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ORIGINAL ARTICLE

Adjustment of observation accuracy harmonisation parameters in optimising the network's reliability

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Abstract

Appropriate precision and low cost are the basic conditions that have to be fulfilled by a project of a geodetic network. Reliability, translating into the ability to detect gross errors in the observations and higher certainty of the obtained point position, is an important network characteristic. The principal way to provide appropriate network reliability is to acquire a suitably large number of redundant observations. Optimisation of the observation accuracy harmonisation procedure allowing for the acquisition of an appropriate level of reliability through modification of the observation a priori standard deviations is the focus of this study. Parameterisation of the accuracy harmonisation is proposed. Furthermore, the influence of the individual parameter operation on the effectiveness of the harmonisation procedure is tested. Based on the results of the tests an optimal set of harmonisation parameters which guarantees the maximal efficiency of the harmonisation algorithm is proposed.

Key words: network reliability, accuracy harmonisation

1 Introduction

The purpose of a geodetic network design is to provide it with optimal parameters. Those parameters define the essential characteristics of a network, that is: accuracy, reliability and cost. The first of these parameters is the result of the top-down requirements for the given geodetic study. The requirements in this regard possess the nature of an inequation. Economic constraints are the result of a natural pursuit of fulfilling the task at the lowest possible cost. The network's reliability is a characteristic that makes it possible to detect an occurrence of a gross error, its location, i.e. indication of the place of the distortion and its elimination.

In this regard the act of designing a geodetic network is an optimisation task that ultimately aims to find an appropriate compromise between the previously mentioned, competitive, characteristics.

According to Grafarend (1974) the following orders can be identified in the process of designing a geodetic network:

- zero order design (choosing an appropriate reference system),
- first order design (choosing an optimal network configuration),
- second order design (choosing an optimal accuracy for observations),
- third order design (choosing a way to improve the existing network).

The scope of this study is linked to network's reliability. This topic has been the subject of studies since the second half of the 60s of the previous century (Baarda, 1967, 1968). Reliability is connected with a method of network parameters estimation. Traditionally method of least squares (LS) were used to adjust geodetic observations. Many authors consider it as prone to gross errors in observations but Prószyński (1997) showed that LS method has also its robustness potential. Another approach to the network reliability proposed by Vaníček et al. (1990) called "robustness analysis" expresses the strength of net points from the strain point of view. In the recent years, the term "robustness" of the network is commonly

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used alongside the term “reliability” (Prószyński and Kwaśniak, 2002).

In this paper the authors limit the range of the study to the issues connected to the internal reliability, meaning the ability to detect distortions in a given set of observations. The reliability (redundancy) matrix R obtained from the equation (1) is commonly used as a measure of internal reliability:

$$R = I - \left(A^T PA\right)^- A^T P \tag{1}$$

where A is the factor matrix of observation equations, P is the weight matrix of observations, whereas I represents the identity matrix. R is an idempotent ($R = R^T R$) and singular ($det(R) = 0$) matrix. Trace of the R matrix corresponds to the number of redundant observations ($Tr(R) = n-u$). From the perspective of the network's reliability the main diagonal of the R matrix is of the highest significance. Its individual elements (R_{ii}) define the reaction of an observation correction to the occurrence of a gross error in the i-th observation. Zero corresponds to uncontrolled observations occurring in unambiguous constructions (without redundant observations). In turn, high value of R_{ii} corresponds to a situation, when the observation is well-controlled by the other observations. A gross error in such observation will manifest itself in the increased observation correction for the same observation. The authors, along Prószyński (1994), treat the inequation:

$$R_{ii} > 0.5 \tag{2}$$

as a sufficient criterion for the observation reliability index.

The level of internal reliability depends on the size and the value of the A matrix elements. The number of rows in the matrix A corresponds to the number of observations, whereas values of its elements depend on the type of observation and network's geometry. The weight matrix P is diagonal and its elements are functions of designed observation's a priori standard deviations.

Following the abovementioned orders of network design proposed by Grafarend (1974) it can be stated that the first (number and type of observations, network's geometry) and the second (a priori observation standard deviations) orders decide the network's (internal) reliability.

