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EXPERIMENTAL VERIFICATION OF METHOD TO SYNTHESIZE THE TRACK VERTICAL IRREGULARITIES

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Rezumat

Lucrarea are ca scop verificarea experimentală a rezultatelor obţinute prin simulări numerice pe baza unui model al sistemului boghiu-cale în care neregularităţile verticale ale căii sunt introduse sub forma unei funcţii pseudoaleatoare. Această funcţie este obţinută printr-o metodă originală de sintetizare a neregularităţilor verticale ale căii în funcţie de calitatea geometrică a căii şi viteză. Pentru verificarea metodei se compară rădăcina medie pătratică (RMS) a acceleraţiilor osiilor şi ale cadrului de boghiu în dreptul fiecărei osii obţinute prin simulări numerice cu cele obţinute experimental în domeniul de frecvenţă pentru lungimi de undă ale neregularităţilor verticale ale căii cuprinse între 3 m şi 120 m. Rezultatele prezentate arată o bună concordanţă între acceleraţiile RMS obţinute prin simulări numerice pentru o cale de calitate slabă şi acceleraţiile RMS măsurate.

Cuvinte cheie: neregularităţi verticale ale căii, analiză experimentală, vibraţiile boghiului, rădăcina medie pătratică a acceleraţiei

Abstract

The paper focuses on the experimental verification of the results derived from numerical simulations, based on a model of the bogie-track system, where the vertical track irregularities are introduced in the form of a pseudorandom function. This function comes from an original method of synthesizing the vertical track irregularities, depending on the geometric quality of the track and on the velocity. To verify the method, the root mean square (RMS) of the simulated accelerations in the axles and the bogie frame against each axle is compared to the experimental accelerations within the frequency range of wavelengths of the track vertical irregularities from 3 to 120 m. The results have shown a good correlation between the simulated RMS accelerations for a low quality track and the measured RMS accelerations.

Keywords: track vertical irregularities, experimental analysis, bogie vibration, root mean square of acceleration

1. INTRODUCTION

The numerical simulations represent basic tools within the research as they are used since the designing stage in order to estimate the dynamic behaviour of the railway vehicle and the optimization of its dynamic performance, and in the investigation of the problems arising during exploitation [1, 2].

In view of the track trials that are expensive, time- and effort-consuming and are affected by a series of unpredictable variables, the numerical simulations have the advantage that they enable the investigation of the dynamic behaviour of the vehicle even in the vicinity of certain extreme situations that cannot be pointed out under real testing conditions [3].

As a matter of fact, certain models of the vehicle-track system underlie the development of the simulation programs, where the typical inputs are represented by the track irregularities that are the main cause for the vibrations of the rail vehicles. It is, on the one hand, about the track geometric irregularities that mainly come from the construction imperfections, track exploitation, change in the infrastructure due to the action of the environment factors or soil movements [4], and, on the other hand, about the irregularities in the rolling surfaces of railway rails. To these, the discontinuities of the rails are added - joints, switches, crossings [5].

The track irregularities exhibit deviations from the design geometry, which are a consequence of the lateral and vertical deviations of each rail from the nominal position. Typically, these deviations are combined to give an alternative set of four independent irregularities: track alignment or lateral alignment, longitudinal level or vertical alignment, cant variation and gauge variation [6, 7].

Rolling on a track with deviations from the ideal geometry triggers vibrations in an axle that are further transmitted to the suspended masses of the vehicle, thus generating and maintaining their vibrations [8, 9], so that the dynamic response of the entire vehicle is affected by the track irregularities [10, 11].

In order to include the track irregularities in the simulation programs of the dynamics in the railway vehicles, the review literature features two distinct approaches. One considers the track irregularities as absolute values defined as a function of distance, obtained by measuring the track geometry [4, 7, 12, 13]. An alternative approach consists of the introduction of the track irregularities in the numerical model as random data defined by the power spectral density [14 - 16]. In this case, the track irregularities are not looked at as absolute values defined by distance, but they are represented as functions of wavelength or frequency. Power spectral densities can be used either directly as an input for power spectral analysis or they can be transformed into track irregularities as a function of distance.

The paper features a new approach, in which the vertical track irregularities can be introduced in the numerical model of the vehicle-track system in the form of a pseudorandom function that depends on the track geometric quality and on the velocity. This function has been obtained from an original method [17] of synthesizing the track vertical irregularities, starting from the power spectral density recommended by ORE B176 [18] for the average statistical properties of the European railways and the specifications in the UIC 518 Leaflet [19] regarding the track geometric quality for the vertical alignment.

The paper aims for an experimental verification of the method above, for which the simulated RMS (root mean square) accelerations in the axles and on the bogie frame against each axle and the experimental ones are being compared.

2. METHOD FOR THE SYNTHESIZING OF THE VERTICAL TRACK IRREGULARITIES

This section introduces an original method to synthesize the track vertical irregularities [17], a method underlain by the power spectral density of the track irregularities as defined by ORE B176 [18] and described in the specifications in the UIC 518 [19] Leaflet regarding the track geometric quality for the vertical alignment.

For the average statistical properties of the European railways, the following equation of the power spectral density is considered representative

$$\Phi(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}, \tag{1}$$

where Ω is the wavelength, Ω_c = 0.8246 rad/m, Ω_r = 0.0206 rad/m, and A is a coefficient depending on the track quality. For a high level quality track, A = $4.032 \cdot 10^{-7}$ radm, whereas for a low level quality, the coefficient A is $1.080 \cdot 10^{-6}$ radm.

According to the UIC 518 Leaflet, a document used to organize the instructions regarding the homologation trials of the railway vehicles in terms of their dynamic behaviour, namely safety, fatigue and the running quality, the geometric description of the track is done through three quality levels - QN1, QN2 and QN3. The quality levels QN1 (high quality level) and QN2 (low quality level) are defined, depending on the reference speed, for the standard deviation for vertical alignment and separately, for the ones in the lateral alignment of the track. Likewise, for the two track quality levels, the peak values of isolated track errors are mentioned depending on the reference speed.

The reference speed is calculated in a differentiated manner, according to whether the track section is in alignment or in a curve. In other words, if the track section is in alignment or in a large-radius curve, the reference velocity is the vehicle limit speed itself; for medium-radius curves, V ranges from 160 to 200 km/h; in curves with small and very small radii, 80 km/h < $V \leq$ 120 km/h.

