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Productivity Development in Selected Central European Countries Measured by the Sato Production Function

Lenka Roubalová,[1] Lenka Viskotová[2]

Abstract: In this paper, we investigate the relationship between economic output, labour and capital in the Visegrád Four, Austria and Germany. The main objective is to determine the type of technological progress in these countries over time, specifically in the period 1995–2015. The Sato production functions (a special case of the linearly homogeneous production function) for all the aforementioned countries are estimated using linear and nonlinear techniques. In addition to the original Sato production function, we propose modifying it in using a time variable, which allows us to analyse the development of productivity over time. Based on the NLS estimates of this modification, we create isoquant maps and calculate the value of the marginal rate of technical substitution of labour for capital to identify the nature of technological progress typical for each country. We also compare the properties of both the OLS and NLS estimates. The results are quite specific to individual countries, but there is some room for generalization.

Key words: production function, Sato production function, isoquants, nonlinear least squares method, technological progress

JEL Classification: C51, D24, O47

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Introduction

The idea that production is a function of factor inputs comes from classical economists, mainly Malthus and Ricardo (Blaug, 1985). In fact, it was Turgot that recognized and described the law of diminishing returns in agriculture, which is generally understood to

[1]Mendel University in Brno, Faculty of Business and Economics, Department of Statistics and Operation Analysis, Zemědělská 1, 613 00 Brno, Czech Republic,
e-mail: xkravack@node.mendelu.cz
[2]Mendel University in Brno, Faculty of Business and Economics, Department of Statistics and Operation Analysis, Zemědělská 1, 613 00 Brno, Czech Republic,
e-mail: lenka.viskotova@mendelu.cz

be the basis of the production function theory. As Humphrey (1997) describes, Malthus later presented his logarithmic production function, Ricardo introduced his quadratic production function and Thünen described the exponential production function based on the statement that marginal product declines geometrically. He also discovered that output per labour unit can be defined as a function of capital per labour unit. We should also emphasize Marshall, who based a prototypal neoclassical growth model on the aggregate production function, and economists like Wicksteed, Walras and Wicksell, who used the production function to show how the total product is exhausted by the sum of factor payments distributed according to marginal productivity. The currently well-known and frequently used Cobb-Douglas production function was originally described by Wicksell (Humphrey, 1997).

In our field, standard production function specifications have been commonly used to analyse the relationship between inputs, typically labour (L), capital (K) and production (Q), at both the microeconomic and macroeconomic level. Inter alia, the Cobb-Douglas function, the Constant Elasticity of Substitution (CES) function in the form described by Kmenta, the linear function and the Leontief production function are all widely used as tools of economic analysis in various areas of the neoclassical tradition. Aggregate production functions have been used to investigate the productivity of capital and labour, the distribution of income at a national level, business cycles and also economic growth. The production function used for economic growth analysis pertains to an approach mainly related to the Keynesians. The concept of technological progress in the long run does not treat the production function as fixed, but it introduces changes in the production process. Specifically, technological progress decreases the quantity of inputs needed to achieve a given output. Economic theory classifies technological progress into three types, based on Hicks (1932). In the case of neutral technological progress, the ratio between the marginal product of labour and the marginal product of capital is not affected. Labour-saving technological progress is typical when technical advances in capital are experienced and they increase the marginal product of capital relative to the marginal product of labour. When educational or skill levels rise, marginal productivity of labour increases relative to the marginal product of capital and capital-saving technological progress can be observed (Besanko and Braeutigam, 2007).

With suitable parametrization, the aforementioned production functions satisfy the assumption of linear homogeneity that corresponds to the economic assumption of constant returns to scale. These functions are characterized by some unique properties: both the average product of labour and capital and the marginal product of labour and capital can be expressed as a function of the capital-labour ratio, $k = K/L$, only. Thus, the average products and the marginal products will remain the same as long as the capital-labour ratio remains constant. Therefore, these products are homogeneous of degree zero in the variables K and L. Euler's theorem

$$K \frac{\partial Q}{\partial K} + L \frac{\partial Q}{\partial L} = Q$$

implies that the total quantity of output Q is exhausted by the distributive shares of all the inputs if each input factor is paid the amount of its marginal product and the pure economic profit is zero.

