

PARAMETRIC STUDY ON HOW THE LATERAL FORCES ACTING ON A DEEP FOUNDATION INFLUENCES THE AXIAL FORCES DEVELOPING IN THE PILES

Radu Adrian Iordănescu, PhD candidate eng., UTCB, e-mail: r_iordanescu@yahoo.com

Rezumat

O piesa lipsa in automatizarea proiectarii infrastructurilor de poduri este faptul ca relatia din mecanica constructiilor cu care se evalueaza solicitarile in coloanele unei fundatii indirecte nu tine cont si de efectul fortelor laterale. In practica, solicitarile coloanelor sunt determinate cu ajutorul unui program de tip MEF. Aceasta metoda inasa, nu poate fi usor incorporate intr-un program automat care efectueaza optimizari locale si globale ale unei structuri. In primul rand, deoarece este deosebit de consumatoare de resurse computationale. Avand in vedere ca un pod poate avea un numar de fundatii indirecte, care trebuie calculate pentru diverse combinatii de actiuni, care trebuie optimizate, recalulate in diverse scenarii si apoi reiterat intregul proces pentru toate solutiile structurale, costul computational poate deveni prohibitiv. In al doilea rand, din cauza lipsei unei relatii intre toti parametrii si dimensiunile care influenteaza comportarea unei fundatii indirecte, optimizarea locala a acestora este dificila.

In acest scop, s-a efectuat in cadrul lucrarii de fata un studiu parametric care sa investigheze ce parametrii influenteaza relatia dintre fortele laterale aplicate pe fundatie si fortele axiale ce se dezvoltă in piloti, iar in final sa propuna o relatie care sa ia in considerare inclusiv fortele laterale si care poata fi aplicata direct. Studiu se desfasoara prin utilizarea datelor experimentale obinute pe modele utilizind analiza prin metoda elementelor finite cu ajutorul programului SAP 2000(v.15).

Cuvinte cheie: infrastructura, fundatii indirecte, piloti, forta axiala, forte laterale, seism

Abstract

A missing piece in the design of bridge substructure is that the equation given in structural mechanics that assesses the axial forces in the piles of a deep foundation does not take into account the effect of lateral forces acting on the pile cap. In practice, pile forces are determined using a FEA software. This method, however, can not be easily incorporated into an automated program that performs local and global optimizations of a structure. One of the reasons is that this method is particularly demanding on the computational resources. Since a bridge can have a number of deep foundations, which must be verified for various combinations of actions, which need to be optimized, recalculated in various scenarios and

then the entire process reiterated for all structural solutions, computational cost can become prohibitive. Another reason is that due to the lack of a relation between all the parameters and dimensions that influence the behaviour of a deep foundation, their optimization is difficult.

For this purpose, a parametric study has been carried out to investigate what parameters influence the relation between the lateral forces applied to the foundation and the axial forces that develop in the piles, and ultimately propose an equation that takes into account the lateral forces. The study is carried out using experimental data obtained on models using the finite element analysis method using SAP 2000 (v.15) software.

Keywords: bridge substructure, deep foundation, piles, axial force, shear force, earthquake

1. THE ANALITICAL EQUATION VS. THE FEA METHOD

The following model illustrates the differences between the results obtained using the analytical equation and using the FEA method. The example consists of a single deep foundation with six piles, Figure 1. The structure must be verified against an earthquake acting in the direction of the longitudinal axis of the bridge. The piles are 15.00m long and 1.20m in diameter. They are arranged in 2 rows that are 4.00m apart and 3 columns that are 3.75m apart. The soil consists of a single layer with a lateral stiffness of $k=1,000tf/m$. The superstructure and the live loads are modelled as a point mass.

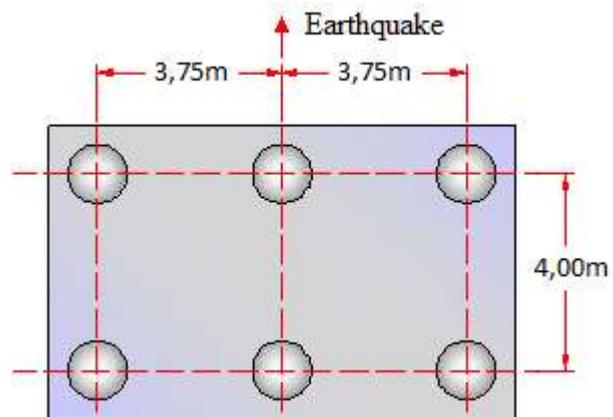


Figure 1. Aerial view of the deep foundation.

Following the seismic analysis the loads in the upper centre of the pile cap are:

Axial force: $N = 750tf$ Bending moment: $M = 1500tfm$ Shear force: $T = 300tf$

1.1. Axial forces in the piles obtained using the analytical equation

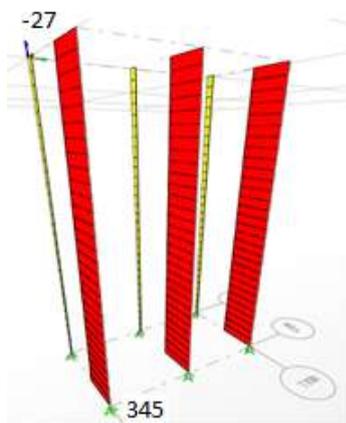
The analytical equation for assessing the axial forces in the tip of the piles:

$$S_i = \frac{N}{n_p} + G_p \pm \frac{M}{d_r \times n_{p.1sir}} \quad (1)$$

where: S_i = axial force in the tip of the piles;
 n_p = total number of piles;
 G_p = weight of a single pile;
 d_r = distance between the piles axes in the earthquake's direction;
 $n_{p.sir}$ = number of piles in a row.

