DE GRUYTER OPEN

Reports on Geodesy and Geoinformatics vol. 97 /2014; pages 103-111 DOI: 10.2478/rgg-2014-0015

PRELIMINARY UNIFICATION OF KRONSZTADT86 LOCAL VERTICAL DATUM WITH GLOBAL VERTICAL DATUM

Adam Łyszkowicz, Joanna Kuczyńska-Siehień, Monika Biryło

Department of Land Surveying and Geomatics,
University of Warmia and Mazury in Olsztyn
Olsztyn, Poland

Abstract

The study concerns computation of the gravity potential difference between the Kronsztadt86 datum and the global vertical datum. This method is based on the use of ellipsoidal heights from satellite observations, normal heights obtained from the conducted leveling campaign and quasigeoid/ellipsoid separations computed based on the EGM2008 model. The obtained results indicate that there are substantial differences in the estimated value of the parameter ΔW, computed from three different satellite networks: POLREF, EUVN-DA and ASG-EUPOS. The parameter was determined with sufficient accuracy and the applied systematic error model has low efficiency. The computations reveal that the best value of ΔW for the territory of Poland is 0.43 m^2s^{-2}.

Keywords: vertical datum unification, geopotential differences, gravity potential

1. Introduction

The study, understanding and modelling of all global changes requires suitable geodetic datum with an accuracy level higher than the values of the studied phenomena, which is coherent and reliable over the entire terrestrial globe (the same accuracy everywhere) and stable over a long period of time (the same accuracy level each time). The International Terrestrial Reference System (ITRS) guarantees global uniform geometric datum with millimetre accuracy (Petit & Luzum, 2010). At the same time, there is no equivalent, in terms of accuracy, of physical vertical datum with such high precision.

Work on the definition and creation of global vertical datum is underway in the “Vertical Datum Standardisation” working group which directly depends on the GGOS Theme 1 and is supported by the IAG Commissions 1 (Reference Frames) and 2 (Gravity Field), as well as by the International Gravity Field Service (IGFS). The main purpose of the working group is to provide a reliable geopotential value W_0 to be introduced as the conventional reference level for the realization of a Unified Global Height System (Sanchez, 2012).

If the global value of W_0 is determined, then the problem of transformation of local vertical datums into the global datum will arise. The parameter ΔW is necessary for this purpose (Fig. 1.). This subject was raised in a number of papers, among others, by Hayden et al. (2013), Hoa (2013), Kotsakis (2011).

The aim of this paper is to determine the parameter ΔW for the Kronsztadt86 vertical datum. The value recommended by the IERS Conventions 2010 was adopted as the global value of W_0, which is the value of 62 636 856.00 m^2 s^{-2}.

2. Theoretical basis of the method

Every vertical reference system is defined by the datum surface and height. The physical vertical system is defined by the gravity potential W_0 of the level surface and geopotential height C_P, which is converted to orthometric or normal height. In theory, there is a known relationship $h - H - N = 0$ between physical heights H, ellipsoidal heights h and the geoid-ellipsoid separation N.

The unification of local vertical datums comes down to the determination of the potential difference ΔW

$$\Delta W = W_0 - W_0^L \tag{1}$$

between the potential W_0 of the global vertical datum and the potential W_0^L of the local vertical datum.

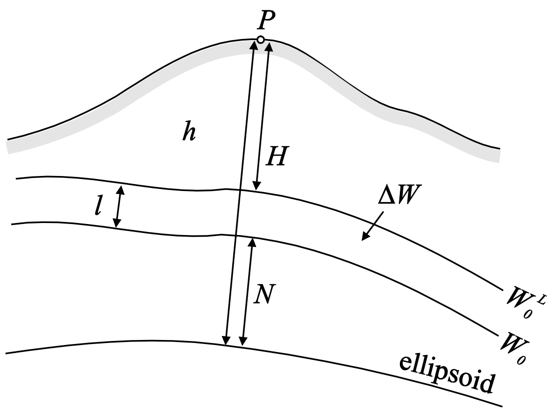


Fig. 1. The principle of vertical datum unification

The difference ΔW can be determined, among others, from GNNS satellite observations, precise leveling and a geoid model. This approach will be presented in this paper.

