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ORIGINAL ARTICLE

Impact analysis of observation coupling on reliability indices in a geodetic network

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Abstract

An optimally designed geodetic network is characterised by an appropriate level of precision and the lowest possible setup cost. Reliability, translating into the ability to detect blunders in the observations and higher certainty of the obtained point positions, is an important network characteristic. The principal way to provide appropriate network reliability is to acquire a suitably large number of redundant observations. This approach, however, faces limitations resulting from the extra cost. This paper analyses the possibility of providing appropriate reliability parameters for networks with moderate redundancy. A common problem in such cases are dependencies between observations preventing the acquisition of the required reliability index for each of the individual observation.
The authors propose a methodology to analyse dependencies between observations aiming to determine the possibility of acquiring the optimal reliability indices for each individual observation or groups of observations. The suggested network structure analysis procedures were illustrated with numerical examples.

Key words: geodetic networks, network design, second order design, network's reliability

1 Introduction

The design and optimisation of a measurement network is a common element when carrying out geodetic tasks. A well-designed network must satisfy a number of requirements. The most important are:

- appropriate point placement,
- required accuracy,
- reliability, translating into the ability to detect blunders in the observations,
- low stabilisation and measurement cost.

The accuracy prerequisite is most often a result of top-down requirements imposed on geodetic studies. These requirements usually take the form of an inequality. Network's reliability is a characteristic which allows to detect a blunder, identify the location of the distortion and eliminate it. The optimisation aims to design a network, so that the assigned task could be realised at the lowest possible cost.

The design of an optimal geodetic network means finding an appropriate compromise between the previously mentioned, competitive, characteristics and it was the subject of many investigations collected in the classic book by Grafarend and Sansò (2012).

According to Grafarend (1974) the optimization problems of designing a geodetic network are classified into the following orders:

- zero order design (identification of an appropriate reference system),
- first order design (prospecting for the optimal network configuration),
- second order design (prospecting for the optimal observation accuracy),
- third order design (prospecting for ways to improve the existing network).

W. Baarda is considered the precursor of research on network's reliability (Baarda, 1967, 1968). In recent years, the term blunder "robustness" of the network is commonly used alongside the term "reliability" (Prószyński and Kwaśniak, 2002).

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The scope of this study focuses on the issues connected to the network's internal reliability, that is the ability to detect distortions in a given set of observations. This characteristic is described by a variance-covariance matrix of the offset corrections **R** obtained from equation (1).

$$\mathbf{R} = \mathbf{I} - \mathbf{A}\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-}\mathbf{A}^T\mathbf{P}, \quad (1)$$

where **A** is the factor matrix of observation equations, **P** is the observation weight matrix, whereas **I** represents the identity matrix. As this paper deals with the case of uncorrelated observations, we can assume that **P** is a diagonal matrix.

Network's reliability characteristic is represented by the main diagonal of the **R** matrix. Its individual elements ($\mathbf{R}_{ii}$) define the reaction of an observation correction to the occurrence of a blunder in the i^{th} observation. These elements will be referred to as reliability indexes for the given observation. Zero corresponds to uncontrolled observations occurring in unambiguous constructions (without redundant observations). In turn, high value of the index means that the observation is well-controlled by the other observations or constraints for the unknowns. A blunder in such observation will manifest itself in the results of a correction in the form of increased observation correction for the same observation.

Trace of the **R** matrix corresponds to the number of redundant observations ($Tr(\mathbf{R}) = n - u$) and is not dependant on weight matrix **P**.

Reliability requirement is considered fulfilled if the inequality (2):

$$\mathbf{R}_{ii} > 0.5 \quad (2)$$

is satisfied (Prószyński, 1994).

The mean value of the diagonal elements of the **R** matrix (3) can be assumed as the global reliability index for the network

$$\mathbf{R}_{avg} = \frac{n-u}{n}, \quad (3)$$

where n is the number of observations and u is the number of unknowns. Due to a diversified structure of the network resulting from various determinants, the individual observations will have different reliability indices.