Substituting the load values in equation (1): $S_{max} = 293tf$ and $S_{min} = 43tf$

1.2. Axial forces in the piles obtained using Finite Element Analysis



$$S_{max} = 345tf \quad S_{min} = -27tf$$

There is an important difference between the results obtained using equation (1) and those obtained using FEA, the ones obtained using FEA being greater.

By analysing equation (1) it becomes apparent that it doesn't take into account the effect of the shear force, the interaction between the piles and the soil and neither the stiffness of the pile cap.

Figure 2. Axial force diagrams in the piles using SAP 2000.

2. ASSESING THE EFFECTS OF THE 3 LOADS SEPARATLY

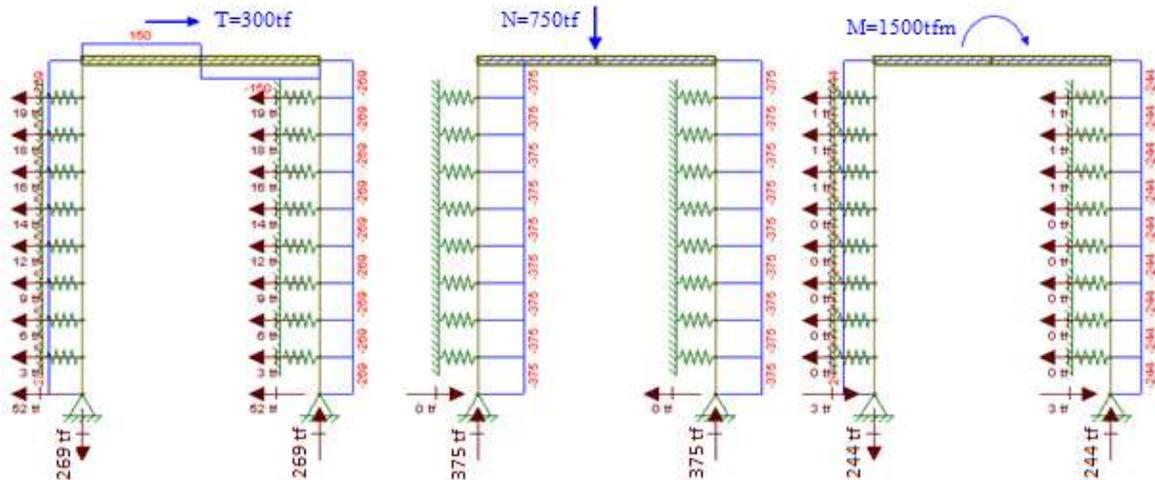


Figure 3. Models of the foundation loaded separately with the 3 loads.

Thus, following this analysis, the following conclusions can be drawn:

1. The effect of the axial force is accurately described by equation (1).
2. The effect of the bending moment is slightly different, the probable cause being that by bending the piles, reactions in the horizontal springs that model the effect of the soil's lateral stiffness develop, which in turn give rise to a bending moment of opposite sign to the external load. Thus, resulting in axial forces in the piles slightly reduced from those described by equation (1).
3. The shear force has a significant effect that equation (1) does not take into account.

From this analysis it can be concluded that the differences between the results obtained using equation (1) and those obtained using FEA are mainly the outcome of the fact that equation (1) does not take into account the effects of the shear force.

3. ASSESING THE EFFECTS OF THE SHEAR FORCE DEPENDING ON THE CONFIGURATION OF THE DEEP FOUNDATION

3.1. Assessing the influence of the piles length 'l'

The length of the piles 'l' is increased from 3m to 30m with a step of 2m, while the rest of the elements remain constant.