Fig. 1. shows that the geometric quantity l corresponds to the potential difference ΔW and can be computed from

$$l \approx h - H - N \approx h - H^n - \zeta \tag{2}$$

for orthometric or normall heights.

It is known from gravity potential theory that there is the relationship

$$\Delta W \approx l \cdot g \approx l \cdot \gamma \tag{3}$$

between ΔW, l and gravity acceleration g, where γ is the normal acceleration on the ellipsoid. For this linear relationship to be true, all the quantities must be small.

A simplification by using γ instead of g, assuming that $\Delta g = g - \gamma$ does not exceed 500 mGal, gives an error below 1 mm, which can be ignored in most geodetic applications.

Taking into account the formula (3), it can be written

$$\frac{1}{\gamma} \Delta W = l \tag{4}$$

The relationship (4) should satisfy certain conditions. The quasigeoid/ellipsoid separation ζ present in Eq. (2) should include a zero-degree term ζ_0, which results from the difference between the actual mass of the Earth (GM) and the mass of the reference ellipsoid (GM_e) and from the difference between the potential of the reference ellipsoid and the potential W_0 on the geoid (Heiskanen & Moritz, 1967, p. 101). The zero-degree term ζ_0 entails *a priori* selection of the value of the reference potential W_0. The geometric heights h and the quasigeoid/ellipsoid separations ζ should relate to the same reference ellipsoid.

The tidal effect causing the Earth's crust deformation and gravity field changes due to the solid tide as well as other contemporary height changes should be uniformly taken into account in the observed heights.

If the quantities h, H^n and ζ are obtained from observations, the quantity l can also be treated as an "observation" and then the observation equation for any point (benchmark) assumes the form

$$\frac{1}{\gamma_i} \Delta W = l_i + v_i \tag{5}$$

For n observations, the unknown parameter ΔW from a system of equations of the type (5) is determined by the least squares method.

3. Data

3.1. Ellipsoidal heights

The POLREF network is a densification of the EUREF-POL network and consists of 360 points measured during three campaigns in 1994 - 1995 and computed by the Department of Planetary Geodesy, Space Research Center of the Polish Academy of Sciences in the EUREF-89 coordinate system (Zieliński et al., 1997). The stations in the POLREF network were connected to the Polish precise levelling network in Kronszadt86 datum. It is estimated that the mean error of the normal height of the POLREF network does not exceed ± 1.3 cm, the mean error of ellipsoidal heights is ± 1.0–1.5 cm, which gives a mean error of the geometric quasigeoid/ellipsoid separation $\zeta^{gps/niw}$ in the order of ± 2 cm. The ellipsoidal heights h of the POLREF network were computed as non-tidal heights and the heights H^n from the leveling are also in the non-tidal system (Wyrzykowski, 1988).

The EUVN97 network consists of 11 points in Poland (Pażus, 2002). The final solution was computed in ITRF-96 coordinate system and then transformed into EUREF-89. The estimated standard deviations of adjusted points are at the level of ± 1 – ± 2 mm (IGWiAG, 2000).

The EUVN-DA network is a densification of the EUVN97 network and consists in Poland of 52 points measured in the campaign in 1999. The points of the EUVN-DA network were located at the benchmarks of the precise leveling network, which ensured the precise determination of their normal heights in the Kronsztadt86 vertical datum. The adjustment was performed in ITRF-96 epoch 1997.4 and transformed into ETRF-89 (Pażus, 2002).

The ASG-EUPOS network, established for surveying practice, has been operating fully since 2008. The network is based on 98 reference stations situated in Poland and 22 foreign stations. The ASG-EUPOS network provides a signal for both the positioning of geodetic control points and for land, air and marine navigation. The complete set of coordinates of the domestic reference stations of the ASG-EUPOS system used in this paper are in ETRF-2000 coordinate system and was downloaded from the website www.asgeupos.pl. The ellipsoidal heights of the network points were determined with an accuracy of 1 cm. The normal heights of these points were determined in the Kronsztadt86 datum by tying them to the domestic precise leveling network.