The quality level QN3 is defined as a function of the peak value of an isolated error corresponding to QN2, namely

$$\text{QN3} = 1.3 \cdot \text{QN2}. \tag{2}$$

Should the quality of a track section correlate with the level QN3, then it will be excluded from the analysis, as not being representative in terms of track standard geometry. Figure 1 presents the standard deviations for vertical alignment and the Figure 2, the peak values of isolated errors, corresponding to the quality levels QN1 and QN2, representative for the quality of an European network. It should be mentioned that the values for the alignment standard deviations correlate with a wavelength range between 3 and 25 meters. Peak values are greater than the standard deviation of the vertical alignment of 4-5.2 times for QN1 level and 5.7-6.2 for QN2 level.

To synthesize the vertical track irregularities, the starting point is in the power spectral density equation mentioned as follows:

$$\Phi(\Omega) = \frac{A_{\text{QN1,2}}\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}, \tag{3}$$

where the coefficient $A_{\text{QN1,2}}$ will be calculated so that the value of the alignment standard deviation due to the components with the wavelength between Λ_1 = 3 m and Λ_2 = 25 m should correspond to the stipulations in the UIC 518 Leaflet, according to the track quality level.

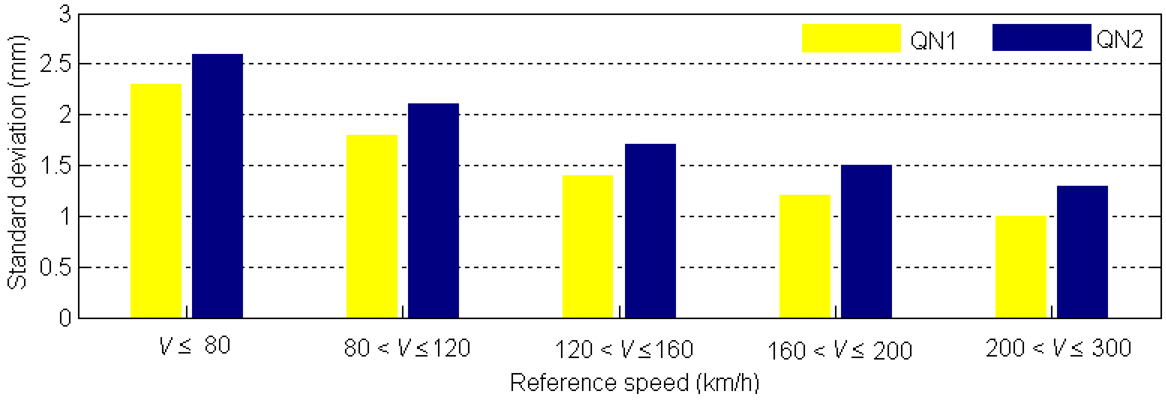

Figure 1. Standard deviation for vertical alignment for track quality levels QN1 and QN2.

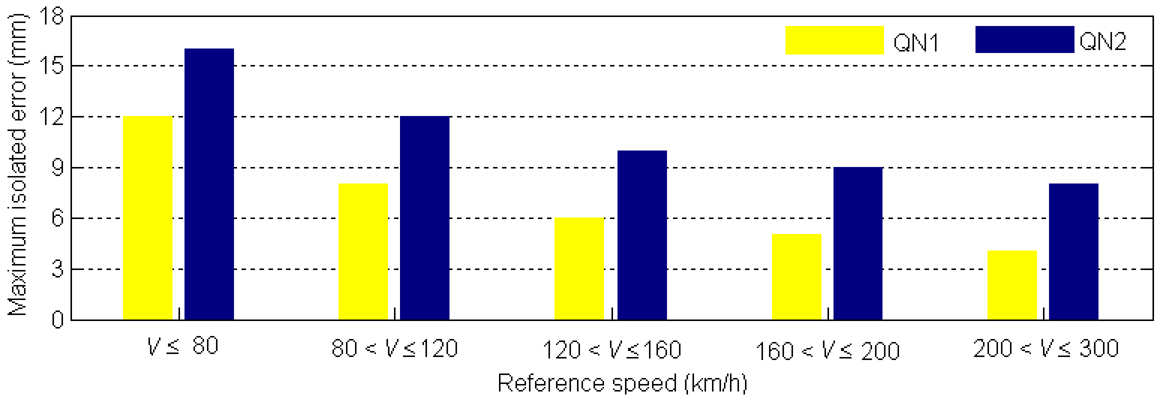

Figure 2. The peak value for vertical alignment for track quality levels QN1 and QN2.

To this purpose, the standard deviation for vertical alignment between the wavelengths Λ_1 and Λ_2 needs to be calculated

$$\sigma_{\eta\mathrm{QN1},1} = \sqrt{\frac{1}{2\pi}\int_{\Omega_2}^{\Omega_1}\Phi(\Omega)\mathrm{d}\Omega}\,, \quad \text{for } \Omega_{1,2} = 2\pi/\Lambda_{1,2}. \tag{4}$$

From (3) and (4), results:

$$A_{\mathrm{QN1},2} = 2\pi\frac{\sigma^2_{\eta\mathrm{QN1},2}}{\Omega_c^2 I_0}, \tag{5}$$

where $$I_0 = \int_{\Omega_2}^{\Omega_1} \frac{\mathrm{d}\Omega}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}. \quad (6)$$

The Fig. 3 shows the values of $A_{QN1,2}$ calculated via relation (5) where $\sigma_{\eta QN1,2}$ has been assigned various values corresponding to a QN1 and to a QN2 quality tracks, as in the UIC 518 Leaflet (see Fig. 1). The same figure includes the values of the coefficient *A* for high level and for low quality level tracks. The values of the coefficient *A*, as in ORE B176, can be noted as being much lower than the ones derived from the stipulations in the UIC 518 Leaflet.