Appropriate reliability level should be ensured during the first order design (FOD). The easiest way to achieve that is to design a large number of redundant observations, which also has a positive impact on the accuracy characteristics of the given network. Nonetheless, this approach entails an increase in the cost of measurements, which is not always acceptable (Prószyński, 2014).

In practice, the design of a network will take a form of a compromise between the drive for high degree of reliability and economic constraints. Apart from cases of exceptional importance, observation redundancy in a network will usually only slightly exceed the minimum. In such situations, the second order design (SOD) becomes increasingly important, as it decides whether both the accuracy and reliability requirements are met.

While the trace of the **R** matrix for a given set of observations is independent of the weight distribution in **P** matrix, the individual diagonal elements are not. The main objective of the actions performed during SOD is the selection of observation weights (accuracies) so that the accuracy requirements are met with the lowest possible diversification of reliability indices for each individual observation. There are two possible strategies:

- analytical method,
- trial and error method.

The first of the methods suggested in studies Amiri-Simkooei (2004) and Amiri-Simkooei and Sharifi (2004) makes it possible to obtain a set of observation (errors) weights generating optimal (the same) reliability indices for all the observations. The accuracy harmonization procedure proposed by Nowak (2011) is another variant of this method.

The trial and error method is based on a successive (iterative) search for a set of observation weights meeting the imposed criteria to the optimal or targeted degree. The procedure utilising the simulated annealing algorithm (Berné and Baselga, 2004; Baselga, 2011) is an example of such method.

Regardless of the applied method and algorithm, the observation weight optimisation aimed at obtaining the desired network's reliability indices must take into consideration the network-specific system of dependencies resulting from the network's structure and generating a corresponding to that system reaction pattern (changes in diagonal elements of the **R** matrix) to the changes in diagonal elements of the weight matrix. This issue is the main topic of this study.

2 Dependencies resulting from the distribution of reliability indices in the network

As previously indicated, the reliability of observation in a geodetic network is characterized by reliability matrix **R**. Equation (1) which describes it can be presented by the formula (4):

$$\mathbf{R} = \mathbf{I} - \mathbf{D}, \quad (4)$$

where:

$$\mathbf{D} = \mathbf{A}\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-}\mathbf{A}^T\mathbf{P}. \quad (5)$$

D is a variance-covariance matrix of the corrected observation. Moreover, similarly to the **R** matrix, it is symmetric and idempotent. Due to the fixed relation between **D** and **R** matrices, matrix **D** can be, on equal terms with matrix **R**, used to characterise the observation reliability in a network. Additional advantage of the said matrix lies in the fact, that it is defined by a slightly less complex equation (5). In the coming part of this study matrix **D** will be used to analyse network reliability. Thus, the inequality (2) will take the form of (6):

$$\mathbf{D_{ii}} < 0.5. \quad (6)$$

Any of its elements is determined by the formula (7):

$$\mathbf{D}_{i,k} = \mathbf{A}_i\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-}\mathbf{A}_k^T\mathbf{P}. \quad (7)$$

The main objective of the actions undertaken as part of SOD is to minimise the diversification of reliability indices for the observations. Those actions, independent of the applied method are based on modifying the weights (standard deviations) of the observations. The impact of the change in the standard deviation for the k^{th} observation will be examined. The standard deviation change factor will be determined by the formula (8):

$$\gamma = \frac{\sigma_{old}}{\sigma_{new}}, \quad (8)$$

where σ_{old}, and σ_{new} represent the standard deviation value before and after change. This type of change will cause modification of the standardised factor matrix of observation equations

A. The new matrix is defined by the equation (9):

$$\overline{\mathbf{A}}_i = \left\{ \begin{array}{ll} \mathbf{A}_i & \text{for } i \neq k \\ \mathbf{A}_i \gamma & \text{for } i = k \end{array} \right\}. \tag{9}$$

The new matrix of normal equation system coefficients will take the form of (10):

$$\overline{\mathbf{A}^T\mathbf{A}} = \mathbf{A}^T\mathbf{A} + \left(\gamma^2 - 1\right) \mathbf{A}_k^T \mathbf{A}_k. \tag{10}$$