3.2. Leveling heights

The vertical datum called Kronsztadt86, is the result of the adjustment of the third leveling campaign measured in 1974-1982. The network consists of 15 827 sections, 371 lines and 135 loops. The total length of the leveling lines is 17 015 km. The leveling lines were measured by automatic levels: Opton Ni 1 and Zeiss Ni002. The following corrections were implemented to the raw data: rod scale corrections, rod temperature corrections, tidal corrections and normal Molodensky corrections. The final adjustment of the entire network was carried out in several versions. In 1985, the accepted solution was obtained as a least squares approach with station constraints. The heights of 23 benchmarks with their estimated accuracy (from the new UPLN solution) were incorporated into the adjustment. After the adjustment, the standard deviation of height difference was $\pm 0.844\, mm\sqrt{km}$ and the standard deviation of adjusted height changes between ± 6.5 mm and ± 11 mm (Wyrzykowski, 1988).

3.3. Quasigeoid model

EGM2008 is a spherical harmonic model of the Earth's gravitational potential, developed by a least squares combination of the ITG-GRACE03S gravitational model and its associated error covariance matrix, with the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined on a 5 arc-minute equiangular grid (Pavlis et al., 2012). This grid was formed by merging terrestrial, altimetry-derived and airborne gravity data. Over areas where only lower resolution gravity data was available, the spectral content was supplemented with gravitational information implied by the topography. EGM2008 is complete to degree and order 2159, and contains additional coefficients up to degree 2190 and order 2159. Over areas covered with high quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/leveling values are in the order of ± 3 to ± 10 cm. EGM2008 vertical deflections over Poland, for example, are within ± 0.6 to ± 0.7 arc-seconds of independent astrogeodetic values. These results indicate that EGM2008 performs comparably with contemporary detailed regional geoid models. The model in the non-tidal version was used for computations.

4. Computations and obtained results

The quasigeoid heights were computed from the general formula:

$$\zeta(r,\phi,\lambda)=\zeta_0+\frac{GM}{r\gamma}\sum_{n=2}^{n_{max}}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}\left(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda\right)P_{nm}(\sin\phi) \tag{6}$$

where C_{nm}, S_{nm} are fully normalized spherical harmonic coefficients of degree n and order m, n_{max} is the maximum degree of the geopotential model, GM is the product of the Newtonian gravitational constant and mass of the geopotential model, r, ϕ, λ are spherical coordinates, a is the equatorial radius of the earth and P_{nm} are the fully normalized associated Legendre functions.

Height anomalies were computed from the EGM2008 model at the points of the POLREF, EUVN-DA and ASG-EUPOS networks using the GEOCOL program (Tscherning et al., 1999).

The term ζ_0 is the zero-degree term due to the difference between the mass of the Earth used in the IERS Conventions and of the GRS80 ellipsoid. It is computed according to the well-known formula:

$$\zeta_0=\frac{GM-GM_0}{R\gamma}-\frac{W_0-U_0}{\gamma} \tag{7}$$

where the parameters GM_0 and U_0 correspond to the normal gravity field on the surface of the normal ellipsoid. For the GRS80 ellipsoid, we have GM_0 = 398,600.5000 x 10^9 m^3s^{-2} and U_0 = 62636860.85 m^2s^{-2}. The Earth's parameter GM used in quasigeoid computation from geopotential models and the constant gravity potential W_0 on the quasigeoid were set to the following values according to the IERS Conventions: GM = 398,600.4415 x 10^9 m^3s^{-2}, W_0 = 6,2636,856.00 m^2s^{-2}. The mean Earth radius R and the mean normal gravity γ on the reference ellipsoid were taken equally to 6,371,008.771 m and 9.798 ms^{-2}, respectively (GRS80 values). Based on the above conventional choices, the zero-degree term from Eq. (7) yielded the value ζ_0 = -0.442 m, which was added to the quasigeoid heights obtained from the corresponding spherical harmonic coefficient series expansions of all geopotential models.

Next, the computation of the value of the term l for the individual satellite networks was started according to Eq. (2).