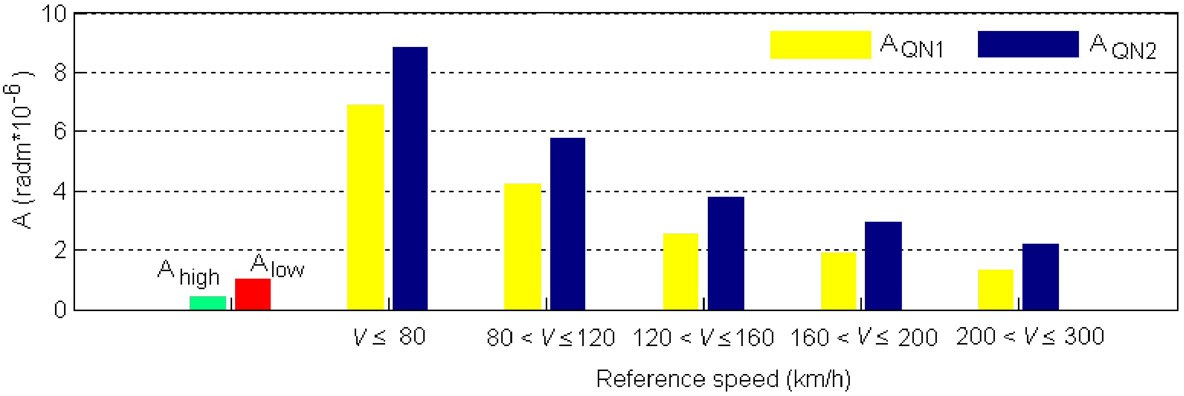

Figure 3. Coefficient of the track quality.

The vertical track irregularities' synthesization underlies on the transformation of the alignment power spectral density $\Phi(\Omega)$, which is a continuous spectrum, with an infinity of spectral components, into the spectrum of the amplitudes $U(\Omega)$, a discrete spectrum with a finite number of spectral components (see Fig. 4). The transformation criterion consists of the power retention of the two spectra in the wavelength range under study.

A certain interval of wavelengths of the track lateral irregularities will be considered, ranging from the minimum wavelength Λ_{min} and the maximum one Λ_{max}. For this interval, a constant step partition of the wave number $\Delta\Omega$ is considered,

$$\Omega_k = \Omega_0 + k\Delta\Omega, \quad k = 0, 1, 2, ..., N, \quad (7)$$

where $N + 1$ is the number of spectral components.

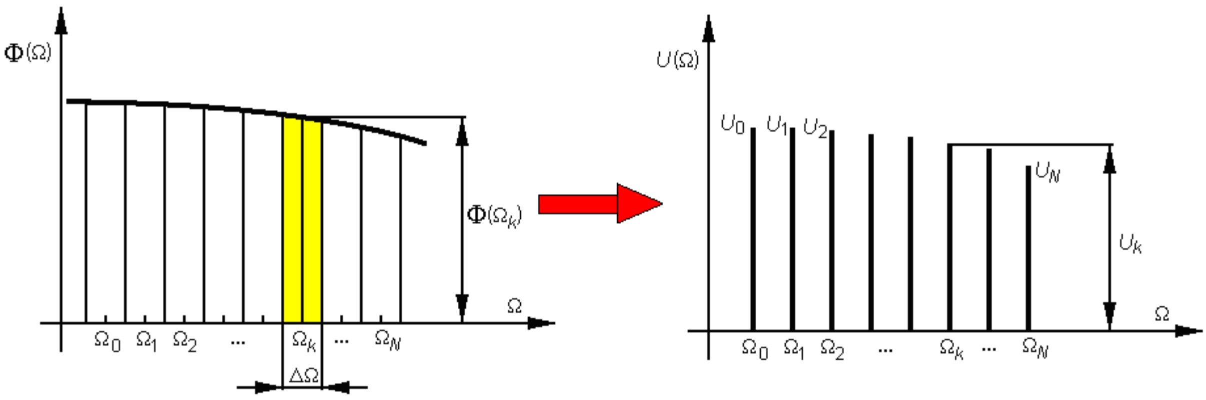

Figure 4. Discretization of the power spectral density.

The following correlations are satisfied

$$\Omega_0 = \frac{2\pi}{\Lambda_{\max}}, \ \Omega_N = \frac{2\pi}{\Lambda_{\min}}. \tag{8}$$

The amplitude of each spectral component is obtained, if considering that the alignment standard deviation comes from relation (4) for the interval between $\Omega_k - \Delta\,\Omega/2$ and $\Omega_k + \Delta\,\Omega/2$, while having the spectral density constant for this interval

$$\sigma_{\eta QN1,2} \approx \sqrt{\frac{1}{2\pi}\Phi(\Omega_k)\Delta\Omega}\,. \tag{9}$$

The amplitude of the spectral component corresponding to the wave number Ω_k is as below

$$U_k = \sqrt{\frac{1}{\pi}\Phi(\Omega_k)\Delta\Omega}\ \text{,where } k = 0, 1, 2, \ldots N. \tag{10}$$

A first analytical form of the vertical track irregularities will result, which preserved the form of the spectral density as in eq (3) and the value of the standard deviation in the vertical alignment as in UIC 518 Leaflet, which is

$$\eta(x) = \sum_{k=0}^{N} U_k \sin(\Omega_k x + \varphi_k), \tag{11}$$

where φ_k is the spectral component shifting ‚k'. To give the track irregularities a stochastic nature, a uniform stochastic repartition will be selected for φ_k, with values between $-\pi$ and $+\pi$.

The synthesized form of the vertical track irregularities given by eq (11) is adjusted so that the peak values of the isolated defects are included within the defined limits (see Fig. 2). This adjustment is done via the scaling of the track lateral irregularities, with a coefficient equalling a ratio between the value of the admitted isolated error $\eta_{\mathrm{admQN1,2}}$ and the absolute maximum value of the lateral irregularities ($\max|\eta(x)|$).

$$K_\eta = \frac{\eta_{\mathrm{admQN1,2}}}{\max|\eta(x)|}. \tag{12}$$

The following analytical form of the track irregularities thus results

$$\eta(x) = K_\eta f(x) \sum_{k=0}^{N} U_k \cos(\Omega_k x + \varphi_k), \tag{13}$$

in which the levelling function $f(x)$ was introduced, to have the connection to the track without irregularities

$$f(x) = \left[6\left(\frac{x}{L_0}\right)^5 - 15\left(\frac{x}{L_0}\right)^4 + 10\left(\frac{x}{L_0}\right)^3\right] H(L_0 - x) + H(x - L_0), \tag{14}$$

where L_0 is the connection length, and $H(.)$ is Heaviside step function

Upon implementing the above method, the vertical track irregularities have been synthesized on a 2-km distance for a QN1 quality level track, as well as for a QN2 one (see Fig. 5). To this purpose, the values of the vertical alignment standard deviations and the peak values of isolated errors for the reference speed between 120 and 160 km/h (see Fig. 1 and Fig. 2).