Optimisation of designed observation weights for the purpose of obtaining acceptable network's reliability indices is the prime focus in this publication. This issue was assigned to the second order design. Final result of the optimisation affects on measurement methods and choice of instruments which must be used to do real measurements.

2 Accuracy harmonisation as a method of improving the network's reliability

Obtaining an acceptable level of reliability when designing a network is, for the most part, dependent on the number of redundant observations. Increase in the number of observations has a positive effect on both the reliability and the accuracy of the network. However, this causes an increase in the measurement cost, which may not always be acceptable. This issue was elaborated on in the study by Prószyński (2014).

The mean value of the diagonal elements of the R matrix can be assumed as the global reliability index for the network. It is defined by Equation (3):

$$R_{avg} = \frac{n-u}{n} \tag{3}$$

Where:

n – number of observations,
u – number of the unknowns.

Due to the diversified structure of the network resulting from its practical application the individual observations will have different reliability indices. Designing a network in which the inequation (2) would be fulfilled for all of the observations may be, in practice, very difficult to achieve, even if R_{avg} determined using Equation (3) is significantly higher than 0.5.

In this case the accuracy harmonisation procedure suggested by Nowak (2011) may serve as a solution to this problem. The process takes advantage of the fact, that the modification of an observation a priori standard deviation also changes the reliability index. An increase in the observation accuracy (reduction of an a priori standard deviation) causes a decrease in the reliability index and vice versa – weakening of the observation causes the reliability index to increase.

By denoting h as standard deviation change index for the i^{th} observation ($m_i' = hm_i$) formula (4) can be used to calculate a new reliability index R_{ii}' (Nowak, 2011):

$$R_{ii}' = \frac{R_{ii}h^2}{1 - R_{ii} + R_{ii}h^2} \tag{4}$$

The h coefficient can be selected in such a way that any value for the R_{ii}' can be obtained:

$$h^2 = \frac{R_{ii}'\left(1 - R_{ii}\right)}{R_{ii}\left(1 - R_{ii}'\right)} \tag{5}$$

In that case the h coefficient can be described as a harmonising coefficient.

Particularly, the h (H) coefficient can be selected so that the value $R_{ii}' = R_{avg}$ is obtained:

$$R_{ii}' = R_{avg} = \frac{R_{ii}H^2}{1 - R_{ii} + R_{ii}H^2} = \frac{n-u}{n} \tag{6}$$

The value of the harmonising coefficient can be determined based on Equation (5):

$$H^2 = \frac{R_{avg}\left(1 - R_{ii}\right)}{R_{ii}\left(1 - R_{avg}\right)} \tag{7}$$

or by using Equation (6):

$$H^2 = \frac{1 - R_{ii}}{R_{ii}} \frac{n-u}{u} \tag{8}$$

Boundary condition for the success of the harmonisation process lies in the fulfilment of the inequation:

$$R_{avg} > 0.5 \tag{9}$$

Due to the structure of the network, the modification of the observation a priori standard deviations leading to obtaining $R_{ii} = R_{avg}$ will not always be possible.

Moreover, it should be noted that a change of an a priori standard deviation for one observation changes the reliability indices for the other observations. The change is described by the Equation (10):

$$R_{kk}' = R_{kk} + \frac{R_{ik}^2\left(1 - h^2\right)}{1 - R_{ii} + R_{ii}h^2} \tag{10}$$