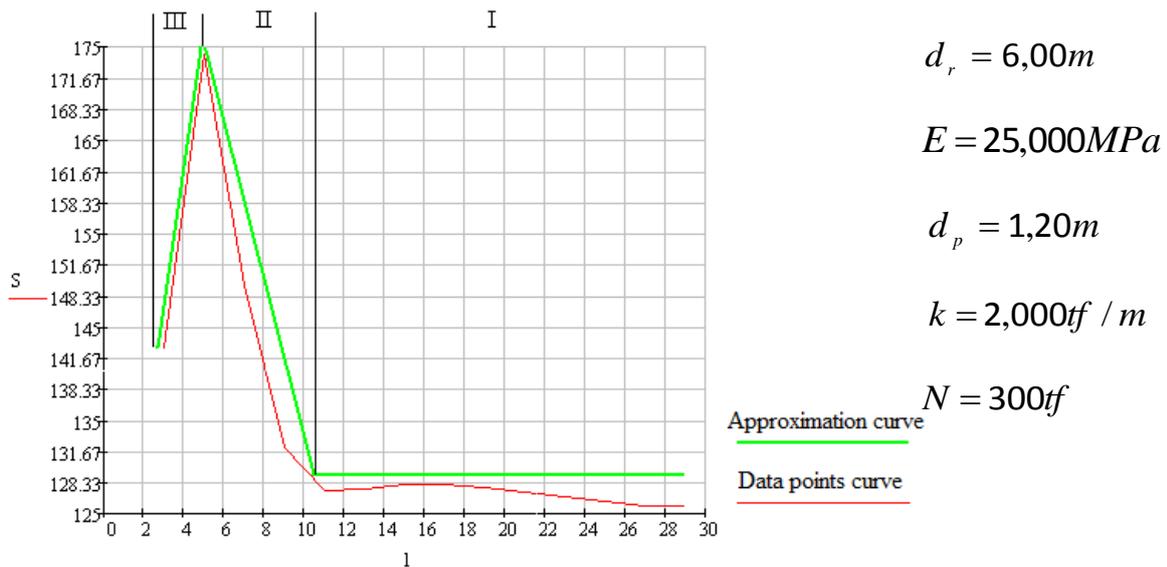


Figure 4. Axial force graph depending on the pile length.

- The function of the axial force with the variable pile length is divided into 3 domains.
- The function of each domain is approximated by a linear variation. For domain 1, the maximum residual value is $3t$, and the maximum error is 2.3%. For domain 2, the maximum residual value is $6t$, and the maximum error is 4.5%.
- Domain 3 is of no further interest, as it is not applicable in practice.

3.2. Assessing the variation in the boundary limit between domain I and domain II depending on the distance between the pile rows ‘ d_r ’

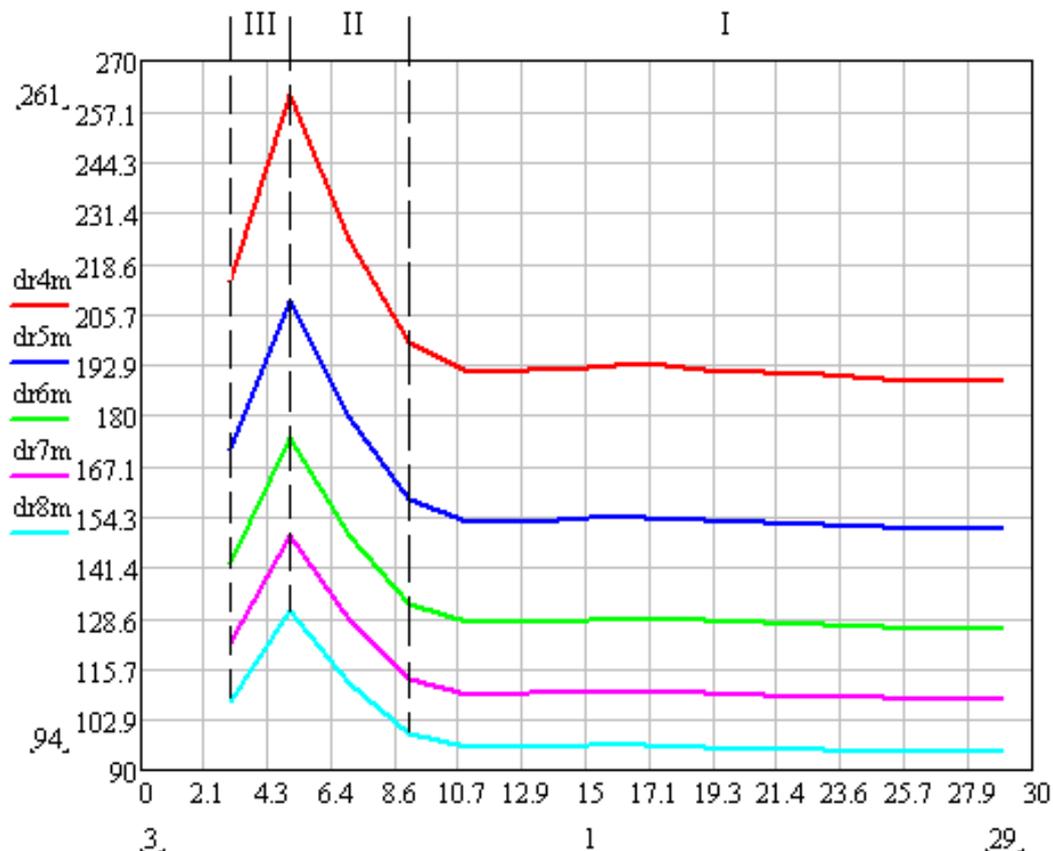


Figure 5. The graph of the axial force depending on the pile length and on the distance between the pile rows in the earthquakes direction.

It can be noticed that although the axial force of the piles varies with the distance between the pile rows, the boundary between the 3 domains of the function remains constant.