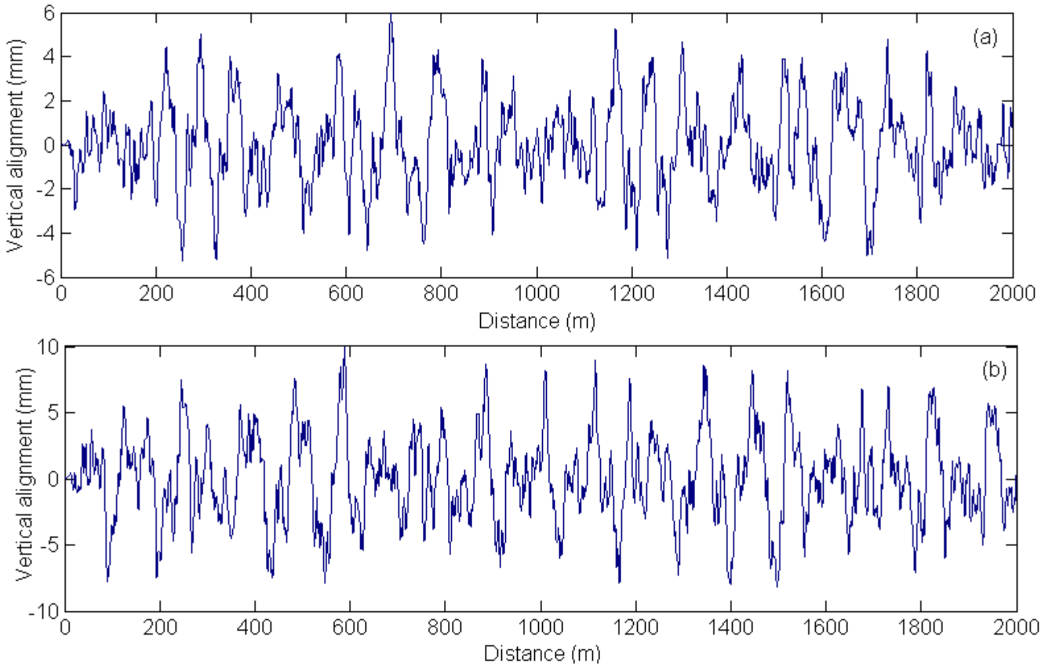

Figure 5. Synthesis of the vertical track irregularities

For the synthesis of the track geometry, the contribution of a number of 300 spectral components (N = 300) has been regarded, with wavelengths between 3 and 120 m. The maximum value of the wavelength has been established by considering that, for a speed of 160 km/h, the minimum frequency of the excitation due to the track irregularities is 0.37 Hz, while the maximum value for this frequency is 14.81 Hz, an interval accommodating the main eigenfrequencies of the vertical vibrations in the rail vehicle. Under the present conditions, the resulted value for the alignment standard deviation is 2 mm for QN1 quality level track, while the value for the QN2, σ_η = 3.3 mm.

3. THE MECHANICAL MODEL OF THE BOGIE-TRACK SYSTEM

Figure 6 shows the mechanical model of a two-axle bogie travelling at a constant velocity V on a track with vertical irregularities in alignment and crosslevel.

The model of the bogie includes 3 rigid bodies that help with modelling the bogie chassis and the two axles connected between them by Kelvin-Voigt type systems that shape the primary suspension corresponding to each axle. The elastic element of the wheelset suspension has the constant $2k_b$ and the damping elements have the constant $2c_b$.

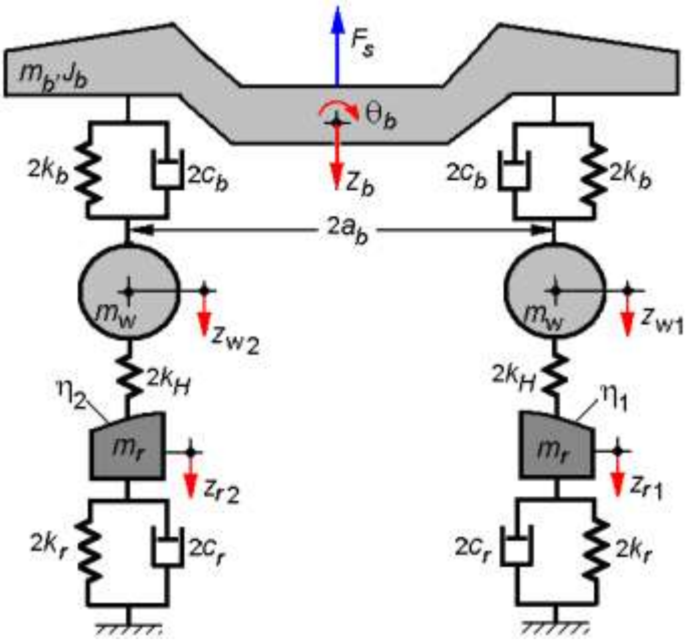

Figure 6. The mechanical model of the bogie-track system.

The rigid vibration modes of the bogie in the vertical plan, namely bounce (z_b) and pitch (θ_b) are considered. The bogie parameters are: m_b – bogie mass, $2a_b$ – bogie wheelset, $J_b = m_b i_b^2$ - inertia moment, with i_b – the gyration radius of the bogie. The axles of mass m_w operates a translation motion on the vertical direction ($z_{w1,2}$).

While overlooking the coupling effects between the wheels due to the propagation of the bending wavelength, in the frequency domain specific to the vertical vibrations of the vehicle, an equivalent model with lumped parameters will be selected. Against each axle, the track is represented as an oscillating system with one degree of freedom that can travel on the vertical direction, with the displacement $z_{r1,2}$. The equivalent model of the track has mass m_r, stiffness $2k_r$ and the damping coefficient $2c_r$.

The vertical irregularities of the track are described against each axle by functions

$$\eta_{1,2}(x_{1,2}) = K_\eta f(x_{1,2}) \sum_{k=0}^{N} U_k \cos(\Omega_k x_{1,2} + \varphi_k), \text{ for } x_{1,2} > 0, \qquad (15)$$

with $x_1 = Vt$ and $x_2 = Vt - 2a_b$.

The elasticity of the wheel-rail contact will be taken into account by introducing certain elastic elements with a linear characteristic. The calculation of the stiffness in the contact elastic elements - $2k_H$ for a wheel-rail pair – is done based on the theory of contact between two Hertz's elastic bodies by applying the linearization of the relation of contact deformation against the deformation corresponding to the static load on the wheel.