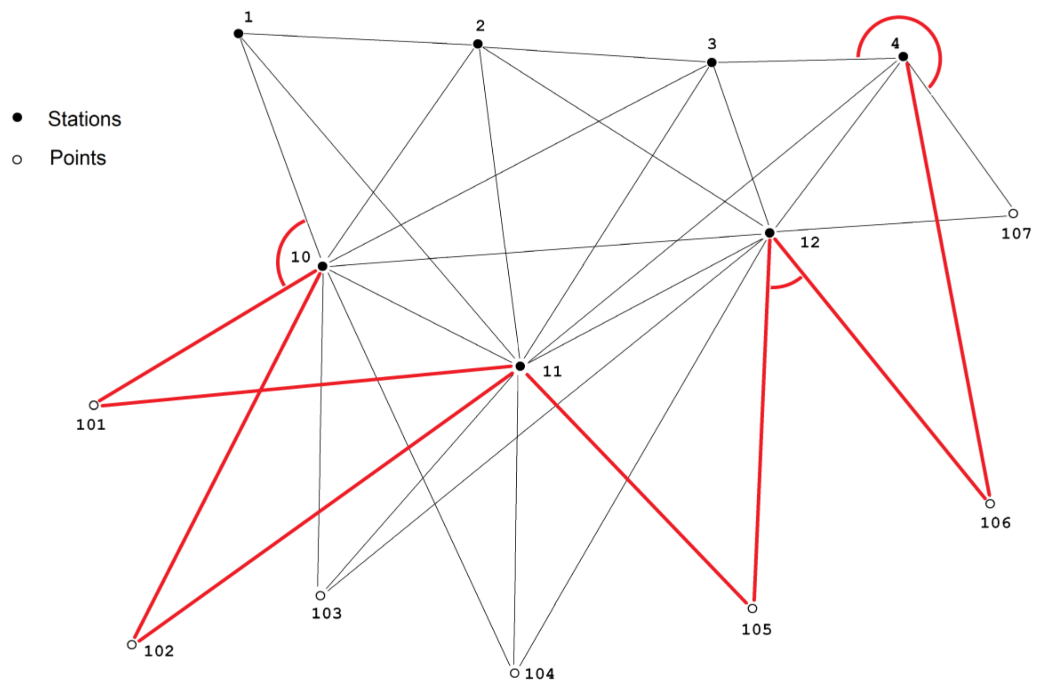

Figure 1. Test network with distribution of the observations that did not fulfil the reliability criterion

Where:

i – (active) observation index for which the a priori standard deviation has been modified,
k – (passive) observation index influenced by the effect of the modification for the i observation.

The aim of the harmonisation procedure is to create a situation in which equation (2) is fulfilled for all observations.

Due to the complex character of the equation and the fact, that the final effect of the procedure was determined using an inequation, the procedure will require an iterative approach.

According to the adopted harmonisation algorithm only the observations for which $R_{ii} < 0.5$ are modified. The modification involves replacement of an implied mean a priori standard deviation in an observation m_i with the $m_i' = H \cdot m_i$, where H is the harmonising coefficient obtained using Equation (7). As a result the modified observation obtains the reliability index equal to $R_{ii}' = R_{avg}$.

3 Practical verification of the effectiveness of the harmonisation algorithm

In order to verify the effectiveness of the accuracy harmonisation algorithm a numerical test was carried out. The test network formed an irregular linear-angular construction (Fig. 1). Angle and distance measurements from stations: 1, 2, 3, 4, 10, 11 and 12 were assumed. The assumed angle and distance measurement standard deviations were equal to ± 10 cc and ± 5 mm respectively. Free datum matrix was used during the calculations. The total number of the performed measurements was equal to 96 (48 angles and 48 distances).

With the total number of the independent unknowns equal to 25 the average reliability index for all observations amounted to $R_{avg} = 0.74$. A detailed analysis of the reliability matrix demonstrated that 11 observations (3 angles and 8 distances) did not fulfil the reliability criterion (2). Their distribution is shown in Figure 1 (marked in red).

To improve the reliability indices for the indicated observation the harmonisation procedure was applied following the algorithm described in Section 2. The attempt resulted in a failure despite performing eight iterations. The iteration process was aborted due to the lack of progress in the number of observations fulfilling criterion (2). Table 1 presents the recorded course of the iteration process.

The second line of Table 1 presents the number of observations not fulfilling the reliability criterion (2) in the subsequent iterations. As it can be observed, only the first iteration resulted in a substantial improvement (reduction from 11 to 5 observations).