3.3. Variation of the boundary depending on the modulus of elasticity ‘E’

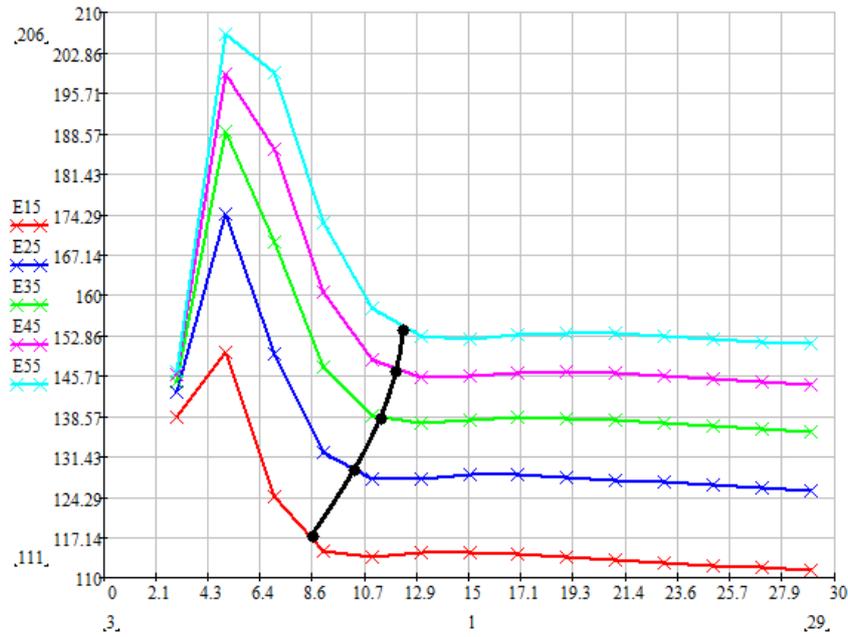


Figure 6. Axial force depending on the pile length and E

The E-limit curve has a function of the type: $f = \sqrt[3]{E}$.

3.4. Variation of the boundary depending on the diameter of the piles ‘d_p’

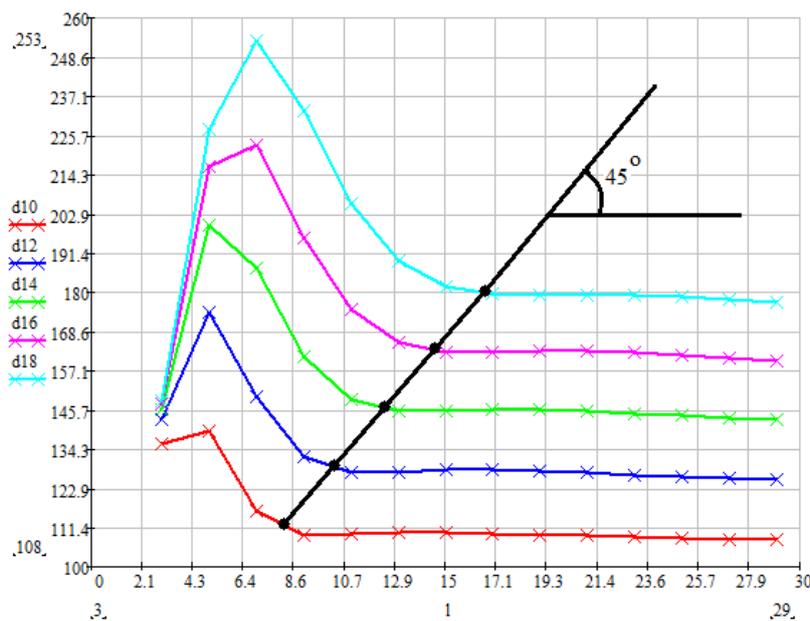


Figure 7. Graph of the axial force depending on the piles length and diameter

The limit curve is a function of the type: $f = x \times d_p$.

3.5. Variation of the boundary depending on the stiffness of the springs 'k'

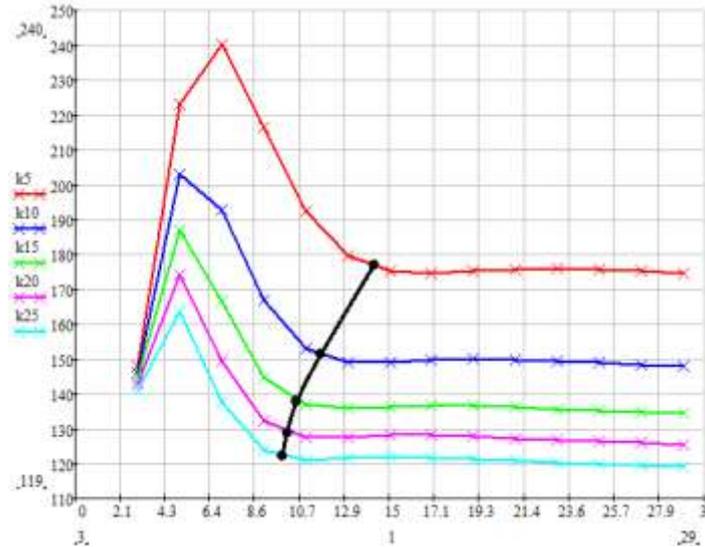


Figure 8. Axial force depending on the pile length and stiffness of the springs

The k-limit curve is a function of the type: $f = x^{\log(k)}$.