The elasticity of substitution in production measures how easy or difficult it is to substitute one input for another, the typical case being the substitution of capital for labour. The elasticity of substitution is very often assumed to be constant. Production functions incorporating this property form the class of so-called constant elasticity of substitution (CES) functions. Among these functions, an important role is played by the CES function published by Kmenta (1967).

Many authors have analysed the class of CES functions and modified it to overcome some interrelated difficulties such as the problem of constancy of the elasticity of substitution between labour and capital and the fact that obstacles are encountered when defining elasticity in the case of more than two inputs. For more detailed information related to the various CES function modifications, see Mishra (2007).

The assumption of constant elasticity of substitution sometimes seems to be very confining, which makes the CES function a bit disadvantageous. Moreover, Henningsen and Henningsen (2012) state that convergence problems and instability are the main problems of the non-linear CES function estimation. These problems often arise due to a non-smooth objective function with large flat areas, or due to the discontinuity of the CES function where the elasticity of substitution is one. In other words, it is quite difficult to estimate the CES function parameters.

To avoid these limitations, we draw our attention to the non-CES production function defined by Sato (1964), see also Sato (1975). Sato based his work on the modification of the CES function and proved that this function can be extended to the *n*-input case or also to the case of the variable elasticity of substitution, where the elasticity depends on the output level. For the Sato function, the elasticity of substitution might vary across the isoquant as the output varies. Therefore, the Sato production function can rather be characterized as a non-CES production function. The Sato production function is linearly homogeneous, so it exhibits constant returns to scale. Makin and Strong (2013) emphasize some advantageous properties of this function; especially, they were able to prove that this function can be applied not only to microeconomic data, but also at the aggregate level. Despite the fact that there is no assumption of constant elasticity of substitution, we also find it useful that the Sato function contains less parameters to be estimated than the CES production function, it can be linearized by inverse transformation instead of the logarithmic transformation that is needed in the case of the CES function and finally, this function is not frequently used, which creates an opportunity for new insight into using production functions to be gained.

The main aim of this paper is to determine the nature of technological progress of the Visegrád Four countries, Austria and Germany. For this purpose, we use the original Sato production function and its modification employing a time variable, which allows us to analyse the development of productivity over time. In this paper, the Visegrád group is extended by two more countries, Austria and Germany. According to Heczková (2014), Slovakia, Poland, Austria and mainly Germany are the most important trading partners of the Czech Republic. The share of these countries in total turnover was about 46.5 % in 2013, which indicates that these countries affect the import and export dynamics of the Czech Republic substantially. Germany has been Hungary’s most important trading partner for years and in 2015, Austria was the third most important (European Commission, 2016). The geographical location of these countries also supports the idea of grouping these countries. As this paper analyses productivity at the country level, we consider these trading relationships to be the main reason for grouping all of these six countries.

Material and Methods

To estimate the parameters of the input factors of the Sato production function, selected methods of non-linear least squares (NLS) and ordinary least squares (OLS) are used. The nonlinear model estimation is computed in iterations using numerical methods; see Levenberg (1944) and Marquardt (1963). For more details related to the NLS method, see Greene (2012). The Sato production function is specified by

$$Y = L^2K^2/(aL^3+bK^3), \tag{1}$$

where Y represents output, K, L are capital and labour inputs, respectively, and a, b are parameters, where $a > 0$, $b > 0$ (Sato, 1975). The original Sato function assumes parameters a and b to be constant. Due to the purpose of the further analysis, which is focused on production function changes over time, we modify the original Sato production function as follows. Parameters a, b, are intended to be a function of time. This means both parameters can vary over time, which implies that the production function and the marginal product of the individual inputs vary as well. Using a linear function, parameters a and b can be expressed as

$$a_t = a_0 + a_1 t, \tag{2}$$

$$b_t = b_0 + b_1 t, \tag{3}$$

where t is time. Then, the time-augmented version of the Sato production function (1) can be expressed as

$$Y = L^2K^2/(a_0L^3+a_1tL^3 + b_0K^3 + b_1tK^3), \tag{4}$$

where a_0, a_1, b_0, b_1 are the parameters of the time-augmented version of the original Sato function. Both types of the function, the original (1) and the augmented (4), are nonlinear in parameters. These functions can be linearized and then all parameters can be estimated using the OLS procedure. Using inverse transformation, the augmented Sato production function is described by the equation