3.1 Modification of the harmonisation algorithm

The analysis of the reliability indices R_{ii} led to a discovery, that the procedure's failure was caused by excessively big changes in the a priori standard deviations conditioned by the value of the harmonisation coefficient obtained through equation (7) resulting from the mean value of the R_{ii} indices for the whole network.

"Improvement" for the R_{ii} reliability index for one observation causes a "deterioration" of that index for a several other connected observations following Equation (10).

As a result, the observations for which the R_{ii} index only marginally exceeded the critical value of 0.5 were not modified in the given iteration and were often influenced by the result of modification of the neighbouring observations obtaining the R_{ii} index lower than 0.5. Figure 2 presents the operation of equation (10) for the selected observations in the test network.

Here, the distance 12–105 is the active observation. An increase of the reliability index for this observation results in a decrease of indices for the other observations. The graph presents the changes in R_i for three of the passive observations: distance 11–105 as well as angles (C-L-P) 11–105–104 and 11–12–105.

In order to avoid the destructive action of the harmonising coefficient H resulting from the mean value of the reliability in-

Table 1. Course of the accuracy harmonisation – original variant

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of R_{ii} < 0.5	5	7	4	5	8	6	5	6	7	5	8	4	7	8	9

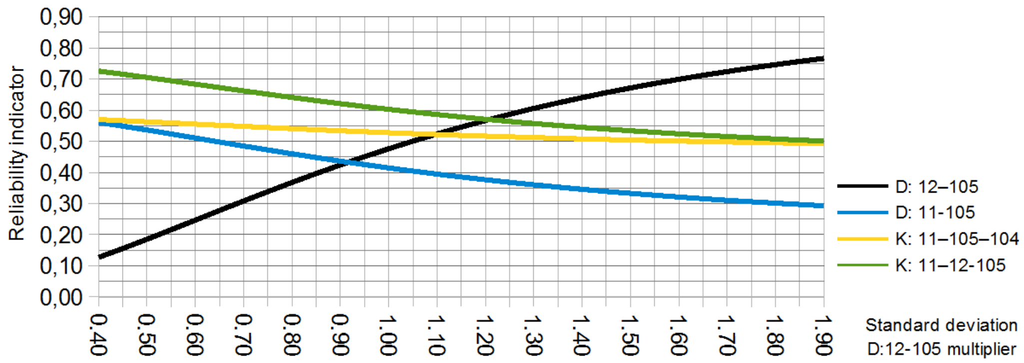

Figure 2. The influence of the 12–105 distance's reliability on the reliability of strongly connected observations

dex R_{avg} a modification was introduced to the initial algorithm. The modification concerned two aspects:

- extension of the scope of the modified observations,
- reduction of the degree of modification for each of the observations, which should alleviate the side effects of the modification in the form of changed reliability index for each of the observations.

The extension of the scope of the modified observations applies to two groups of observations:

- observations for which R_{ii} marginally exceeds 0.5,
- observations, for which R_{ii} significantly exceeds R_{avg}.

The reliability index is reduced for the second group. Given the constancy of the trace of the reliability matrix R, it should facilitate achieving the aim of (R_{ii} > 0.5) for all of the remaining observations.

For easier control over the iteration process three parameters were introduced:

R_T – the target value of the reliability index. The harmonizing coefficient h for the modified observations is determined so that the result of the modification results in $R_{ii} = R_T$. $R_T = R_{avg}$ in the initial variant,

R_{MIN} – the minimal value of the reliability index. The observations for which $R_{ii} < R_{MIN}$ are modified. R_{MIN} = 0.5 in the initial variant,

R_{MAX} – the maximal value of the reliability index. The observations for which $R_{ii} > R_{MAX}$ are modified. R_{MAX} = 1.0 in the initial variant.

In order to evaluate the effects of operating each of the individual parameters a number of tests were carried out. The aim of the first series of tests was to examine the impact of changing the target reliability index – R_T parameter. Because R_{avg} value was considered to be difficult (and in some cases impossible) to obtain, the iteration process for lower values of R_T was investigated. The remaining parameters were not modified and were equal to R_{MIN} = 0.5, R_{MAX} = 1.0. The test results are presented in Table 2.

