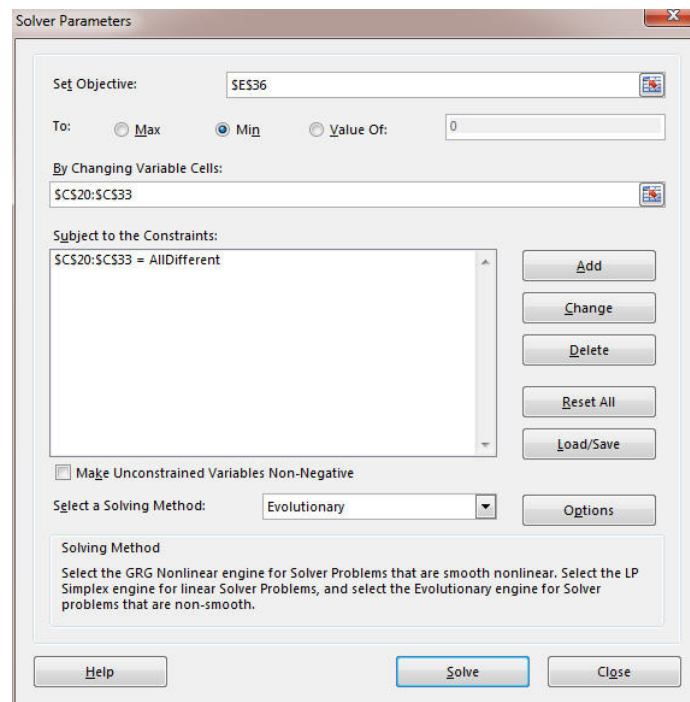


Figure 4: Solver Parameters Bottleneck Traveling Salesman Problem

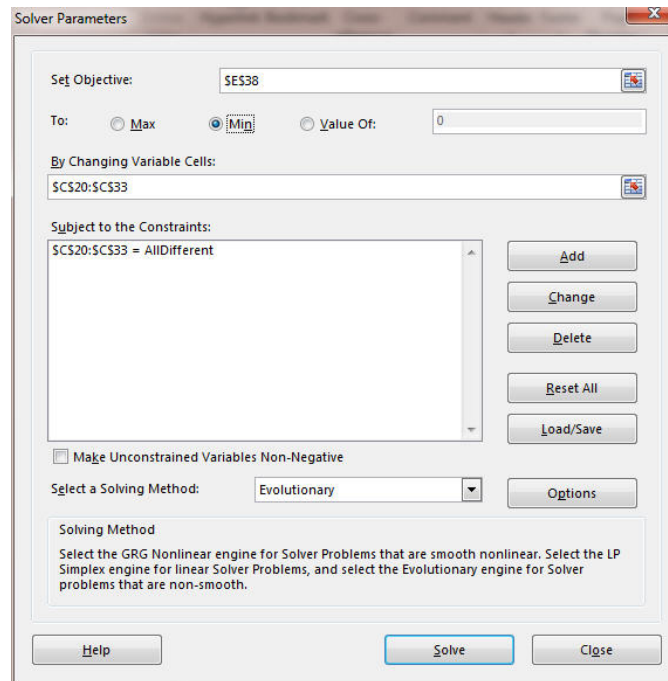


Figure 5: Solver Parameters Variation Traveling Salesman Problem

Table 7 displays a summary of the suggested tours. The first suggested tour is the baseline and could be considered as a random trip since it is in alphabetical order. For this baseline tour, the total distance is 5,508 miles, the bottleneck trip is 730 and the standard deviation is 191.2 miles. In comparison the classic TSP solution reduces the total trip distance by almost 3,000 miles to 2,549. The Bottleneck TSP solution results in the longest single intercity trip of 344 miles. Multiple runs indicate multiple optimal solutions with the same result of 344 miles which is the distance from Lubbock to El Paso. The Variation TSP, which minimizes the standard deviation, results in a tour with a standard deviation of 26.69. Figure 5 displays the visual map of the suggested Bottleneck tour. Figure 6 displays the suggested Variation Bottleneck tour, which attempts to provide the most uniform intercity distance between cities.