3.6. Variation of the boundary depending on the value of the shear force

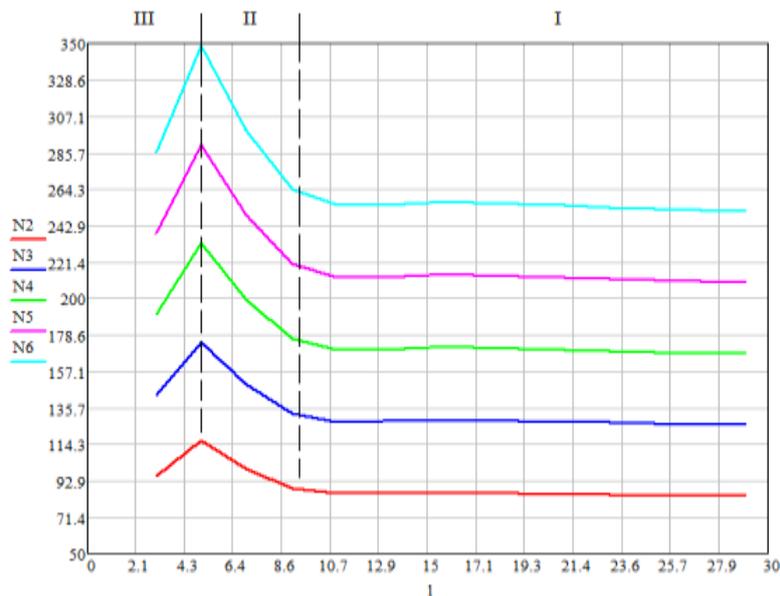


Figure 9. Graph of the axial force depending on the pile length and shear force

It can be seen that although the axial force in the piles varies with the value of the shear force, the boundary between the 3 domains remain constant.

3.7. Conclusions on the limits of the domains of the axial force function

1. Boundary limit between domain I and domain II:

a) It does not vary depending on:

- the distance between the pile rows.
- the value of the shear force.

b) It varies depending on:

- the modulus of elasticity 'E' of the piles concrete, exponentially.
- the piles diameter 'd_p', linearly.
- the stiffness of the springs 'k' in a logarithmic manner.

2. Taking into account the nature of the values on which piles are placed either in domain I or domain II, the domains can be defined as:

Domain I → The domain of flexible piles.

Domain II → The domain of stiff piles.

3. Analyzing how the boundary between domains I and II varies depending to the 3 parameters described in 3.7(1-b), a classification equation can be written:

$$\left| \begin{array}{l} \text{if } \frac{\sqrt[3]{E} \times d_p}{l_p \times 1,75^{\log\left(\frac{k}{50 \times d_{res}}\right)}} \leq 1,5 \quad \text{Flexible} \quad \text{otherwise} \quad \text{Stiff} \end{array} \right. \quad (2)$$

The pile's minimum length to be classified as flexible can be evaluated:

$$l_{p.\min} = \frac{\sqrt[3]{E} \times d_p}{1,5 \times 1,75^{\log\left(\frac{k}{50 \times d_{res}}\right)}} \quad (3)$$

3.8. Checking the general validity of the pile classification equation

The values of the minimum lengths of the piles at the border between domain I and domain II obtained using equation (3) will be checked against FEA results. The checks will be carried out for 5 random cases.

Table 1. The proposed cases to check equation (3).

Case 1	Case 2	Case 3	Case 4	Case 5
E=40000MPa	E=35000MPa	E=30000MPa	E=25000MPa	E=20000MPa
k=500t/m	k=1.000t/m	k=1.500t/m	k=2.000t/m	k=2.500t/m
d _p =1,80m	d _p =1,60m	d _p =1,40m	d _p =1,20m	d _p =1,00m

The values obtained using eq.(3) for the minimum length of the flexible pile:

$$l_{p,\min} = \frac{\sqrt[3]{E \times d_p}}{1,5 \times 1,75^{\log\left(\frac{k}{50}\right)}} = \begin{pmatrix} \text{case1} = 23,45 \\ \text{case2} = 16,85 \\ \text{case3} = 12,69 \\ \text{case4} = 9,54 \\ \text{case5} = 6,99 \end{pmatrix}$$

The values obtained using FEA for the minimum length of the flexible pile:

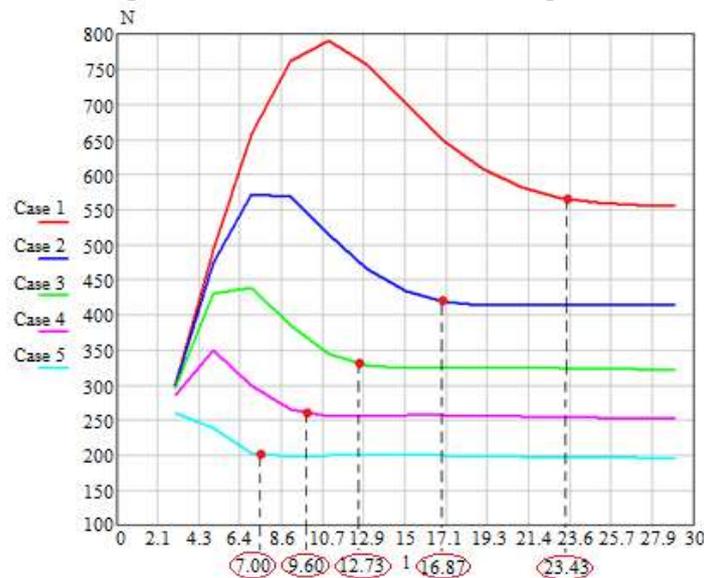


Figure 10. Axial forces for the 5 test cases depending on the pile's length

Table 2. Comparison of the values for ‘ $l_{p,min}$ ’ obtained using eq.(3) and FEA.

	Case 1	Case 2	Case 3	Case 4	Case 5
Eq.(4)	23,45	16,85	12,69	9,54	6,99
FEA	23,43	16,87	12,73	9,60	7,00

4. EFFECT OF THE DISTANCE BETWEEN THE PILE ROWS ‘ d_r ’

The distance between the pile rows in the earthquakes direction is varied from 4m to 8m with a 1m step. The rest of the parameters are kept constant.