As it can be observed, limiting the R_T results in an apparent improvement in the convergence of the iteration process. For the test network the best effect was achieved for R_T = 0.60.

The second series of tests aimed at examining the effect of changing the R_{MIN} parameter. The values of the remaining parameters were set to their default: $R_T = R_{avg}$ = 0.74, R_{MAX} = 1.0. As the result of the algorithm modification, in the subsequent iterations the R_{ii} indices were changed also for the observations for which this indices fell within range $\langle 0; R_{MIN}\rangle$. The test results are shown in Table 3.

As it can be seen, application of too low (R_{MIN} = 0.55) or too high values (R_{MIN} = 0.63) did not prove to be effective. R_{MIN} = 0.57 turned out to be the optimal value for the test network.

Examining the results of R_{MAX} modification was limited in scope and reduced to a single value of R_{MAX} = 0.80. The course of the iteration process is presented in Table 4.

The decrease of the reliability index for the observation with the highest value of the index did cause any improvement in the convergence when compared to the initial variant (Tab. 1). Lack of visible effects of such algorithm modification stems from the fact, that the harmonisation pertained the observation with the best reliability, in majority not connected to the problematic observations(R_{ii} < 0.5).

Based on the performed tests the authors have proposed a hypothesis, that a further improvement in the algorithm's effectiveness can be expected if all three parameters are tuned simultaneously. To this end the authors propose:

- decreasing, in reference to R_{avg}, the value of R_T parameter,
- limiting the scattering of the R_{ii} reliability parameter for all of the observations.

The test results show, that the best result was obtained through operating the R_T parameter. The highest effectiveness of the harmonisation algorithm was achieved for R_T = 0.60. This value, in approximation, corresponds to:

$$R_T = \frac{0.5 + R_{avg}}{2} = 0.62 \tag{11}$$

and this value was used in the final version of the algorithms.

In the case of R_{MIN} and R_{MAX} parameters, their significance for the harmonisation algorithm varies. Significantly more important role is played by R_{MIN}. This parameter serves the role of a "guardian" for the weakly controlled observations with R_{ii} index values marginally exceeding the boundary value of 0.5. The value derived from the equation:

$$R_{MIN} = \frac{0.5 + R_T}{2} = \frac{1.5 + R_{avg}}{4} = 0.56 \tag{12}$$

Table 2. Course of accuracy harmonisation – R_T modification

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
R_T = 0.70	4	5	7	8	5	7	6	6	5	4	4	4	4	3	5	3	3	4	4	6
R_T = 0.65	8	3	3	2	2	1	1	1	1	1	2	1	1	0						
R_T = 0.60	8	1	1	1	0															
R_T = 0.55	9	6	1	1	0															

Table 3. Course of accuracy harmonisation – R_{MIN} modification

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
R_{MIN} = 0.55	5	6	7	3	5	3	2	8	6	4	6	4	3	2	2	4	4	3	3	4
R_{MIN} = 0.57	6	6	5	5	3	5	2	1	1	1	0									
R_{MIN} = 0.60	9	6	5	5	5	5	5	4	5	4	5	4	3	1	0					
R_{MIN} = 0.63	5	4	3	3	4	4	3	4	2	4	5	4	4	3	3	2	2	2	2	2

was adopted as the R_{MIN}.

The R_{MAX} parameter is of lesser importance for the progression of the iteration process. Two versions were tested for the final version of the algorithm. The first one does not take this parameter into account (R_{MAX} = 1.0). In the second version, the value derived from the equation:

$$R_{MAX} = \frac{1.0 + R_{avg}}{2} = 0.87 \tag{13}$$

was adopted. The course of the iteration process for the adopted assumptions is presented in Table 5.

The hypothesis was fully supported by the results of the performed computational tests. For both parameter value combinations, the iteration process of accuracy harmonisation turned out to be characterised by fast convergence. Only two iterations were necessary to achieve success, irrespective of the R_{MAX} parameter value.

