Articles

The triangular binary clock

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Abstract: *A new idea for a binary clock is presented. It displays the time using a triangular array of* 15 *lamps each representing a certain amount of time. It is shown that such a geometric, triangular arrangement is only possible because our system of time divisions is based on a sexagesimal system in which the number of minutes in* 12 *hours equals the factorial of a natural number* ($720 = 6!$)*. An interactive applet allows one to "play" with the clock.*

Key-words: binary, clock, time, sexagesimal system.

1 Introduction

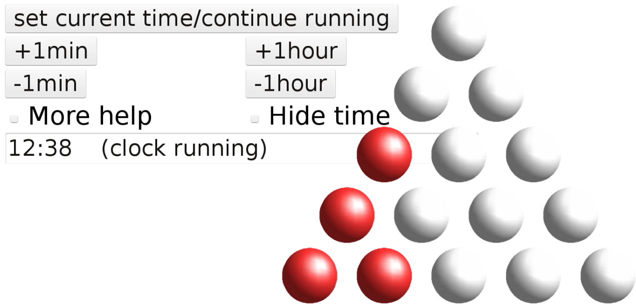
There are many ways to display the time. For example the familiar analog display with a dial and clock hands or digital displays using numerals. In addition there are binary displays which are a bit more difficult to read. Here a lamp lit corresponds to a certain amount of time. The binary display presented in this article has the special feature that the lamps are arranged in form of a triangle.

In section 2 the clock is presented with the help of an interactive javascript applet. Section 3 illuminates the mathematical background and shows that the triangular display is only possible because our system of time divisions is based on a sexagesimal system rather than a decimal system (see also [1]).

2 The triangular clock

The following applet illustrates the idea. To read off the time you have to know that every lamp lit in the ...
... 1st line (top line) corresponds to 6h
... 2nd line corresponds to 2h
... 3rd line corresponds to 30 min
... 4th line corresponds to 6 min
... 5th line corresponds to 1 min
green is for AM and red for PM. You can play with the clock by using the buttons on the left. The allow one to advance or to move back the clock in steps

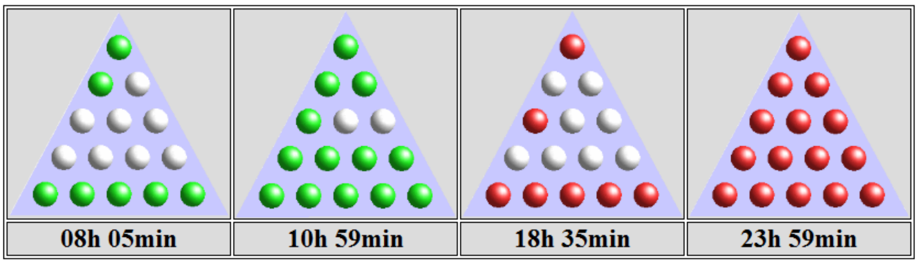
of one minute or one hour. To set the time back to the present time just push the top button. The two checkboxes allow for more help or to hide the time given in numerals in case you would like to train yourself. You will find out that all lamps on corresponds to 11:59 (green) or 23:59 (red). So this arrangement perfectly fits for a twelve hour display.



Applet online:

`http://web.physik.rwth-aachen.de/user/pretz/binary_clock/ludus/jpretz_binary_clock.htm`.

A few examples are shown below.



3 Mathematical background

We now come to the question why our system of time divisions allows such a triangular display? First note, that the amount of time, T_{n-1}, a lamp corresponds to in the $(n-1)$th row equals (m_n+1) times the amount, T_n, in the nth row where m_n is the number of lamps in the nth row. Thus,

$$T_{n-1} = (m_n + 1) \times T_n.$$

Here are two examples:

- The 3rd row has $m_3 = 3$ lamps representing 30 minutes each. Thus for the 2nd row one finds $(3 + 1) \times 30$ min $= 2$ h.
- The 5th row has $m_5 = 5$ lamps representing 1 minute each. Thus for the 4th row one finds $(5 + 1) \times 1$ min $= 6$ min.

The triangular display has the special feature that $m_n = n$, i.e. in the nth row there are also n lamps This is just the condition for a triangular display. But what does $m_n = n$ mean for the number of states which can be represented by the display? A row with n lamps allows one to display $n + 1$ states, from all lamps off to all lamps on. (Note that in the way the display is used here, not all of the 2^n possible states are used because if a lamp is on, also the lamps left to it are on in the same row.) For a triangular display with n lamps in the bottom row and the number of lamps decreasing by one in every following row the total number of states is thus,

$$(n + 1) \times ((n - 1) + 1) \times ((n - 2) + 1) \times \cdots \times (1 + 1) = (n + 1)!$$

i.e the number of states equals always a factorial of a natural number. Now, note that our system of time measurement is based on numbers which have many divisors, e.g. $12 = 3 \times 4$ and $60 = 1 \times 2 \times 5 \times 6$. For a 12 hour display with a precision of one minute the number of states one has to display is thus,

$$12 \times 60 \text{ minutes} = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \text{ minutes} = 6! \text{ minutes.}$$

which perfectly fits in a triangular display with five rows! Thus the whole concept works because our system of time divisions is based on a sexagesimal system, dating back to the Babylonian [2, 3], rather than a decimal system, as proposed during the French Revolution [4, 5].

References

[1] Pretz, J. "A different way to read off the time – A new idea for a binary clock", *Journal of Recreational Mathematics*, Vol. 35(4), 327–330, arXiv:1006.1373v1 [math.HO], 2006.

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[5] Wikipedia article. "Decimal time": `http://en.wikipedia.org/wiki/Decimal_time`