Table 7: Summary of Suggested Tours Traveling Salesman Problem

Alphabetical				Classic			
	From\To				From\To		
1	Abilene	Miles	Dev.	7	Houston	Miles	Dev.
2	Amarillo	266	127.43	4	Corpus Christi	207	24.93
3	Austin	478	84.571	10	McAllen	152	30.07
4	Corpus Christi	192	201.43	8	Laredo	143	39.07
5	Dallas	377	16.429	12	San Antonio	154	28.07
6	El Paso	617	223.57	3	Austin	79	103.1
7	Houston	730	336.57	1	Abilene	213	30.93
8	Laredo	311	82.429	6	El Paso	439	256.9
9	Lubbock	498	104.57	11	Odessa	274	91.93
10	McAllen	618	224.57	9	Lubbock	137	45.07
11	Odessa	565	171.57	2	Amarillo	119	63.07
12	San Antonio	336	57.429	14	Wichita Falls	225	42.93
13	Waco	181	212.43	5	Dallas	136	46.07
14	Wichita Falls	198	195.43	13	Waco	91	91.07
1	Abilene	141	252.43	7	Houston	180	2.071
	Total	5508			Total	2549	
	Intercity	730			Intercity	439	
	Mean	393.4			Mean	182.1	
	Std. Dev.	191.2			Std. Dev.	91.21	
Bottleneck				Variation			
	From\To				From\To		
11	Odessa			12	San Antonio		
6	El Paso	274	43.714	14	Wichita Falls	336	27.21
9	Lubbock	344	113.71	7	Houston	371	7.786
14	Wichita Falls	208	22.286	10	McAllen	345	18.21
2	Amarillo	225	5.2857	13	Waco	401	37.79
1	Abilene	266	35.714	8	Laredo	334	29.21
5	Dallas	180	50.286	1	Abilene	373	9.786
13	Waco	91	139.29	4	Corpus Christi	387	23.79
4	Corpus Christi	287	56.714	5	Dallas	377	13.79
7	Houston	207	23.286	2	Amarillo	361	2.214
12	San Antonio	197	33.286	6	El Paso	418	54.79
10	McAllen	236	5.7143	9	Lubbock	344	19.21
8	Laredo	143	87.286	3	Austin	368	4.786
3	Austin	232	1.7143	11	Odessa	334	29.21
11	Odessa	334	103.71	12	San Antonio	336	27.21
	Total	3224			Total	5085	
	Intercity	344			Intercity	418	
	Mean	230.3			Mean	363.2	
	Std. Dev.	68.98			Std. Dev.	26.69	