S- Curve obtained by data points
R-Logarithmic function used to approximate the real curve

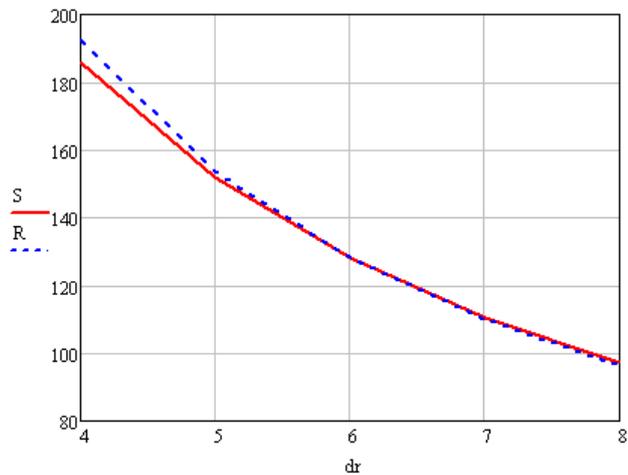


Figure 11. Graph of the axial force depending on the distance between the pile rows

5. ASSESSING THE EFFECT OF THE MODULUS OF ELASTICITY ‘E’ OF THE PILE’S CONCRETE OF THE AXIAL FORCE

The modulus of elasticity ‘E’ is varied from 15,000MPa to 55,000MPa with a 10,000MPa step, while the rest of the parameters are kept constant.

S- Curve obtained by data points
R-Logarithmic function used to approximate the real curve

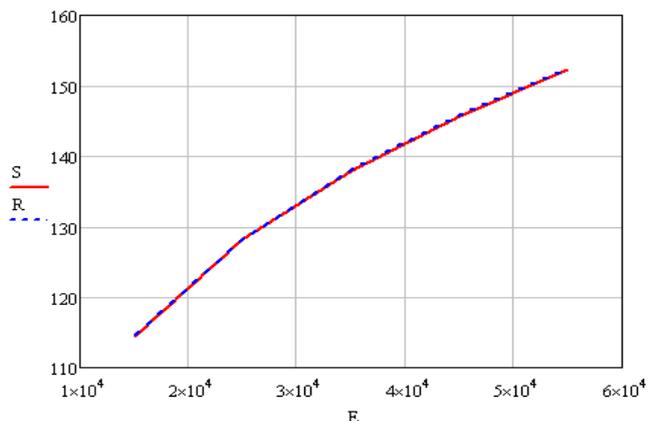


Figure 12. Graph of the axial force depending on the modulus of elasticity of the pile’s concrete.

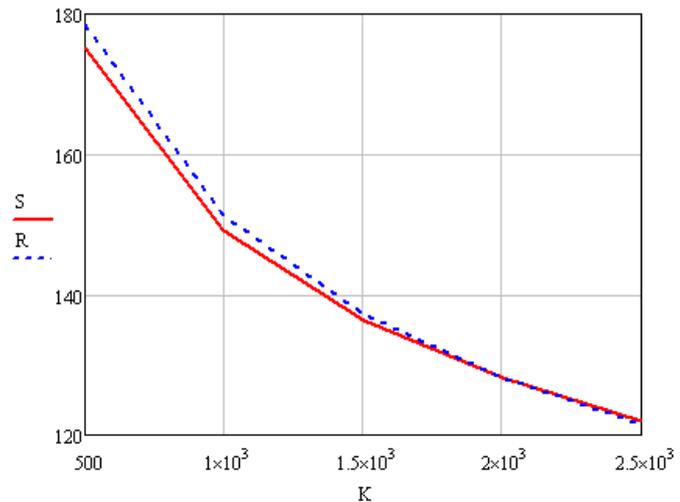
6. ASSESSING THE EFFECT OF THE SOIL'S LATERAL STIFFNESS

The stiffness of the springs 'k' are varied from 500tf/m to 2,500tf/m with a 500tf/m step, while the rest of the parameters are kept constant.

S- Curve obtained
by data points

R- Logarithmic function
used to approximate
the real curve

Figure 13. Graph of the axial force depending on the stiffness of the springs k.



7. ASSESSING THE EFFECT OF THE PILES DISMETER 'd_p'

The piles diameter 'd_p' is varied from 1.00m to 1.80m with a 20cm step, together with the stiffness of the springs (as it has been observed that the function of 'd_p' is influenced by 'k'), the rest of the parameters remain constant.

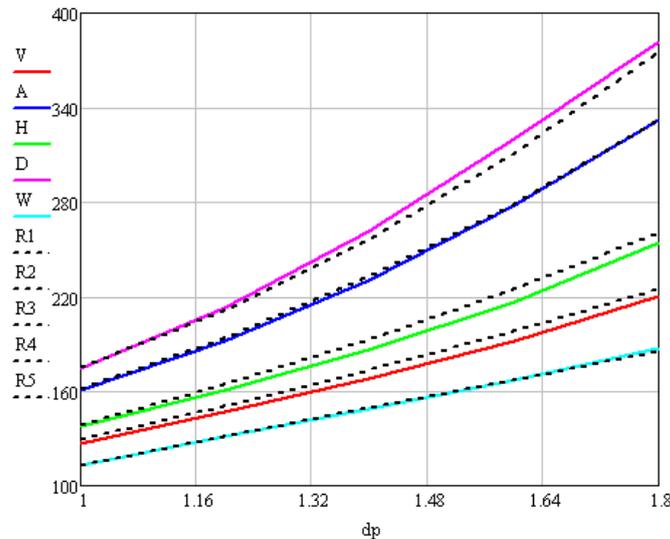


Figure 14. Axial force depending on the piles diameter 'd_p' and 'k'.