$$\frac{1}{Y} = a_0\, L/K^2+a_1\, tL/K^2+b_0\, K/L^2+b_1\, tK/L^2. \tag{5}$$

If the time variable is omitted, we obtain a simplified model representing the linearized version of the original Sato production function

$$\frac{1}{Y} = aL/K^2+bK/L^2. \tag{6}$$

The quality of the model is evaluated by the coefficient of determination R^2. Theil's U (Theil, 1966) is used to compare the models to each other from the point of view of the overall accuracy of the forecast. Theil's U has a minimum of 0, the naive models yield $U = 1$, the lower the value, the better the accuracy of the forecast. We also use information criteria AIC, BIC, HQC to compare models of the original and the time augmented function (with a different number of parameters).

Considering the map of isoquants of the estimated Sato production function in the *LK*-plane, we can find the economically efficient region of production, where the isoquants are downward-sloping and production costs are minimized. The suitable segment of each isoquant stretches between the two points where the slope of the tangent line is either zero (the tangent line is horizontal) or equal to ∞ (the tangent line is vertical).

These points satisfy the assumption

$$MPL = 0, \tag{7a}$$

$$MPK = 0, \tag{7b}$$

where *MPL* is the marginal product of labour and *MPK* is the marginal product of capital. The marginal products are defined as the partial derivatives of the production function. In the case of the Sato production function, the borders of the economically efficient region are defined by the formulas

$$K = \sqrt[3]{\frac{2a}{b}}L, \tag{8a}$$

$$K = \sqrt[3]{\frac{a}{2b}}L. \tag{8b}$$

The plots of these equations are the straight lines passing through the origin of the isoquant map. The region where backward-bending or upward-sloping isoquants are situated corresponds to the uneconomic region of production.

With respect to the objective of the technological progress classification, the marginal rate of technical substitution (*MRTS*) of capital for labour is given by the ratio of the marginal product of labour *MPL* and the marginal product of capital *MPK*. When calculating *MRTS* for the given production function, a straight line from the origin can be constructed, which represents a ray identifying the tangent point of the isoquant and its tangent. The *MRTS* can then be found for individual isoquants during the period under observation.

If the marginal rate of technical substitution is decreasing during this period, the absolute value of the isoquant slope is decreasing, and the isoquants become flatter. This indicates that *MPK* increases more rapidly than *MPL* and labour-saving technological progress is identified. Likewise, if the marginal rate of technical substitution is increasing during the period being observed, the absolute value of the isoquant slope is increasing, and the isoquants become steeper. *MPK* therefore increases at a slower pace than *MPL* and capital-saving technological progress can be identified.

In the case of neutral technological progress, lower amounts of labour and capital are needed; therefore, the isoquant corresponding to the given level of output shifts inward, but in contrast to the two previous situations, in this case, the marginal rate of technical substitution is unchanged along the ray from the origin. Neutral technological progress keeps the isoquant shape unchanged. For a more detailed microeconomic explanation, see Besanko and Braeutigam (2007).

The methods outlined above are applied to data sourced from the EU KLEMS database. Gross output at current basic prices is used for economic output *Y*, capital *K* is represented by the nominal gross fixed capital formation, both in millions of national currencies, and labour L is represented by the total number of employees in thousands. The data are in the form of annual time series for the period 1995–2015, except for Poland, where the period 2003–2015 is used due to limited data availability.

In our case, *MRTS* is determined based on the isoquant corresponding to the average output and the axis of the intersection of the economically efficient regions in all given years. There is a problem due to the empty intersection for Germany, which is why there are no values of *MRTS* for this country for 1995–1999.

All calculations were run in the MATLAB R2017b computational system. The level of significance was set to 0.05, as usual.