The vertical motions of the bogie-track system are described by six motion equations, corresponding to the vibration modes of the bogie – bounce and pitch, the vertical displacements of the wheels and of the rails.

The equations defining the bounce and pitch motions of the bogies are

$$m_b \ddot{z}_b = \sum_{i=1}^{2} F_{bi} - F_s ; \tag{16}$$

$$J_b \ddot{\theta}_b = a_b \sum_{i=1}^{2} (-1)^{i+1} F_{bi} , \tag{17}$$

where F_s represents the force due to the secondary suspension of the vehicle and F_{bi} are the forces from the primary suspension of the axles i (for i = 1, 2),

$$F_{b1,2} = -2c_b(\dot{z}_b \pm a_b \dot{\theta}_b - \dot{z}_{w1,2}) - 2k_b(z_b \pm a_b \theta_b - z_{w1,2}) . \tag{18}$$

The equations of the motions on the vertical direction of each axle are

$$m_w \ddot{z}_{w1,2} = 2Q_{d1,2} - F_{b1,2} , \tag{19}$$

where $Q_{d1,2}$ are the dynamic forces of contact; the dynamic forces on the wheels of an axle is considered to be equal.

To calculate the dynamic forces, the hypothesis of the Hertzian linear contact between the wheel and the rail has been opted for

$$Q_{d1,2} = -k_H [z_{w1,2} - z_{r1,2} - \eta_{1,2}], \tag{20}$$

in which k_H is the stiffness of the wheel-rail contact.

The vertical displacements of the rails are to be found in

$$m_w \ddot{z}_{r1,2} = F_{r1,2} - 2Q_{d1,2} , \tag{21}$$

where $F_{r1,2} = -2c_r \dot{z}_{r1,2} - 2k_r z_{r1,2}$. (22)

The equations of motion for the bogie-track system become

$$m_b \ddot{z}_b + 2c_b[2\dot{z}_b - (\dot{z}_{w1} + \dot{z}_{w2})] + 2k_b[2z_b - (z_{w1} + z_{w2})] - F_s = 0 \tag{23}$$

$$J_b \ddot{\theta}_b + 2c_{b1} a_b [2a_b \dot{\theta}_b - (\dot{z}_{w1} - \dot{z}_{w2})] + 2k_b a_b [2a_b \theta_b - (z_{w1} - z_{w2})] = 0 \tag{24}$$

$$\begin{aligned} & m_w \ddot{z}_{w1,2} + 2c_b(\dot{z}_{w1,2} - \dot{z}_b \mp a_b \dot{\theta}_b) + 2k_b(z_{w1,2} - z_b \mp a_b \theta_b) + \\ & + 2k_H(z_{w1,2} - z_{r1,2} - \eta_{1,2}) = 0 \end{aligned} \tag{25}$$

$$m_r \ddot{z}_{r1,2} + 2c_r \dot{z}_{r1,2} + 2k_s z_{r1,2} + 2k_H(z_{r1,2} - z_{w1,2} + \eta_{1,2}) = 0. \tag{26}$$

A system of six differential equations of second order was thus obtained, in which the variables of state, displacements and velocities are introduced,

$$q_{2k-1} = p_k, \; q_{2k} = \dot{p}_k, \text{ for } k = 1 \ldots 6, \tag{27}$$

where $p_1 = z_b$, $p_2 = \theta_b$, $p_{3,4} = z_{w1,2}$, $p_{5,6} = z_{r1,2}$. The result will be a system of 12 differential equations of first order that can be written in a matrix-like form,

$$\dot{\mathbf{q}} = \mathbf{A}\mathbf{q} + \mathbf{B}, \tag{28}$$

where **q** is the vector of the state variables, **A** the matrix of the system and **B** – the vector of the non-homogeneous terms. The system of equations (28) can be solved by a numeric integration, applying the Runge-Kutta algorithm.

4. EXPERIMENTAL MEASUREMENTS

The experimental measurements were made during a running on a current track, available for 160 km/h, on a track section in alignment and crosslevel. The measurements were done for the vertical accelerations of the axles and bogies on a passenger vehicle fitted with Minden-Deutz bogies (Fig. 7), meant for average and long lead traffic. The maximum speed of the vehicle is 140 km/h.

Figure 7. The Minden-Deutz bogie.

The experimental measurements were carried out using a measuring chain incorporating, on the one hand, the components of the measurement, acquisition and processing system for the vertical accelerations – namely four of 4514 Brüel & Kjær piezoelectric accelerometers and the set of the NI cDAQ-9174 chassis for data acquisition and the NI 9234 module for acquisition and synthesizing the data from accelerometers, and, on the other hand, the NL-602U type GPS receiver for monitoring and recording the vehicle velocity.

Figure 8. Mounting accelerometers on the bogie frame and the axle boxes.

The four accelerometers were installed on a side of the bogie, with one accelerometer on each axle box and one accelerometer on the bogie frame against each axle (Fig. 8).

Recordings were made of the accelerations measured at a constant speed on a distance of circa 60 km/traffic direction, for 20-second sequences, at the sampling frequency of 2048 Hz. The maximum running velocity reached during the measurements was 137 km/h on direction 1 and 117 km/h on direction 2.