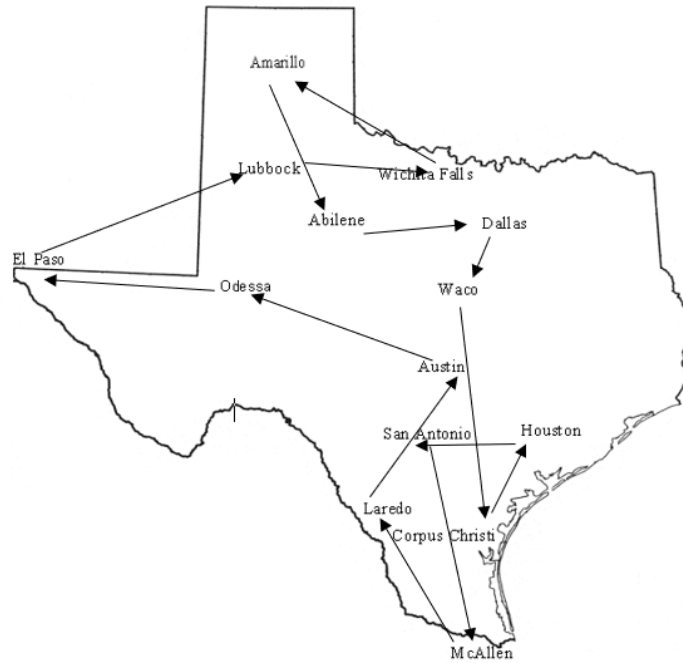


Figure 5: Texas Solution Map of Bottleneck Traveling Salesman Problem

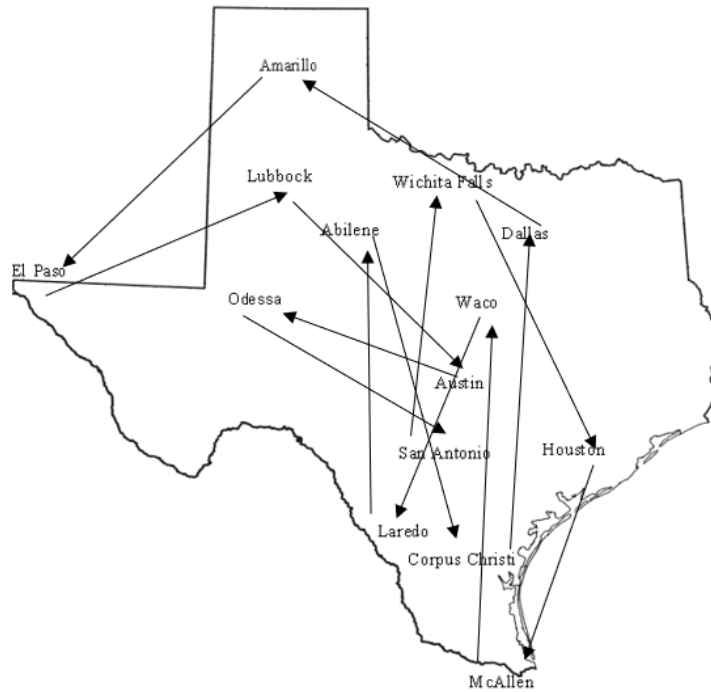


Figure 6: Texas Solution Map of Variation Traveling Salesman Problem

The presence of multiple optimal solutions to the BTSP led the authors to further explore the model formulation. Using the minimize objective function for the maximum intercity distance found in a tour efficiently located the bottleneck to be the trip between El Paso and Lubbock. With the current formulation, any tour which included this trip and satisfied the constraints would be considered optimal. The authors determined that the trip between El Paso and Lubbock could be determined to be mandatory by the inclusion of a constraint which required that the maximum intercity distance be required to be equal to the distance between these two cities, i.e. 344 miles. With this additional constraint, the objective function was then modified to minimize the total tour distance. Thus it accomplishes two objectives. First, the tour must include the route from Lubbock to El Paso. Secondly, the model identifies the shortest total tour distance, which includes the bottleneck route. Figure 7 displays the Solver parameters.