Results

To avoid a spurious regression, we tested for cointegration of the time series used for this analysis (ADF and KPSS tests applied on the cointegration regression residuals) and we can conclude that the time series are cointegrated. Even though heteroskedasticity occurs rarely in time series regressions, it was tested using the Breusch-Pagan test. Homoskedasticity was confirmed with the exception of three cases (the OLS model of the original function (6) for the Czech Republic and for Slovakia and the OLS model of the augmented function (5) for Hungary), where p-values were slightly lower than the significance level – this can be the consequence of the relatively short time series. Graphical analysis of the residuals did not indicate the problem of heteroskedasticity. The Lilliefors test proved the normality of residuals in all cases. Autocorrelation of the residuals was investigated for up to a maximum lag of 10; there are no significant autocorrelations for any residuals of all models. As the form of the functions is determined by economic theory, we do not focus on specification tests. The most important results of t-tests and F-tests are presented and discussed below in the text regarding individual models.

The Sato production function parameters obtained via NLS and OLS for the original production function, given by equations (1) and (6), are very similar, as well as the values of Theil's U. The parameters meet the condition of $a > 0$ and $b > 0$. The coefficients of determination explained more than 90 % of the variability of output. The results summarized in Tab. 1 and 3 show that the original Sato production function parameters estimated via NLS and OLS do not differ dramatically and the models are very similar from the point of view of its quality. Despite the fact that not all parameters are statistically significant, the p-values of the F-test are under 0.001 for all countries, all these OLS models can thus be considered statistically significant. These results prove that the Sato production function is suitable for describing the output and input relationship. In Tab. 2 and Tab. 4, the p-values of the t-test for the original Sato production function parameters estimated via NLS and OLS are presented. Tab. 2 shows that parameters a, b estimated via the NLS method are statistically significant for the original Sato function models of all six countries. Tab. 4 shows that, in the case of the OLS estimation, the parameters a and b are statistically significant for Poland, the Czech Republic, Slovakia and Hungary; in the case of Austria and Germany, the estimated parameter a is not significant.

Table 1 Original Sato production function (1) parameters estimated via NLS, coefficient of determination, Theil's *U* and information criteria.

	PL	CZ	SK	HU	AT	DE
a	1.383	17.197	0.587	108.521	2.161	1.375
b	7.107×10^{-5}	7.504×10^{-7}	4.684×10^{-4}	1.525×10^{-8}	3.347×10^{-5}	9.206×10^{-5}
R^2	88.53 %	97.02 %	90.59 %	94.11 %	96.67 %	78.04 %
U	1.025	0.661	1.356	1.526	1.023	2.135
AIC	331	604	464	704	480	597
HQC	330	605	464	704	480	598
BIC	332	606	466	706	482	600

Source: own processing

Table 2 P-values of the *t*-test for the original Sato production function (1) parameters estimated via NLS.

	PL	CZ	SK	HU	AT	DE
a	4.26×10^{-6}	1.54×10^{-15}	5.81×10^{-10}	1.22×10^{-10}	2.34×10^{-15}	3.89×10^{-9}
b	5.23×10^{-5}	6.35×10^{-8}	7.69×10^{-5}	7.23×10^{-7}	0.0290	0.0407

Source: own processing

Table 3 Original Sato production function (6) parameters estimated via OLS, coefficient of determination, Theil's *U* and information criteria.

	PL	CZ	SK	HU	AT	DE
a	1.168	16.130	0.396	49.410	2.291	1.384
b	8.772×10^{-5}	9.299×10^{-7}	1.035×10^{-3}	3.645×10^{-8}	1.324×10^{-5}	1.029×10^{-4}
R^2	92.22 %	97.32 %	94.41 %	84.05 %	97.09 %	67.95 %
U	1.029	0.759	2.375	2.205	1.118	2.105
AIC	-379	-713	-473	-719	-612	-674
HQC	-379	-712	-473	-719	-612	-674
BIC	-378	-711	-471	-717	-610	-672

Source: own processing

Table 4 P-values of the *t*-test for the original Sato production function (6) parameters estimated via OLS.