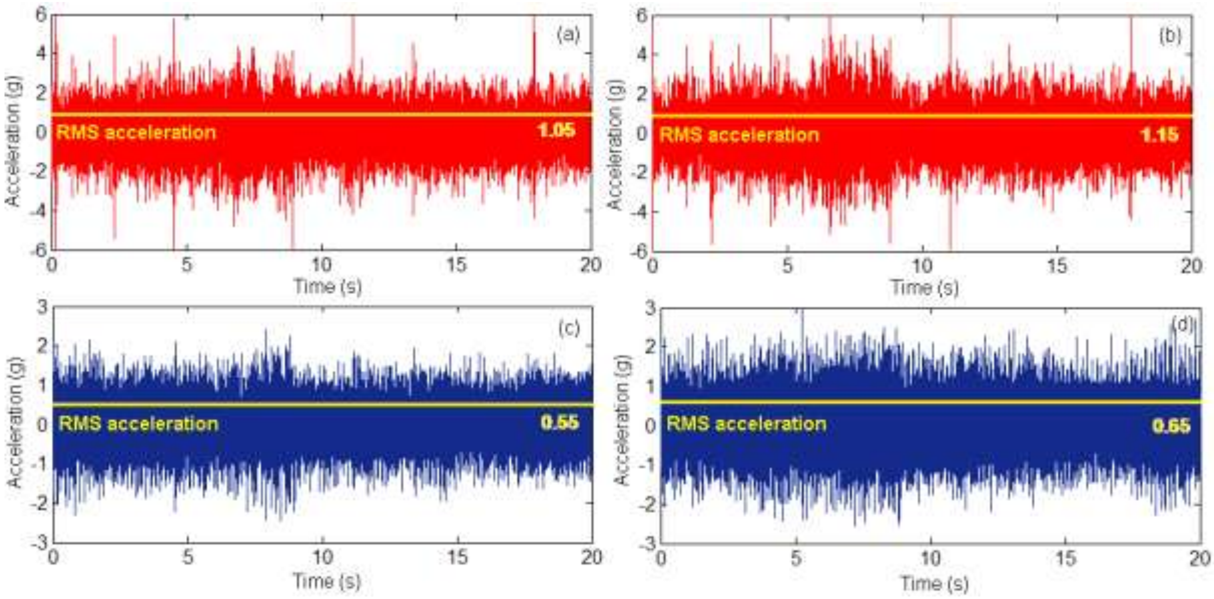

Figure 9. Accelerations registered on a measured sequence at speed 56 km/h: (a) in the axle box 1; (b) in the axle box 2; (c) on the bogie frame above axle 1; (d) on the bogie frame above axle 2.

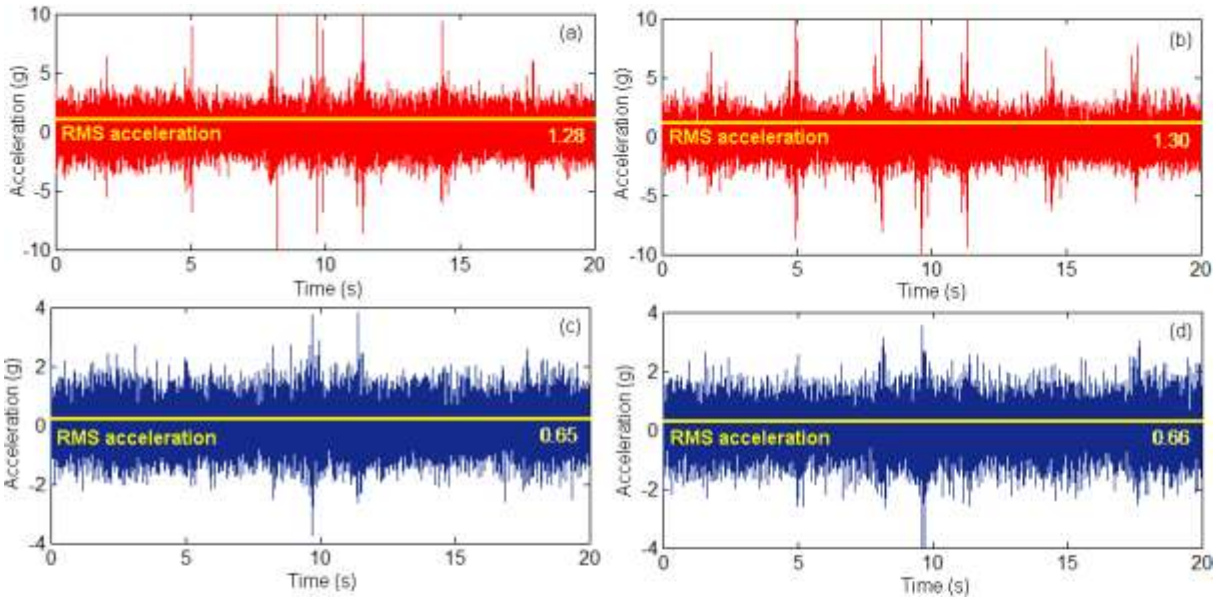

Figure 10. Accelerations recorded on a measured sequence at speed 137 km/h: (a) in the axle box 1; (b) in the axle box 2; (c) on the bogie frame above axle 1; (d) on the bogie frame above axle 2.

Fig. 9 features the accelerations recorded by the four accelerometers for a measurement sequence at speed 56 km/h, while Fig. 10 is for 137 km/h. The RMS acceleration measured in the axle boxes in both cases is circa 2 times higher than the acceleration on the bogie frame. Similarly, the results also show that the RMS acceleration goes up along with the velocity. For the axle boxes, the increase is around 22% for axle 1 and 13% for axle 2. On the bogie frame, the RMS acceleration grows by 18% against axle 1 and the percentage against axle 2 is insignificant. In the diagrams featured in Fig. 9 and Fig. 10, the acceleration is noticed to be higher in axle 2 and on the bogie frame above axle 2.

5. VERIFICATION OF METHOD TO SYNTHESIZE THE TRACK VERTICAL IRREGULARITIES

A comparison between the simulated and the experimental results based on the model in section 3 is made in order to verify the analytical method for the synthesization of the track vertical irregularities. In practice, the RMS accelerations of the axles and the RMS accelerations of the bogie above the two axles derived from the numerical simulations and measurements are examined in contrast. To this end, the measured accelerations are bandpass filtered in the frequency interval that corresponds to the wavelengths of the track irregularities for 3 to 120 m and to the velocity.