In order to determine its inverse the Sherman-Morrison-Woodbury (Woodbury, 1950) formula will be used. In a version given by Rao (1982) it can be shown as follows:

$$\left(\mathbf{M} + \mathbf{BCB}^T\right)^{-1} = \mathbf{M}^{-1} - \mathbf{M}^{-1}\mathbf{B}\left(\mathbf{B}^T\mathbf{M}^{-1}\mathbf{B} + \mathbf{C}^{-1}\right)^{-1}\mathbf{B}^T\mathbf{M}^{-1}, \tag{11}$$

where $\mathbf{M}_{(m\times m)}$ and $\mathbf{C}_{(n\times n)}$ are nonsingular matrices and $\mathbf{B}$ is $m \times n$ matrix. By substitution we obtain

$$\begin{aligned} \left(\overline{\mathbf{A}^T\mathbf{A}}\right)^{-1} &= \left(\mathbf{A}^T\mathbf{A} + \left(\gamma^2 - 1\right)\mathbf{A}_k^T\mathbf{A}_k\right)^{-1} = \\ &= \left(\mathbf{A}^T\mathbf{A}\right)^{-1} - \Delta\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}_k^T\mathbf{A}_k\left(\mathbf{A}^T\mathbf{A}\right)^{-1}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \Delta &= \left(\mathbf{A}_k\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}_k^T + \left(\gamma^2 - 1\right)^{-1}\right)^{-1} = \\ &= \left(\mathbf{D}_{kk} + \left(\gamma^2 - 1\right)^{-1}\right)^{-1} = \frac{\gamma^2 - 1}{(\gamma^2 - 1)\,\mathbf{D}_{kk} + 1} = \left(\gamma^2 - 1\right)\delta. \end{aligned}$$

By taking into consideration the above dependencies, the elements of the new $\mathbf{D}$ matrix can be presented as the value function of the appropriate elements of the old $\mathbf{D}$ matrix and the coefficient of the change in standard deviation γ:

$$\overline{\mathbf{D}}_{i,j} = \left\{ \begin{array}{l} \mathbf{D}_{i,j} - \Delta\mathbf{D}_{i,k}\mathbf{D}_{k,j} = \mathbf{D}_{i,j} - \left(\gamma^2 - 1\right)\beta\mathbf{D}_{i,k}\mathbf{D}_{k,j} \text{ for } i,j \neq k, \\ \gamma\mathbf{D}_{i,k} - \Delta\gamma\mathbf{D}_{i,k}\mathbf{D}_{k,k} = \gamma\mathbf{D}_{i,k} - \left(\gamma^2 - 1\right)\beta\gamma\mathbf{D}_{i,k}\mathbf{D}_{k,k} = \\ \qquad = \gamma\beta\mathbf{D}_{i,k} \quad \text{for } i \neq k; j = k, \\ \gamma^2\mathbf{D}_{k,k} - \Delta\gamma^2\mathbf{D}_{k,k}^2 = \mathbf{D}_{k,k} + \left(\gamma^2 - 1\right)\beta\left(1 - \mathbf{D}_{k,k}\right)\mathbf{D}_{k,k} = \\ \qquad = \gamma^2\beta\mathbf{D}_{k,k} \quad \text{for } i = j = k. \end{array} \right\} \tag{13}$$

Observation reliability indices are dependent on the diagonal elements of the $\mathbf{D}$ matrix. By analysing the diagonal element of the new $\mathbf{D}$ matrix

$$\overline{\mathbf{D}}_{k,k} = \mathbf{D}_{k,k} + \frac{\left(\gamma^2 - 1\right)\left(1 - \mathbf{D}_{k,k}\right)\mathbf{D}_{k,k}}{(\gamma^2 - 1)\,\mathbf{D}_{kk} + 1} = \frac{\mathbf{D}_{k,k}\gamma^2}{(\gamma^2 - 1)\,\mathbf{D}_{kk} + 1},$$

one obtains the equation (14).