	PL	CZ	SK	HU	AT	DE
a	1.98×10^{-8}	7.70×10^{-20}	6.63×10^{-10}	6.46×10^{-12}	0.3978	0.1453
b	7.52×10^{-7}	5.42×10^{-9}	5.90×10^{-5}	1.00×10^{-6}	2.95×10^{-17}	2.92×10^{-7}

Source: own processing

Our results also prove that both estimating techniques yield similar values of estimated parameters, so from this point of view, linearization could be considered an alternative approach that is equivalent to nonlinear estimation. The information criteria AIC, BIC, HQC are used to compare models of the original Sato production function (1) and (6) with the models of the time augmented function (4) and (5), see Tab. 5 and Tab. 7. It is also necessary to highlight that the values of the estimated parameters *a* and *b* differ significantly between countries. This is caused by the fact that the variable *K* is expressed in thousands of national currencies.

Table 5 The augmented Sato production function (4) parameters estimated via NLS, coefficient of determination, Theil's *U* and information criteria.

	PL	CZ	SK	HU	AT	DE
a_0	0.860	16.201	6.060×10^{-1}	91.922	2.117	1.251
a_1	0.041	-0.202	-1.445×10^{-2}	-1.817	6.861×10^{-2}	-0.047
b_0	1.507×10^{-4}	1.601×10^{-6}	1.188×10^{-3}	4.041×10^{-8}	1.239×10^{-4}	0.003×10^{-2}
b_1	-7.603×10^{-6}	-2.400×10^{-8}	-2.223×10^{-5}	-9.018×10^{-10}	4.098×10^{-6}	4.733×10^{-6}
R^2	97.56 %	99.32 %	97.12 %	97.36 %	99.21 %	97.86 %
U	0.427	0.368	0.852	1.08	0.457	0.620
AIC	316	577	443	691	453	553
HQC	315	578	444	692	454	553
BIC	318	581	447	695	458	557

Source: own processing

Table 6 P-values of the *t*-test for the augmented Sato production function (4) parameters estimated via NLS.

	PL	CZ	SK	HU	AT	DE
a_0	1.01×10^{-4}	1.08×10^{-14}	2.14×10^{-8}	2.15×10^{-7}	2.46×10^{-10}	3.38×10^{-6}
a_1	0.3852	0.0262	0.0376	0.2182	1.49×10^{-6}	0.0005
b_0	3.09×10^{-5}	1.50×10^{-9}	5.90×10^{-6}	1.33×10^{-6}	0.0088	0.0022
b_1	0.0038	0.0134	0.0317	0.0031	0.0202	0.0269

Source: own processing

Based on a comparison of the original and the augmented versions, we can state that the value of Theil's *U* is higher for the original Sato function models. This indicates that the model with the time-augmented function is better. The value of Theil's *U* is considerably higher than 1 in most cases of the original function; meanwhile, in the case of the time-augmented version, Theil's *U* is $0 < U < 1$ in most cases; in the case of Hungary, it is slightly higher than 1. The coefficient of determination proves that the nonlinear models explain a higher percentage of the variability of the dependent variable. For NLS (see Tab. 5), results were comparable with the results obtained via OLS models (see Tab. 7); in the majority of cases, the coefficients of determination and the Theil's *U* values coming from OLS are very close to the ones obtained from the nonlinear models.

The only exception is Slovakia, where NLS produces a visibly higher coefficient of determination and lower Theil's U than OLS. Further, the values of the estimated augmented Sato function parameters differ considerably from the parameters obtained via NLS (we observe very similar values of estimated parameters only in the case of Poland). The p-values of the *F*-test (p-value=8.161×10^{-5} for Poland, p-value=4.182×10^{-8} for the Czech Republic, p-value=2.430×10^{-8} for Slovakia, p-value=5.660×10^{-6} for Hungary, p-value=3.870×10^{-17} for Austria, p-value=3.102×10^{-15} for Germany) show that all these OLS models are statistically significant.

Table 7 The augmented Sato production function (5) parameters estimated via OLS, coefficient of determination, Theil's *U* and information criteria.