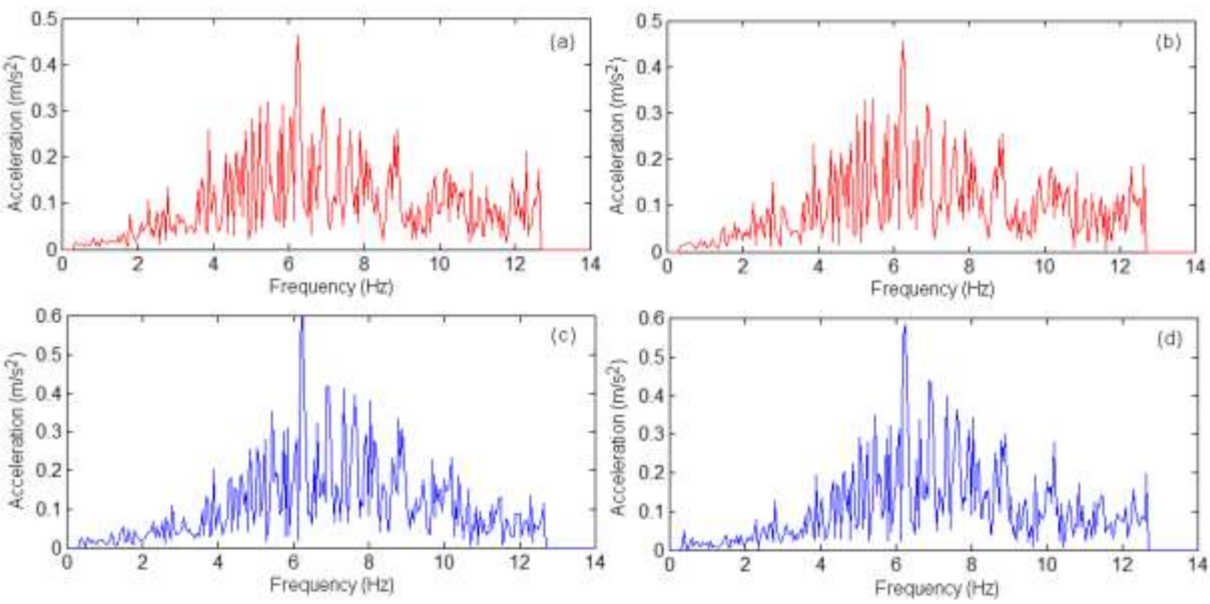

Figure 11. Spectra of the acceleration measured at speed 137 km/h: (a) in axle 1; (b) in axle 2; (c) on the bogie frame above axle 1; (d) on the bogie frame above axle 2.

For instance, Fig. 11 shows the spectra of the accelerations measured on a time sequence at speed 137 km/h, filtered in the 0.32 – 12.68 Hz frequency range. Due to the fact that this interval includes the eigen frequencies of the bogie (bounce and pitch), the accelerations are seen to be higher on the bogie frame than in the axle boxes, unlike the situation above in Fig. 9 and Fig. 10.

As seen in Table 1, the parameters of the numerical model were specially provided for Minden-Deutz bogie, while the contribution of 300 spectral components with wavelengths between 3 and 120 m was considered to synthesize the vertical irregularities in the track. The standard deviations for the vertical alignment and the peak values of the isolated defects were chosen for the maximum velocity of 140 km/h, namely $\sigma_{\eta QN1}$ = 1.4 mm; $\sigma_{\eta QN2}$ = 1.7 mm; η_{admQN1} = 6 mm; η_{admQN2} = 10 mm (see. Fig. 1 and Fig. 2).

Table 1. The parameters of the numerical model

Bogie mass	m_b = 2700 kg
Axle mass	m_w = 1400 kg
Rail mass (under the axle)	M_r = 175 kg
Bogie wheelset	$2a_b$ = 2.5 m
Inertia moment	$J_b = 1.73 \cdot 10^3$ kg·m^2
Elastic constant of the suspension corresponding to a wheel	k_b = 0.616 MN/m
Damping constant of the suspension corresponding to a wheel	c_b = 9.05 kNs/m
Rail vertical stiffness	k_r = 70 MN/m
Vertical damping of the track	c_r = 20 kNs/m
Stiffness of the wheel-rail contact	k_H = 1500 MN/m

Figure 12 shows the RMS accelerations for 25 measurement sequences at the constant speed of 117 km/h and the RMS accelerations derived from the numerical simulations for a QN1 quality track and QN2 quality track, respectively. The measured accelerations were band pass filtered in the range frequency of 0.27 – 10.83 Hz. Each measurement sequence is described by a certain value of the RMS acceleration, between 0.75 m/s^2 and 1.23 m/s^2 for axles, with a medium value of 0.96 m/s^2; for the bogie frame, the values are between 0.80 m/s^2 and 1.84 m/s^2, with an average value of 1.15 m/s^2 and 1.19 m/s^2. This interval also contains the value of the RMS acceleration from numerical simulations for a QN2 quality track. In all the cases under study, this value is very close to the average value of the measured RMS accelerations.

The difference between the RMS acceleration from numerical simulations and the average of the measured RMS accelerations does not exceed 10%. But instead, the RMS acceleration from numerical simulations for a QN1 quality track is outside the intervals of values corresponding to the measured RMS acceleration.

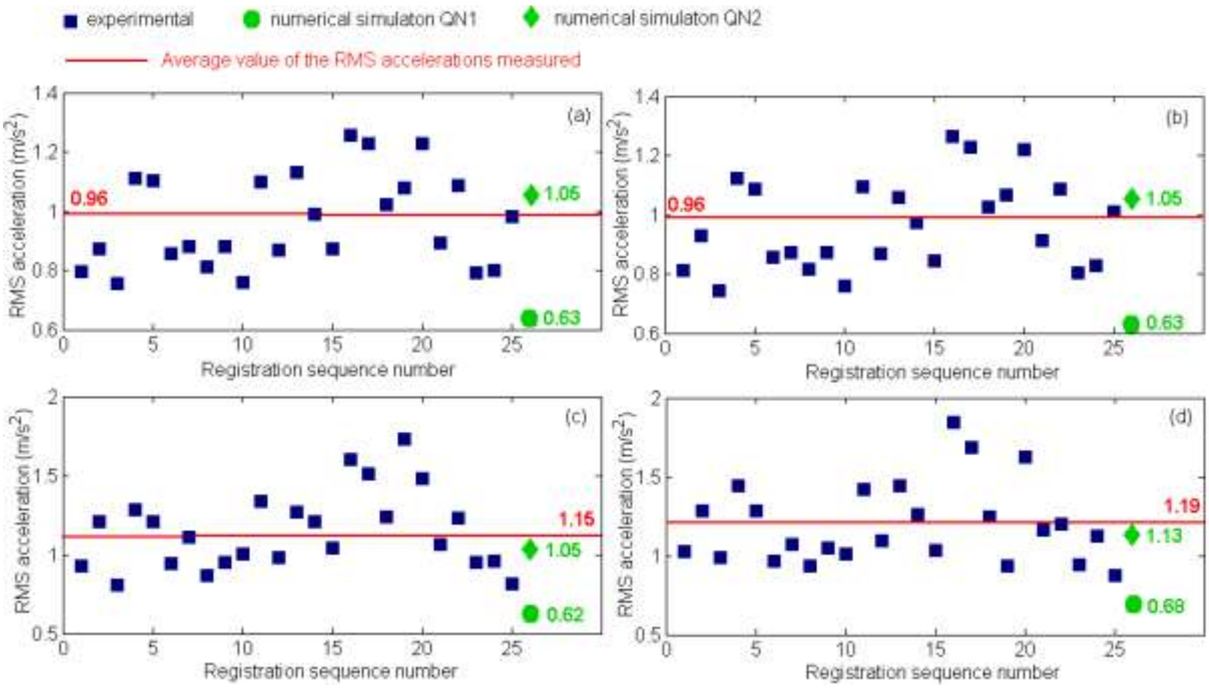

Figure 12. Comparison of the RMS accelerations from numerical simulations to the RMS accelerations measured at the speed of 117 km/h: (a) in axle 1; (b) in axle 2; (c) on the bogie frame above the axle 1; (d) on the bogie frame above the axle 2.