- W,V,H,A and D are the data points curves.
- R1,2,3,4 and 5 are functions that approximate the real curves.
- From the analysis of Figure 14 can be observed that de function of 'd_p' of an exponential type, and that it is dependent on the stiffness of the springs 'k'.
- For simplicity the function of 'd_p' can be divided into 2 domains.
- Domain 1, where the exponential effect is negligible and the thus the function can be approximated by a linear variation. For k > 1,600tf/m.
- Domain 2, where the exponential effect is significant and the function needs to be approximated by a power function in which the exponent increases logarithmically with the spring stiffness. For k < 1,600tf/m.

8. ESTABLISHING THE EQUATION FOR ASSESSING THE EFFECT OF THE SHEAR FORCE ON THE AXIAL FORCE IN THE PILES

The equation is determined on the basis of the curves of the axial forces when the independent parameters are varied as determined earlier using an iterative process of successive attempts to overlap an approximate curve over all the curves resulted from the data points obtained using FEA, with the aim that the resulting maximum error must be 5%.

8.1. Domain I for k < 1600tf/m

$$5 \times \left[\left(\frac{d_p}{1,5} \right)^{\log\left(\frac{85000-25k}{k}\right)} + \frac{4 + \log\left[\frac{1500}{\left(\frac{k}{100}\right)^{1,8}} \right]}{10} \right] \times \left(\frac{E}{22000} \right)^{0,22}$$

$$d_r \times 1,8^{\log\left(\frac{k}{50}\right)} \tag{4}$$

8.2. Domain II for $k > 1600\text{tf/m}$

$$\frac{5 \times \left(\frac{d_p}{1,07} + 0,1 \right) \times \left(\frac{E}{22000} \right)^{0,22}}{d_r \times 1,8^{\log\left(\frac{k}{50}\right)}} \quad (5)$$

9. CHECKING THE NEWLY ESTABLISHED EQUATIONS

- The check will be carried out for 12 randomly chosen test cases.
- The objective is that the equation will have a 95% precision for a distance between the springs that have a range of 0.5m to $d_p/2$.
- All test cases are loaded with a shear force $T=300\text{tf}$.

Table 3. Comparison of the axial forces using equations (4)(5) and FEA.

Case	Parameters				FEA	Eq. (4)(5)	Diff.
	d_r	E	d_p	k			
1	6	25.000	0.8	350	127t	133t	4.5%
2	4	37.000	1.2	500	275t	282t	2.5%
3	7	50.000	1.0	700	133t	133t	0 %
4	8	42.000	1.4	2000	119t	119t	0%
5	5	27.000	1.8	2500	198t	206t	3.9%
6	6	30.000	1.2	1700	171t	173t	1.7%
7	6	35.000	1.2	1000	154t	155t	0.8%
8	8	20.000	1.0	750	93t	93t	0%
9	7	35.000	1.3	1500	130t	131t	0.7%
10	5	47.000	1.5	1300	232t	234t	0.8%
11	8	28.000	1.6	1800	126t	126t	0%
12	6	36.000	2.0	2000	223	214t	-4%

10. INTRODUCING THE PARAMETER ‘ K_{med} ’

10.1. Determining the parameter ‘ K_{med} ’

K_{med} is a parameter that represents the average stiffness of the springs that model the effect of the soil in order for the equations (4) and (5) to work with multiple soil strata.

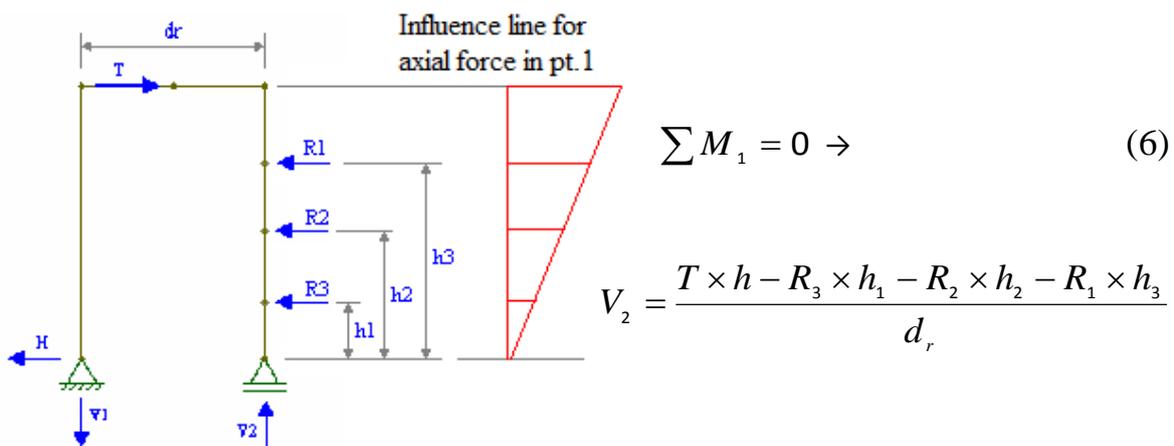


Figure 15. Static scheme of the foundation where R1,2 and 3 are the springs that model the soil.