	PL	CZ	SK	HU	AT	DE
a_0	0.809	14.314	2.871×10^{-1}	31.415	1.559	1.06
a_1	0.032	0.0287	1.741×10^{-2}	11.127	-5.139×10^{-2}	-0.043
b_0	1.643×10^{-4}	2.016×10^{-6}	2.824×10^{-3}	3.576×10^{-8}	2.677×10^{-4}	3.468×10^{-4}
b_1	-8.241×10^{-6}	-6.188×10^{-8}	-1.421×10^{-4}	-2.581×10^{-9}	-2.167×10^{-4}	2.023×10^{-6}
R^2	98.85%	98.89 %	88.93 %	98.60 %	98.98 %	98.30 %
U	0.412	0.441	1.663	1.220	0.573	0.621
AIC	-398	-727	-491	-767	-630	-732
HQC	-399	-726	-491	-766	-629	-731
BIC	-396	-723	-487	-764	-626	-728

Source: own processing

Table 8 P-values of the *t*-test for the augmented Sato production function (5) parameters estimated via OLS.

	PL	CZ	SK	HU	AT	DE
a_0	1.05×10^{-06}	2.05×10^{-16}	8.75×10^{-8}	1.09×10^{-11}	8.75×10^{-8}	5.16×10^{-6}
a_1	0.3506	0.838	0.0004	9.87×10^{-7}	0.0004	0.0009
b_0	9.04×10^{-6}	3.02×10^{-6}	0.0001	0.0018	0.0001	0.0001
b_1	0.0003	2×10^{-4}	0.2477	4.94×10^{-6}	0.2477	0.6345

Source: own processing

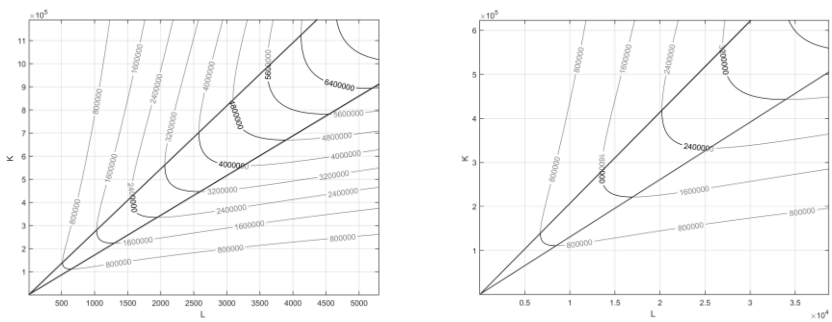
It can be seen that in the case of the augmented version (5), linearization was found to not be suitable. The OLS estimated parameters differ from the parameters obtained via NLS (except for Poland) and in almost every model, one of the parameters is not statistically significant at the 5 % significance level, see Tab. 8. In the case of NLS models of the augmented Sato function (4), the estimation of the a_1 parameter is not significant for Poland and Hungary, see Tab. 6. This may indicate that the contribution of the change of labour input dependent on time is minor in these countries. Considering the AIC, HQC and BIC information criteria, we can conclude that these are systematically lower for models of the augmented Sato function in comparison with the models of the original function – that indicates that the time augmented version is preferred. The isoquant maps based on the OLS estimated parameters produce an isoquant that is not shaped in accordance with economic theory. Thus, the only credible approach is to use the time-augmented function parameters obtained via NLS.

The isoquant maps are depicted based on the parameters coming from NLS. For illustration purposes, we provide pairs of graphs for three years of the given period. The first year of our observed period (1995) is depicted in Fig. 1, the year at the midpoint of the studied period (2005) in Fig. 2 and the last year of the period (2015) in Fig. 3, to briefly show the production development. The black parts of the isoquants represent the economically efficient region; the grey parts show the combinations of inputs that are not economically efficient. These figures also show how the economically efficient region gets wider with the increasing amount of labour and capital inputs for individual countries.

For each year, we can see two isoquant maps related to two countries, the Czech Republic on the left-hand side and Germany on the right-hand side. The same type of development as in the Czech Republic can be observed in Poland, Hungary and Austria. On the contrary, the nature of the development of production detected in Germany is the same as that observed in Slovakia. Therefore, only the figures for the Czech Republic and Germany, representing each group of countries, are plotted.