$$\overline{\mathbf{D}}_{k,k} + \overline{\mathbf{D}}_{k,k}\mathbf{D}_{kk}\gamma^2 - \overline{\mathbf{D}}_{k,k}\mathbf{D}_{kk} = \mathbf{D}_{k,k}\gamma^2. \tag{14}$$

This way one obtains a formula important in reliability design which allows to determine the change in observation accuracy based on the requested results:

$$\gamma^2 = \frac{\overline{\mathbf{D}}_{k,k}\left(1 - \mathbf{D}_{k,k}\right)}{\mathbf{D}_{k,k}\left(1 - \overline{\mathbf{D}}_{k,k}\right)}. \tag{15}$$

2.1 Weight modification result transfer mechanism

The diagonal elements of the $\mathbf{D}$ matrix decide the network's reliability properties. Using the equation (15) the weight (standard deviation) of the observation can be modified in such a way that the desired value of the diagonal element and, as the result, the value of the reliability index is obtained. However, the change will cause a modification of a larger fragment of the $\mathbf{D}$ matrix, including the diagonal elements. This results from the connection between observation functions expressing themselves by non-zero extra-diagonal elements $\mathbf{D}_{i,j}$. Distribution of the results for the observations and the change in the weight for observation k is governed by the reaction coefficient $\delta_{i|k}$ in accordance with the equation (16):

$$\overline{\mathbf{D}}_{i,i} - \mathbf{D}_{i,i} = \left(\overline{\mathbf{D}}_{k,k} - \mathbf{D}_{k,k}\right)\cdot\delta_{i|k}. \tag{16}$$

The value of this coefficient is defined by the formula (17):

$$\delta_{i|k} = \left\{ \begin{array}{ll} 1 & \text{for } i = k \\ -\dfrac{\mathbf{D}_{i,k}^2}{\mathbf{D}_{k,k}\left(1 - \mathbf{D}_{k,k}\right)} & \text{for } i \neq k \end{array} \right\}. \tag{17}$$

From the equation (18)

$$\sum_{i\neq k}\delta_{i|k} = \frac{-1}{\mathbf{D}_{k,k}\left(1 - \mathbf{D}_{k,k}\right)}\sum_{i\neq k}\mathbf{D}_{i,k}^2 = -\frac{\mathbf{D}_{k,k} - \mathbf{D}_{k,k}^2}{\mathbf{D}_{k,k}\left(1 - \mathbf{D}_{k,k}\right)} = -1, \tag{18}$$

it can be concluded that the sum of changes of the reliability indices for the inactive observations ($i \neq k$) is equal to the change in the reliability index for an active observation (k) with the opposite sign. Therefore, the principle that the action is equal to the reaction is in effect.

The value of the $\delta_{i|k}$ coefficient satisfies the inequality $|\delta_{i|k}| \leq 1$, but the equality is fulfilled only for two equally accurate observations. The reaction coefficient rapidly decreases as the distance from the active observation increases. The change in observation's accuracy virtually only influences the reliability indices of the neighbouring observations.

3 Numerical examples

The operation of the described equations will be illustrated in two numerical examples. The first of the examples concerns a levelling network. The network consists of 12 points connected by 26 height difference observations. Study of this network is presented in Fig. 1. Black numbers refer to points whereas blue numbers refer to observations.

From topological perspective the network consists of 5 squares with diagonals. For the level differences corresponding to the "sides of the squares" the standard deviation of $\sigma = 1$ was assumed, whereas for the diagonals $\sigma = 1.15$ was adopted. The values are presented in column 4 of Table ??.

Assuming a single point datum, the mean value of the diagonal element of the $\mathbf{D}$ matrix equals:

$$\mathbf{D}_{avg} = \frac{u}{n} = \frac{11}{26} = 0.423.$$

An overview of diagonal elements of the $\mathbf{D}$ matrix values for all of the observations are presented in column 5 of Table ??.