- The effect of the springs varies linearly with their depth.
- K_{med} can thus, be described as being the weighted mean of the stiffness of all the springs with regard to each spring's depth.

$$K_{med} = \frac{\sum_1^n K_i \times h_i}{\sum_1^n h_i} \quad (7)$$

10.2. Checking the validity of the ‘ K_{med} ’ equation

A comparison is carried out between the structure modelled with the real stiffness of the soil's strata and the structure modelled with a single, equivalent stiffness determined with equation (7).

Radu Adrian IORDĂNESCU

Parametric study on how the lateral forces acting on a deep foundation influences the axial forces developing in the piles.

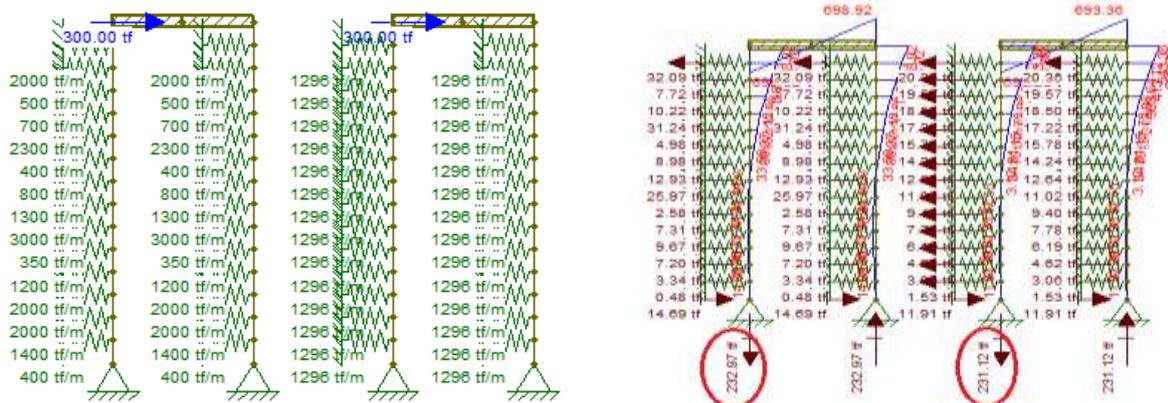


Figure 16. Comparison between the axial forces obtained using the real stiffness of all the soil strata and using the equivalent stiffness K_{med}

The model of the structure with all the soil strata gives an axial force of 232.97t and the model of the structure with the equivalent stiffness gives an axial force of 231.12t. Thus, the residual value is -1.85t and the error is -0.79%.

11. CONCLUSIONS

1. Equation (1) is supplemented with the effect of the shear force and becomes:

$$S_i = \frac{N}{n_p} + G_p \pm \frac{M}{d_r \times n_{p,1sir}} \pm \frac{\rho \times T}{n_{p,1sir}} \quad (8)$$

Where: ρ is the coefficient of the shear force. And it is calculated as follows:

2. The stiffness of the springs "k" that model the effect of the soil for each strata is determined, with a distance between them in the range of 0.5m and $d_p / 2$.

3. ' K_{med} ' is determined using equation (7):

$$K_{med} = \frac{\sum_1^n K_i \times h_i}{\sum_1^n h_i}$$

4. It is determined whether the piles are flexible or stiff using equation (3):

**ROMANIAN JOURNAL
OF TRANSPORT INFRASTRUCTURE**

Radu Adrian IORDĂNESCU

Parametric study on how the lateral forces acting on a deep foundation influences the axial forces developing in the piles.

$$l_{p.min} = \frac{\sqrt[3]{E} \times d_p}{1,5 \times 1,75^{\log\left(\frac{K_{med}}{50}\right)}}$$

5. ρ coefficient is determined using equations (4) or (5) substituting k with K_{med}

REFERENCES

- [1]. “NP 123 Normativ privind proiectarea geotehnica a fundatiilor pe piloti”, UTCB, Contract nr. 314/2007
- [2]. W. F. CHEN, L. DUAN: “*Bridge engineering, substructure design*”, Taylor & Francis Group, LLC, New York, 2003
- [3]. A. J. BOND, B. SCHUPPENER, G. SCARPELLI, T. L.L. ORR: “*Eurocode 7: Geotechnical Design Worked examples*”, Worked examples presented at the Workshop Eurocode 7: Geotechnical Design, Dublin, 13-14 June, 2013
- [4]. R. FRANK: “*Design of pile foundations following Eurocode 7-Section 7*”, Workshop Eurocodes: background and applications, Brussels, 18-20 February 2008
- [5]. “SR EN 1997-1, Eurocod 7: Proiectarea geotehnica, Partea 1: Reguli generale”, ASRO, 2008
- [6]. I. MANOLIU: “*Ghid pentru aplicarea standardelor SR EN 1997-1:2004 + SR EN 1997-1:2004/NB:2007 Proiectarea geotehnica: Partea 1 – Reguli generale si SR EN 1997-2:2007 + SR EN 1997-1:2007/NB:2009 Proiectarea geotehnica: Partea 2 – Investigarea si incercarea terenului*”, UTCB, Contract: nr. 467/2010