Figure 13 features the RMS acceleration for 20 measurement sequences at the constant speed of 137 km/h and the RMS accelerations from numerical simulations for a QN1 quality track and QN2 quality track. In this case, the measured accelerations were band pass filtered in the 0.31 – 12.68 Hz frequency range. The values for the RMS accelerations in the axle 1 are between 0.93 m/s^2 and 1.93 m/s^2, with an average of 1.48 m/s^2, while in the axle 2, the values are 0.96 m/s^2 - 2.02 m/s^2, with an average of 1.53 m/s^2. The difference between the RMS acceleration from numerical simulations for a QN2 quality track and the average value is of circa 8% in axle 1 and 5% for axle 2.

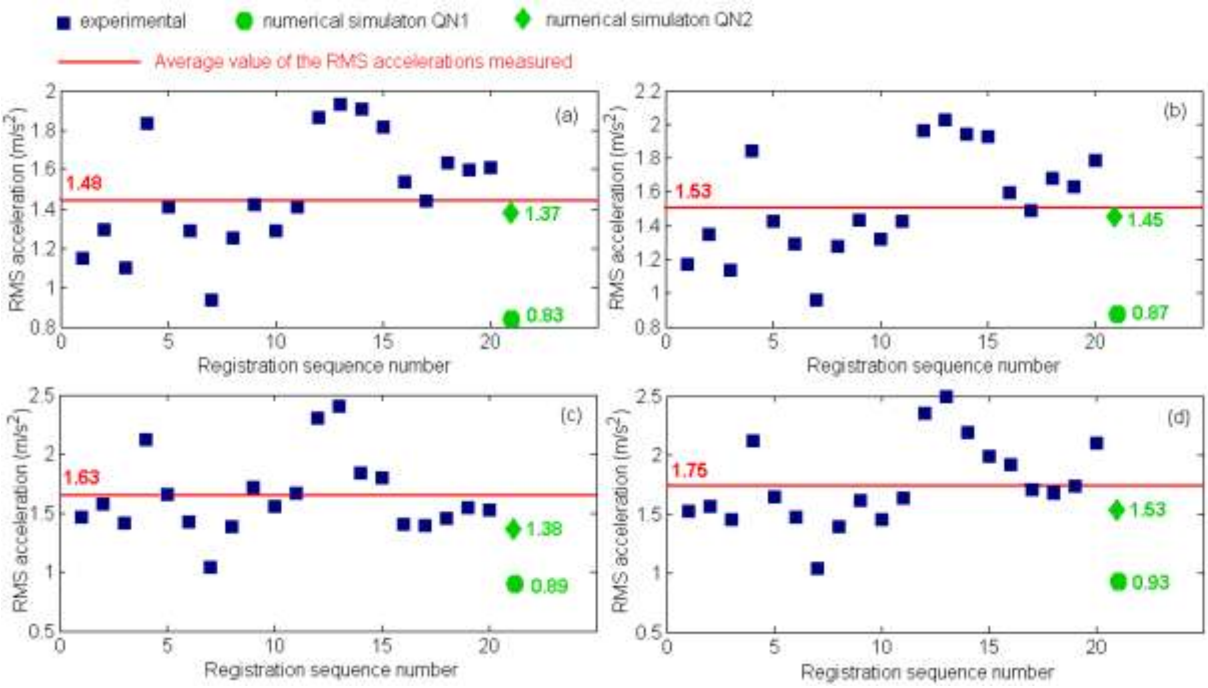

Figure 13. Comparison of the RMS accelerations between thenumerical simulations to the RMS accelerations measured at the speed of 137 km/h: (a) in axle 1;
(b) in axle 2; (c) on the bogie frame above the axle 1;
(d) on the bogie frame above the axle 2.

The RMS accelerations measured on the bogie frame vary from 1.04 m/s^2 to 2.40 m/s^2 – above axle 1, with an average of 1.63 m/s^2, and from 1.03 m/s^2 to 2.49 m/s^2 – above axle 2, with an average of 1.75 m/s^2. Herein, the differences between the RMS acceleration from numerical simulations for a QN2 quality track and the average values are higher than 10%, namely 18% - above axle 1; 14% - above axle 2.

6. CONCLUSIONS

The paper turns to the experiment in order to verify an analytical method to synthesize the track vertical irregularities. The method herein suggested lies on the power spectral density function of the track irregularities, as defined in ORE B176, where the track quality coefficient was changed so that it will match the specifications in the UIC 518 Leaflet regarding the track geometrical quality for the vertical alignment. The vertical track irregularities are analytically described via a pseudorandom function that depends on the track quality level and on the reference speed.

An important advantage of the method herein consists of the fact that it enables the convenient establishment of the limits in the specific domain of the wavelengths in the track irregularities, so that it will be representative for the frequency range of the vertical vibrations in the railway vehicle. Another advantage is related to the idea that the method can be easily adjusted to any other form of power spectral density that describes the track irregularities and be also applied to synthesize the lateral alignment [20, 21].

The verification of the method has been conducted by comparing the RMS accelerations of the axles and of the bogie frames against each axle through numerical simulations and experimental determinations for wavelengths of the track vertical irregularities between 3 and 120 m. The results here have shown a good correlation between the simulated RMS accelerations for a low quality track and the measured RMS accelerations.

The differences between the simulated RMS accelerations of the axle and the average value of the RMS accelerations measured in the axle box do not exceed 10%. For the RMS accelerations in the bogie frame, such differences increased along with the velocity, being able to reach up to circa 18%.

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