Due to the network's structure and the datum, the reliability indices are symmetrical. Condition (6) is not fulfilled for the two extreme observations (1 and 26). Columns 6 and 7 present the relationship between the problematic observation 1 and the rest of the observations. As it can be observed, a significant de-

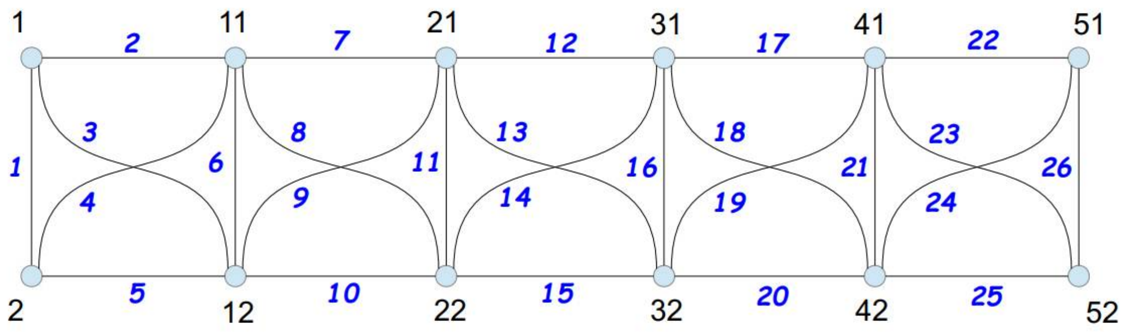

Figure 1. Test 1 network

Table 1. Accuracy harmonization of network 1

No.	Begin	End	Start				Modifications					
							Obs. 1		Obs. 1, 2, 5		Obs. 1, 2, 5, 26, 25, 22	
			σ	$\mathbf{D}_{i,i}$	$\mathbf{D}_{i,1}$	$\delta_{i\|1}$	σ	$\mathbf{D}_{i,i}$	σ	$\mathbf{D}_{i,i}$	σ	$\mathbf{D}_{i,i}$
1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	1.00	0.533998	0.533998	1.00	1.10	0.486398	1.10	0.499083	1.10	0.499083
2	1	11	1.00	0.497526	0.255164	−0.26164	1.00	0.509980	1.05	0.485935	1.05	0.485935
3	1	12	1.15	0.394099	0.242464	−0.23625	1.15	0.405344	1.15	0.417838	1.15	0.417838
4	2	11	1.15	0.394099	−0.242460	−0.23625	1.15	0.405344	1.15	0.417838	1.15	0.417838
5	2	12	1.00	0.497526	−0.255160	−0.26164	1.00	0.509980	1.05	0.485935	1.05	0.485935
6	11	12	1.00	0.364588	0.023670	−0.00225	1.00	0.364695	1.00	0.370258	1.00	0.370258
7	11	21	1.00	0.458845	0.011310	−0.00051	1.00	0.458870	1.00	0.460140	1.00	0.460140
8	11	22	1.15	0.359173	0.010747	−0.00046	1.15	0.359195	1.15	0.360342	1.15	0.360342
9	12	21	1.15	0.359173	−0.010750	−0.00046	1.15	0.359195	1.15	0.360342	1.15	0.360342
10	12	22	1.00	0.458845	−0.011310	−0.00051	1.00	0.458870	1.00	0.460140	1.00	0.460140