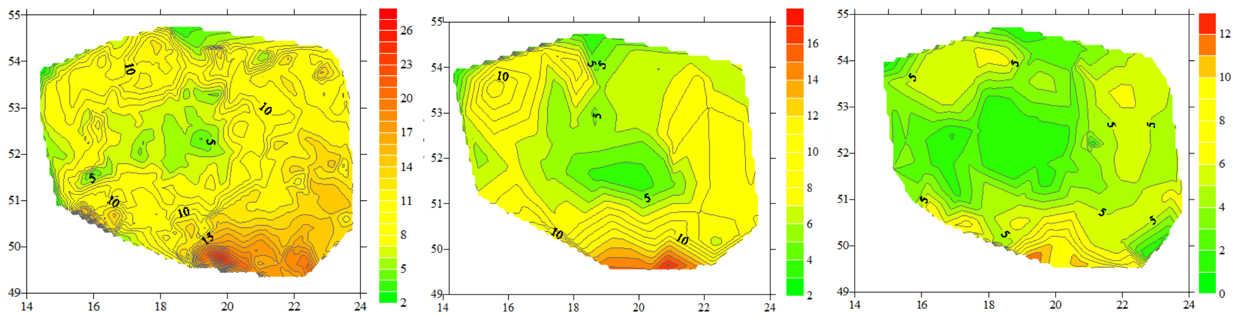
The ellipsoidal heights h of the satellite networks present in Eq. (2) are in the ETRF89 and ETRF-2000 datum and the non-tidal system. Since the normal heights H^n of the leveling network are heights in the non-tidal system, the height anomalies ξ were also computed in the non-tidal system from the EGM08 model.

The terms l_i were computed from Eq. (2) for the POLREF, EUVN-DA and ASG-EUPOS networks and their statistical characteristics are given in Table 1. The spatial distribution of the values of l_i for the successive networks are given in Fig. 2.

Table 1. Numerical values of the term l_i in cm

	POLREF	EUVN-DA	ASG-EUPOS
mean	10.0	7.6	4.3
SD	3.6	2.8	2.4
min	1.2	1.9	0.00
max	27.0	17.5	12.3

The linear displacement between equipotential surfaces (Fig. 1.) should be fairly identical and disturbed only by unavoidable random errors and possibly by systematic errors. The obtained results show that the mean linear displacement between the surfaces computed from the successive networks varies widely from 10 cm to 4 cm. Changes in standard deviation (3.6 – 2.4 cm) which, in this case, are the measure of the consistency and “smoothness” of these surfaces and are much smaller.



a) b) c)

Fig. 2. Spatial distribution of the term l_i for the (a) POLREF, (b) EUVN-DA and (c) ASG-EUPOS network

The values of l_i do not show constant displacement, but spatial inclinations (plane) or even more complex relationships are visible (Fig. 2.). This results from the existence of systematic effects and spatially correlated errors in the height data h, H^n and ζ, due to which the estimation of the parameter ΔW according to Eq. (5) gives a result affected by a systematic factors.

Several types of systematic errors affect the estimation of the parameter ΔW. These errors include geometric distortions in leveling networks, long- and medium-term errors in quasigeoid models, inconsistencies between ellipsoidal and physical heights resulting from the use of different ellipsoids and different approaches and in the treatment of tides in used data.

The elimination of these systematic factors from data is necessary for the correct determination of the parameter ΔW. This task can be performed earlier by introducing proper corrections to height data or parallel with the estimation of ΔW by the least squares method using an extended observation equation:

$$\frac{1}{\gamma_i}\Delta W + \mathbf{a}_i^T \mathbf{x} = l_i + v_i \tag{8}$$

in which additional parameters (nuisance) were introduced: **x** and the vector of coefficients $\mathbf{a}_i$ dependent on the spatial point (benchmark) position.

Examples of parametric models, which were used to describe systematic factors in a mixed set of geometric, orthometric heights and geoid-ellipsoid separations, can be found in the paper (Fotopoulos, 2003).