4 Some practical aspects of accuracy harmonisation

The conducted experiments proved, that even for a less than ideal network in terms of reliability, it is possible to obtain acceptable reliability indices for all of the observations through the modification of a priori standard deviations for some of them.

Tables 6 and 7 present standard deviations of the observations being the result of harmonization. The list deals with the case for R_T = 0.62, R_{MIN} = 0.56, R_{MAX} = 1.0.

Gray colour was used to highlight the observations, for which the a priori standard deviations did change. As it can be observed, out of 96 observation only for 21 the standard deviations were modified. It should be noted, that the changes being the result of harmonisation are not big. This is particularly important since the observation a priori standard deviations result mainly from the surveying technique. Exceedingly big change in the initial accuracy caused by the reliability considerations is not favourable, as it means that the distortion that will cause a noticeable reaction in the compensatory model deviates from the initial – determined technologically – measurement accuracy for the given observation. The risk of such situation increases with the number of iterations. In this context the result of harmonisation obtained after 11 or 15 iterations (Tab. 3) should be treated as questionable.

As a result of accuracy harmonisation for each of the observations their a priori standard deviation is obtained. However, the values of those deviations will generally be diverse. Because of that, the result of harmonisation cannot be treated as a guideline for the creation of a network survey project. In practice, the measurement methods and techniques for particular angles and distances are not differentiated, which leads to unified accuracy characteristics for the actual observations. The results of accuracy harmonisation can be, however, used during the phase of analysis of the network measurement results in order to detect possible gross errors. Operations on the observation a priori standard deviations are the basis of so-called robust methods of geodetic networks adjustment.

Another significant aspect of accuracy harmonisation, especially for heterogeneous networks (i.e. linear-angular), is the relation between the accuracy and reliability. In an ideal situation the reliability indices for all types of observations are similar. It is not always the case. In such situations harmonisation of the mean reliability indices for a group of observations will be justified. This can be achieved through an increase in accuracy for a group of observations characterised by high reliability indices or by decreasing the accuracy for observations with low reliability. A combination of both of these approaches is possible. The option, where the accuracy of observations is increased seems to be more desirable, because it also has a positive effect on the accuracy of points. However, actions concerning harmonisation should be evaluated in context of economic results as well as fulfilment of the boundary conditions in terms of network's accuracy. It may be that the reduction of accuracy for a group of observations does not reduce the measurement cost. Then, the reduction of the measurement accuracy, even if it leads to balanced reliability indices, will not be expedient.

5 Conclusions

As proven by the performed computational experiments, the initial version of the algorithm does not ensure the convergence of the iteration process. This results from the fact, that the change in the R_{ii} reliability index for the selected observation also changes the indices for the other observations. Furthermore, an attempt to bring the R_{ii} index of the modified observations to the mean value for the whole network meant

Table 4. Course of accuracy harmonisation – R_{MAX} modification

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
R_{MAX} = 0.80	4	5	6	4	4	4	3	4	3	6	7	6	5	6	7	6	6	6	7	6

Table 5. Course of accuracy harmonisation – R_{MIN} , R_T, R_{MAX} modification

$R_{MIN}/R_T/R_{MAX}$	1	2
0.56/0.62/1.00	5	0
0.56/0.62/0.87	5	0

Table 6. List of a priori standard deviations in the angular observations being the result of harmonization – initial value m_k = 10^{cc}