11	21	22	1.00	0.364255	0.001049	$-4.4{\cdot}10^{-6}$	1.00	0.364255	1.00	0.364266	1.00	0.364266
12	21	31	1.00	0.458769	0.000501	$-1{\cdot}10^{-6}$	1.00	0.458769	1.00	0.458772	1.00	0.458774
13	21	32	1.15	0.359104	0.000476	$-9.1{\cdot}10^{-7}$	1.15	0.359104	1.15	0.359106	1.15	0.359109
14	22	31	1.15	0.359104	−0.000480	$-9.1{\cdot}10^{-7}$	1.15	0.359104	1.15	0.359106	1.15	0.359109
15	22	32	1.00	0.458769	−0.000500	$-1{\cdot}10^{-6}$	1.00	0.458769	1.00	0.458772	1.00	0.458774
16	31	32	1.00	0.364255	0.000046	$-8.7{\cdot}10^{-9}$	1.00	0.364255	1.00	0.364255	1.00	0.364266
17	31	41	1.00	0.458845	0.000022	$-2{\cdot}10^{-9}$	1.00	0.458845	1.00	0.458845	1.00	0.460140
18	31	42	1.15	0.359173	0.000021	$-1.8{\cdot}10^{-9}$	1.15	0.359173	1.15	0.359173	1.15	0.360342
19	32	41	1.15	0.359173	−0.000021	$-1.8{\cdot}10^{-9}$	1.15	0.359173	1.15	0.359173	1.15	0.360342
20	32	42	1.00	0.458845	−0.000022	$-2{\cdot}10^{-9}$	1.00	0.458845	1.00	0.458845	1.00	0.460140
21	41	42	1.00	0.364588	0.000002	$-1.7{\cdot}10^{-11}$	1.00	0.364588	1.00	0.364588	1.00	0.370258
22	41	51	1.00	0.497526	$9.65{\cdot}10^{-7}$	$-3.7{\cdot}10^{-12}$	1.00	0.497526	1.00	0.497526	1.05	0.485935
23	41	52	1.15	0.394099	$9.55{\cdot}10^{-7}$	$-3.7{\cdot}10^{-12}$	1.15	0.394099	1.15	0.394099	1.15	0.417838
24	42	51	1.15	0.394099	$-9.6{\cdot}10^{-7}$	$-3.7{\cdot}10^{-12}$	1.15	0.394099	1.15	0.394099	1.15	0.417838
25	42	52	1.00	0.497526	$-9.6{\cdot}10^{-7}$	$-3.7{\cdot}10^{-12}$	1.00	0.497526	1.00	0.497526	1.05	0.485935
26	51	52	1.00	0.533998	$1.34{\cdot}10^{-7}$	$-7.2{\cdot}10^{-14}$	1.00	0.533998	1.00	0.533998	1.10	0.499083