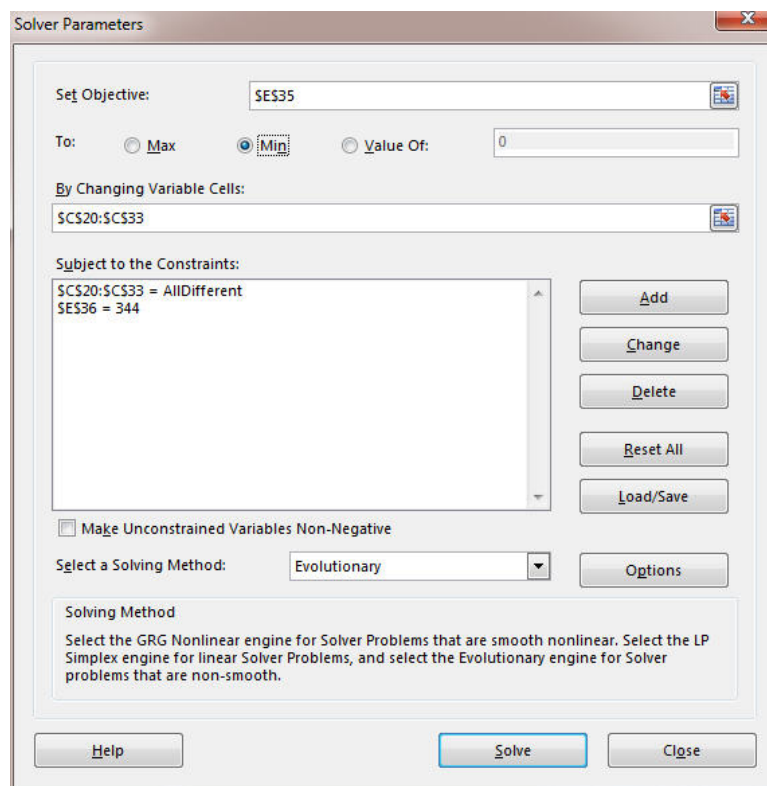


Figure 7: Solver Parameters Second Bottleneck Variation Traveling Salesman Problem

Table 8 displays the Solver solution to the newly formulated BTSP. This solution does include the bottleneck trip from Lubbock to El Paso. Thus, the bottleneck trip is reduced by 609 miles (3224-2615) in comparison to the original BTSP model formulation. The original classic TSP solution resulted in a total tour distance of 2,549 miles. The new

formulation increases the total distance to 2,615, a difference of 66 miles. Figure 8 displays a map of the suggested tour.

Table 8: Summary of Suggested Tour Modified Bottlenecks Traveling Salesman Problem

	From\To		
14	Wichita Falls		
5	Dallas	136	50.79
13	Waco	91	95.79
7	Houston	180	6.786
4	Corpus Christi	207	20.21
10	McAllen	152	34.79
8	Laredo	143	43.79
12	San Antonio	154	32.79
3	Austin	79	107.8
1	Abilene	213	26.21
9	Lubbock	162	24.79
6	El Paso	344	157.2
11	Odessa	274	87.21
2	Amarillo	255	68.21
14	Wichita Falls	225	38.21
	Total	2615	794.6
	Intercity	344	
	Mean	186.786	
	Std. Dev.	72.1997	

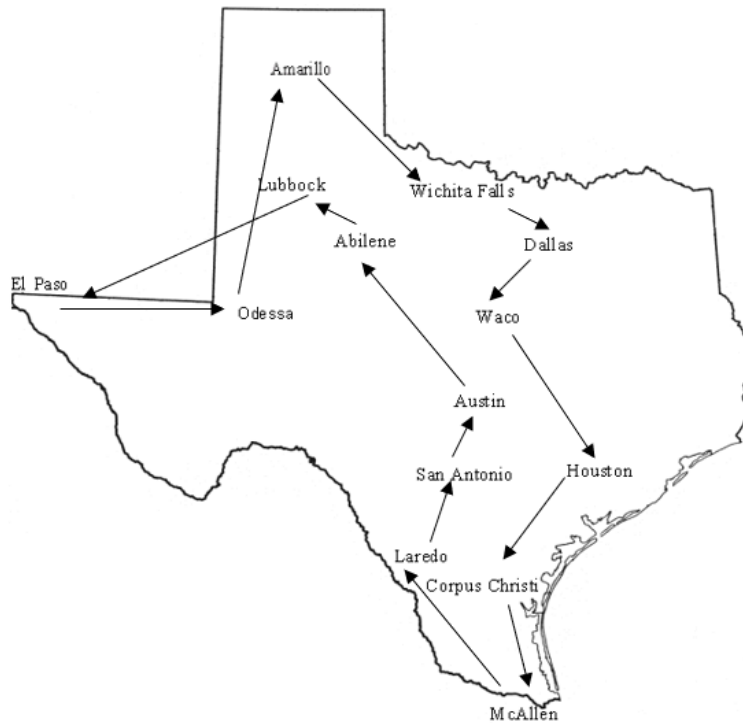


Figure 8: Texas Solution Map of Modified Bottleneck Traveling Salesman Problem

3. Summary and Conclusion

The purpose of the paper is to expand on the Traveling Salesman Problem and present a spreadsheet model which includes alternatives to the objective to minimizing the total distance of the tour. The additional objectives include minimizing the bottleneck distance and the degree of variation around the mean intercity trip difference. The Excel model formulation and solutions represent different approaches to the classic Traveling Salesman Problem.

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