This paper uses only two models from (ibdi.), namely, a simple model without additional parameters, Eq. (5), and a model where the trend is modeled by a plane, i.e. :

$$\mathbf{a}_i^T \mathbf{x} = x_1(\varphi_i - \varphi_0) + x_2(\lambda_i - \lambda_0)\cos\varphi_i \tag{9}$$

where the plane inclination in the N–S direction is represented by the parameter x_1 and in the W–E direction by the parameter x_2. The coordinates φ_0 and λ_0 are the coordinates of the center of the area. An example of a trend computed for the POLREF network is shown in Fig. 3.

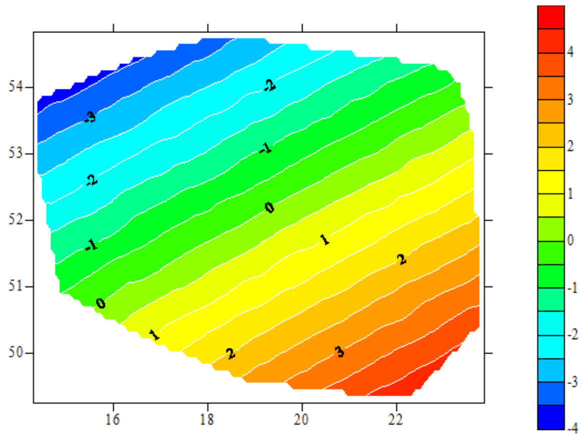


Fig. 3. Trend in centimeters computed from the formula (9) for the POLREF network

The figure shows that the plane representing the trend is inclined in the N–W direction at around 1 cm/100 km.

The parameter ΔW was computed using data from three satellite networks in two variants. In the first variant, the non-existence of systematic errors was assumed: model #1, Eq. (5) and in the variant 2 the presence of systematic errors was taken into account: model #2, Eq. (9). The obtained results are given in Table 2.

Table 2. Parameter ΔW and it mean error computed for three different satellite networks

	POLREF			EUVN-DA			ASG-EUPOS		
	ΔW	m_0	$m_{\Delta W}$	ΔW	m_0	$m_{\Delta W}$	ΔW	m_0	$m_{\Delta W}$
model #1	0.974	0.036	0.019	0.747	0.028	0.035	0.426	0.024	0.023
model #2	0.986	0.027	0.014	0.747	0.024	0.030	0.425	0.022	0.021

5. Conclusions

The ΔW parameter values calculated in this paper for three networks based on ellipsoidal heights from satellite observations, quasigeoid separations computed based on the EGM2008 model and normal heights obtained from the conducted leveling campaign are significantly different from 0.4 to 1.0 m^2s^{-2}. These differences are probably caused by crucial improvements in satellite observations. The first satellite observations on the POLREF network points were conducted in 1994 - 1995, whereas the observations on the ASG-EUPOS network points are contemporary. Furthermore, POLREF and EUVN-DA networks have ellipsoidal heights in the EUREF89 reference frame and ASG-EUPOS has ellipsoidal heights in the ETRF-2000 frame.

The aforementioned reasons indicate that currently the best estimation of the ΔW parameter for the territory of Poland is the value of 0.43 m^2s^{-2}. A similar ΔW value equal to 0.36 m^2s^{-2} was obtained by Burša (Burša et al., 2002). The slight difference is probably caused by the fact that the authors of this study estimated ΔW value for the network consisting of 16 points only, and used older model EGM96 to calculate the separation of the geoid from the ellipsoid.

Attempting to remove systematic errors using the model described by equation (9) does not give any good results. Other systematic errors models given in the work (Fotopoulos, 2003) are not applicable in this case.

Estimated error of calculated ΔW parameter is of the order of 0.02 m^2s^{-2} (2 mm) in the case of POLREF and ASG-EUPOS and 0.03 m^2s^{-2} (3 mm) in the case of EUVN-DA.

It is necessary to conduct further work, which will consist, among others, in the determination of ΔW by other methods and the use of other systematic error models.