Table 2. Accuracy harmonization of network 2

No.	Station	Target	D / A*	Start σ_k = 0.001 σ_d = 0.001 + 0.000004d				Modification Obs. 35, 36, 37	
				σ	$D_{i,i}$	$D_{i,35}$	$\delta_{i\|35}$	σ	$D_{i,i}$
1	2	3	4	5	6	7	8	9	10
1	2	4	A	0.00100	0.31483	0.00064	0.00000	0.00100	0.314931
2	2	3	A	0.00100	0.29529	0.01050	−0.00047	0.00100	0.295670
3	2	7	A	0.00100	0.28282	−0.01062	−0.00048	0.00100	0.283419
4	2	1	A	0.00100	0.38870	−0.00052	0.00000	0.00100	0.388723
5	2	4	D	0.00250	0.26656	−0.00631	−0.00017	0.00220	0.266905
6	2	3	D	0.00350	0.19006	0.00558	−0.00013	0.00300	0.190105
7	2	7	D	0.00306	0.22616	0.00403	−0.00007	0.00265	0.226802
8	2	1	D	0.00622	0.45573	0.00048	0.00000	0.00518	0.455751
9	3	2	A	0.00100	0.33006	−0.02210	−0.00207	0.00100	0.330487
10	3	4	A	0.00100	0.37008	−0.00730	−0.00023	0.00100	0.370184
11	3	5	A	0.00100	0.32225	0.06050	−0.01555	0.00100	0.325683
12	3	1	A	0.00100	0.42680	−0.03110	−0.00411	0.00100	0.428105
13	3	2	D	0.00350	0.19006	0.00558	−0.00013	0.00300	0.190105
14	3	4	D	0.00200	0.30539	0.01701	−0.00123	0.00180	0.305867
15	3	5	D	0.00258	0.24137	−0.02107	−0.00188	0.00226	0.242192
16	3	1	D	0.00610	0.37276	−0.01562	−0.00104	0.00508	0.373753
17	4	1	A	0.00100	0.35219	−0.01831	−0.00142	0.00100	0.353398
18	4	2	A	0.00100	0.46683	−0.01623	−0.00112	0.00100	0.467363
19	4	3	A	0.00100	0.49941	0.00842	−0.00030	0.00100	0.499471
20	4	5	A	0.00100	0.40064	0.12469	−0.06603	0.00100	0.415269
21	4	6	A	0.00100	0.34751	−0.04731	−0.00951	0.00100	0.356598
22	4	7	A	0.00100	0.35336	−0.05125	−0.01116	0.00100	0.360654
23	4	1	D	0.00600	0.28140	−0.00061	0.00000	0.00500	0.281487
24	4	2	D	0.00250	0.26656	−0.00631	−0.00017	0.00220	0.266905
25	4	3	D	0.00200	0.30539	0.01701	−0.00123	0.00180	0.305867
26	4	5	D	0.00171	0.29359	−0.13729	−0.08005	0.00157	0.310582
27	4	6	D	0.00150	0.29365	0.11672	−0.05786	0.00140	0.311148
28	4	7	D	0.00171	0.29503	−0.01183	−0.00059	0.00157	0.310227
29	5	4	A	0.00100	0.39923	−0.05078	−0.01095	0.00100	0.401508
30	5	3	A	0.00100	0.46590	−0.09923	−0.04182	0.00100	0.475561
31	5	6	A	0.00100	0.46703	0.15001	−0.09557	0.00100	0.488163
32	5	4	D	0.00171	0.29359	−0.13729	−0.08005	0.00157	0.310582
33	5	3	D	0.00258	0.24137	−0.02107	−0.00188	0.00226	0.242192
34	5	6	D	0.00150	0.35769	0.08441	−0.03026	0.00140	0.365040
35	6	5	A	0.00100	0.62062	0.62062	1.00000	0.00140	0.499961
36	6	4	A	0.00100	0.49507	0.24784	−0.26088	0.00122	0.495280
37	6	7	A	0.00100	0.61137	0.13154	−0.07349	0.00140	0.491810
38	6	5	D	0.00150	0.35769	0.08441	−0.03026	0.00140	0.365040
39	6	4	D	0.00150	0.29365	0.11672	−0.05786	0.00140	0.311148
40	6	7	D	0.00150	0.35797	−0.00494	−0.00010	0.00140	0.363970
41	7	4	A	0.00100	0.36471	0.01336	−0.00076	0.00100	0.365467
42	7	2	A	0.00100	0.37371	0.03713	−0.00585	0.00100	0.376982
43	7	6	A	0.00100	0.44888	−0.09900	−0.04163	0.00100	0.474086
44	7	1	A	0.00100	0.47930	0.04852	−0.01000	0.00100	0.484827
45	7	4	D	0.00171	0.29503	−0.01183	−0.00059	0.00157	0.310227
46	7	2	D	0.00306	0.22616	0.00403	−0.00007	0.00265	0.226802
47	7	6	D	0.00150	0.35797	−0.00494	−0.00010	0.00140	0.363970
48	7	1	D	0.00553	0.35859	0.01421	−0.00086	0.00462	0.359734

*D – distance, A – angle

pendency between observations in relation to the observation 1 can be discussed only for observations 2 to 5, with satisfactory reliability indices for observations 3 and 4.

The distribution of values $\mathbf{D}_{i,1}$ and $\delta_{i|1}$ is confirmed in the course of accuracy harmonisation presented by columns 8–13. Clearly, the first step involving the change in the value of σ for the first observation caused an improvement in the reliability index for this observation, but, at the same time, it was also the cause of deterioration of the indices for the connected observations, where for observations 2 and 5 condition (6) was exceeded. This fact illustrates the role played by coupled observations characterised by the **D** matrix. Modification of the σ for observations 2 and 5 during the second iteration led to a situation, where condition (6) is fulfilled for all the observations connected to observation 1. Columns 12 and 13 show the final set of σ for all the observations (after changes were applied to the opposite segment of the network connected to observation 26).