Obs. No.	Station	P. Left	P. Right	m_i [^{cc}]	R_{ii}
1	1	2	11	10.0	0.865
2	1	11	10	10.0	0.883
3	1	10	2	10.0	0.770
4	2	1	3	10.0	0.602
5	2	3	12	10.0	0.891
6	2	12	11	10.0	0.881
7	2	11	10	10.0	0.861
8	2	10	1	10.0	0.766
9	3	2	4	10.0	0.582
10	3	4	12	10.0	0.686
11	3	12	11	10.0	0.802
12	3	11	10	10.0	0.922
13	3	10	2	10.0	0.889
14	4	3	107	13.7	0.558
15	4	107	106	13.4	0.612
16	4	106	12	10.0	0.577
17	4	12	11	10.0	0.827
18	4	11	3	10.0	0.839
19	10	101	1	15.7	0.575
20	10	1	2	10.0	0.743
21	10	2	3	10.0	0.886
22	10	3	12	10.0	0.944
23	10	12	11	10.0	0.859
24	10	11	104	10.0	0.730
25	10	104	103	10.0	0.709
26	10	103	102	11.5	0.598
27	10	102	101	12.2	0.533
28	11	10	1	10.0	0.842
29	11	1	2	10.0	0.888
30	11	2	3	10.0	0.870
31	11	3	4	10.0	0.937
32	11	4	12	10.0	0.892
33	11	12	105	12.3	0.584
34	11	105	104	12.1	0.551
35	11	104	103	10.0	0.570
36	11	103	102	10.0	0.531
37	11	102	101	10.0	0.589
38	11	101	10	10.0	0.623
39	12	10	2	10.0	0.918
40	12	2	3	10.0	0.795
41	12	3	4	10.0	0.673
42	12	4	107	11.7	0.616
43	12	107	106	12.2	0.527
44	12	106	105	15.8	0.548
45	12	105	104	10.0	0.551
46	12	104	103	10.0	0.849
47	12	103	11	10.0	0.812
48	12	11	10	10.0	0.907

Table 7. List of a priori standard deviations in the linear observations being the result of harmonization – initial value m_l =5 mm

Obs. No.	P_1	P_2	m_i [mm]	R_{ii}
49	1	2	5.0	0.780
50	1	11	5.0	0.803
51	1	10	5.0	0.808
52	2	3	5.0	0.811
53	2	12	5.0	0.817
54	2	11	5.0	0.863
55	2	10	5.0	0.855
56	2	1	5.0	0.780
57	3	4	5.0	0.809
58	3	12	5.0	0.851
59	3	11	5.0	0.874
60	3	10	5.0	0.841
61	3	2	5.0	0.811
62	4	107	5.0	0.525
63	4	106	9.4	0.527
64	4	12	5.0	0.843
65	4	11	5.0	0.814
66	4	3	5.0	0.809
67	10	1	5.0	0.808
68	10	2	5.0	0.855
69	10	3	5.0	0.841
70	10	12	5.0	0.844
71	10	11	5.0	0.850
72	10	104	5.8	0.593
73	10	103	5.7	0.601
74	10	102	9.7	0.518
75	10	101	8.7	0.515
76	11	1	5.0	0.803
77	11	2	5.0	0.863
78	11	3	5.0	0.874
79	11	4	5.0	0.814
80	11	12	5.0	0.873
81	11	105	9.0	0.575
82	11	104	5.0	0.549
83	11	103	5.0	0.609
84	11	102	9.8	0.517
85	11	101	8.9	0.541
86	11	10	5.0	0.850
87	12	2	5.0	0.817
88	12	3	5.0	0.851
89	12	4	5.0	0.843
90	12	107	5.0	0.537
91	12	106	9.1	0.506
92	12	105	8.2	0.550
93	12	104	5.9	0.616
94	12	103	5.0	0.583
95	12	11	5.0	0.873
96	12	10	5.0	0.844

too extensive changes in the other observations, which in turn caused the lack of convergence of the iteration process.

Among the individually implemented modifications of the harmonisation algorithm, operating the target reliability index proved to be the most effective. “Bottom up” extension of the scope of the modified observations turned out to be slightly less effective (R_{MIN} modification). Definitely the smallest impact was achieved by operating the R_{MAX} parameter, that is extending the scope of the modified observations from the “top-down”.

The best results were obtained by combining the three modifications of the original algorithm. With appropriately selected parameters it was possible to achieve success of the harmonisation process in the second iteration.

The utilised observation accuracy harmonisation method – however effective it turned out to be – can be modified further. According to the authors, one can count on the increase in the effectiveness of the harmonisation algorithm through making the harmonisation parameters dependent on the progress of the iteration process.

However, pursuit of this idea requires further research.

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