References

Burša, M., S. Kenyon, J. Kouba, K. Raděj, V. Vatrt, M. Vojtiškova, and J. Šimek (2002). World height system specified by geopotential at tide gauge stations. In Vertical Reference Systems, edited by H. Drewes, A. H. Dodson, L. P. S. Fortes, L. Sanchez and P. Sandoval.

Fotopoulos, G. (2003). An analysis on the optimal combination of geoid, orthometric and ellipsoidal height data. PhD Thesis, UCGE Report No. 20185, Department of Geomatics Engineering, University of Calgary.

Hayden, T. & Amjadiparvar, B. & Rangelova, E. & Sideris, M. G. (2013). Estimating Canadian vertical datum offsets using GNSS/levelling benchmark information and GOCE global geopotential models. *Journal of Geodetic Science 2 (4)*. doi: 10.2478/v10156-012-0008-4.

Heiskanen, W. A. & Moritz, H. (1967). *Physical Geodesy*. San Francisco, California: W.H. Freeman and Company.

Hoa, H. M. (2013). Estimating the geopotential value W_0 of the local geoid based on data from local and global normal heights of GPS/leveling points in Vietnam. *Geodesy and Cartography (Lithuania) 01/2013, 39 (3).* doi: 10.3846/20296991.2013.823705.

IGWiAG (2000). Extension of the domestic EUVN network by conducting a satellite (GPS) survey at the points of the basic vertical control (Rozwinięcie krajowej sieci EUVN poprzez wykonanie pomiarów satelitarnych GPS na punktach podstawowej osnowy wysokościowej), Warsaw University of Technology, Chair of Geodesy and Geodetic Astronomy, Warsaw, Report for GUGiK.

Kotsakis, C. & Katsambalos, K. & Ampatzidis D. (2011). Estimation of the zero-height geopotential level in a local vertical datum from inversion of co-located GPS, leveling and geoid heights: a case study in the Hellenic islands. *Journal of Geodesy 86 (6).* doi: 10.1007/s00190-011-0530-7.

Pażus, R. (2001). National Report of Poland to EUREF 2001, Report on the Symposium of the IAG Sub-commission for Europe (EUREF) held in Dubrovnik, Croatia, 16–18 May 2001, EUREF Publication No 10, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, Band 23, Frankfurt am Main, 248–253.

Pavlis, N. K. & Holmes, S. A. & Kenyon, S. C. & Factor, J. K. (2012). The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). *Journal of Geophysical Research-Solid Earth 117*. doi: 10.1029/2011JB008916.

Petit, G. & Luzum, B. (eds.) (2010). IERS Conventions (2010). IERS Technical Note No. 36, Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 179 pp., ISBN 3-89888-989-6.

Sánchez, L. (2012). Towards a vertical datum standardisation under the umbrella of Global Geodetic Observing System. *Journal of Geodetic Science 12/2012, 2 (4):325–342*. doi: 10.2478/v10156-012-0002-x.

Tscherning, C. C. & Forsberg, R. & Knudsen, P. (1992). The GRAVSOFT package for geoid determination. First Continental Workshop on the Geoid in Europe: Towards a Precise Pan-European Reference Geoid for the Nineties, Prague, May 11–14, 1992.

Wyrzykowski, T. (1988). Monograph on domestic 1st class precise leveling networks (Monografia krajowych sieci niwelacji precyzyjnej I klasy). Warsaw: Institute of Geodesy and Cartography.

Zieliński, J. B. & Łyszkowicz, A. & Jaworski, L. & Świątek, A. & Zdunek, R. & Gelo, S. (1998). POLREF-96 – The New Geodetic Reference Frame for Poland, in Advances in Positioning and Reference Frames, vol. 118 of International Association of Geodesy Symposia, IAG Scientific Assembly, Rio de Janeiro, Brazil, September 3–9, 161–166, Springer.

Authors:

Prof. Adam Łyszkowicz [1)], adaml@uwm.edu.pl
PhD Joanna Kuczyńska-Siehień [1)], joanna.kuczynska@uwm.edu.pl
PhD Monika Biryło [1)], monika.sienkiewicz@uwm.edu.pl
[1)] Department of Land Surveying and Geomatics, Heweliusza 12, 10-900 Olsztyn