The second example concerns a linear-angular network presented in Fig. 2. The network consists of 7 points, between which 48 observations were designed (24 directions, 24 distances). The assumed constant mean error for the directions took the value of $\sigma_A = \pm 0.001$ gon. The mean error for distances is expressed by the equation $\sigma_D = \pm 1$ mm + 4 ppm. Assuming a non-distorting datum, the mean value of the diagonal element of the **D** matrix is equal to:

$$\mathbf{D}_{avg} = \frac{u}{n} = \frac{17}{48} = 0.354.$$

A list of observations with their mean errors is presented in Table 2. On the basis of the reliability indices shown in column 6 one can conclude that condition (6) is not fulfilled for two directional observations (35 and 37 – in Fig. 2 marked with a thick, red line), where the observation 35 proves to be the most problematic ($\mathbf{D}_{i,i} = 0.62062$). By analysing the summary of the extra-diagonal values $\mathbf{D}_{i,35}$ (col. 7) and the values of the reaction coefficient $\delta_{i|35}$ (col. 8) it can be observed that the strongest connection for observation 35 occurs between observation 36 ($\delta_{i|35} = -0.26088$). In Fig. 2 these observations are marked with a thin, red line. Because the reliability index for that observation lies very close to 0.5, it is expected that the change in σ_{35} will cause the $\mathbf{D}_{i,i}$ for observation 36 to fail criterion (6). In order to avoid such situation, it was decided to change the σ for both observations, for which the criterion is not fulfilled (35 and 37) and, at the same time, for the observation 36. The effects of the change are presented in columns 9 and 10. As it can be observed, condition (6) was satisfied for all the observations in a single iteration.

4 Conclusions

Basing on the carried-out studies and computational tests the following conclusions can be drawn. By utilising formula (15) it is possible – through change in the standard deviation σ for the selected observation – to easily obtain the desired value of the reliability index for that observation. However, this type of change deteriorates the indices for the remaining observations. The reaction coefficient $\delta_{i|k}$ (17) is the measure of the effect a change in standard deviation σ for the observation has on the reliability index for any other observation. Analysis of the value of this coefficient makes it possible to predict the negative effects (deterioration of the reliability indices) for the other observations in the network and “preventively” modify the σ for other “endangered” observations, thereby shortening the process of reaching the final outcome.

The procedure applied in this study possesses, to a large degree, the characteristics of a manual one. However, the presented equations may serve as a base for the creation of an automated algorithm, where consecutive steps will be carried out on the basis on unambiguous rules. Definition of these rules and identification of the connected numerical parameters requires further research.

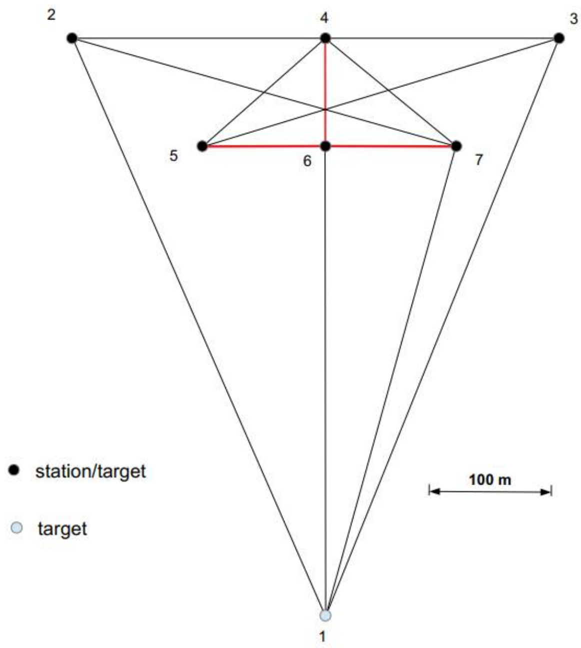

Figure 2. Test 2 network

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