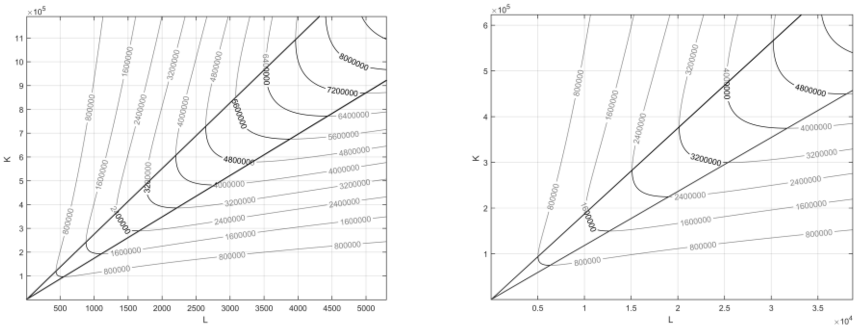
Noticeably, if Fig. 1 and Fig. 3 are compared, a considerable shift of the isoquants is observed during the period. The two rays starting in the origin define the economically effective region (the region where the isoquants are downward-sloping). Focusing on any isoquant representing a certain level of output in the first year, we can find that the isoquant shifted inward in the following years, highlighting the presence of technological progress in all the countries.

Figure 1 Isoquant maps based on the parameters estimated via NLS for the first year of the period, i.e. 1995; the Czech Republic on the left-hand side, Germany on the right-hand side.



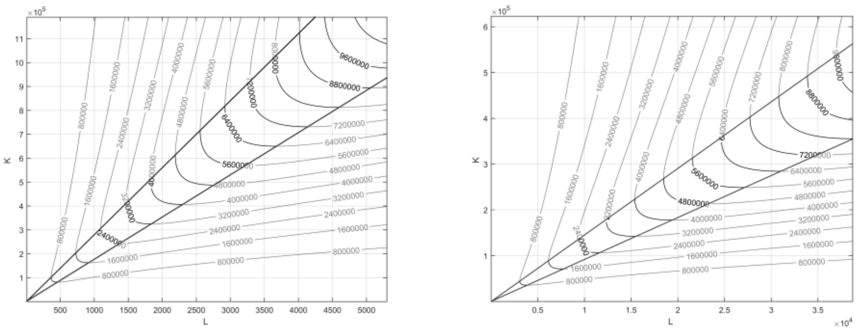
Source: own processing

Figure 2 Isoquant maps based on the parameters estimated via NLS for the year 2005; the Czech Republic on the left-hand side, Germany on the right-hand side.



Source: own processing

Figure 3 Isoquant maps based on the parameters estimated via NLS for the final year 2015; the Czech Republic on the left-hand side, Germany on the right-hand side.

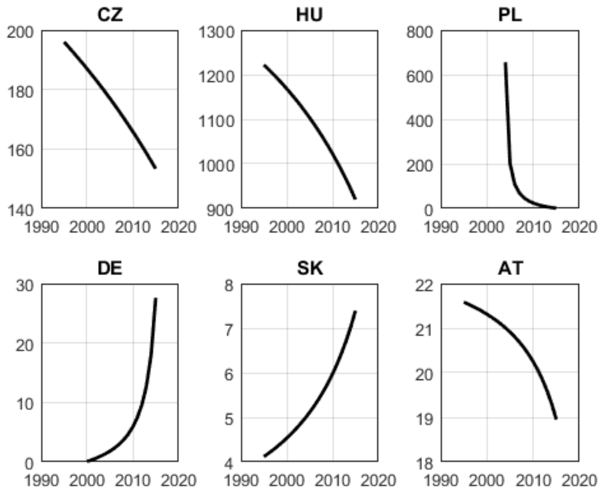


Source: own processing

The values for *K* and *L* are based on the real range of data for each country. In other words, the figures take into account the maximal values of *K* and *L* input for each country. *K* is measured in thousands of national currencies; *L* is approximated by the total number of employees. Note that the scale of *L* for the Czech Republic and Germany is not the same. The figures also show that the isoquant slope changes over time, which means that this is not a case of neutral technological progress either in the Czech Republic or in Germany. Looking at the figures not just from the point of view of time but also from the point of view of the specific countries, we find that the development of production in the group of countries represented by the Czech Republic is not the same as the development of production observed in the second group of countries, represented by Germany, in the same period.

These results imply that the nature of technological progress typical for each group of countries is not the same. Therefore, the *MRTS* time series for each country are always calculated with respect to the same point on each isoquant. The *MRTS* trend in each country is described in Fig. 4. It can be viewed as the key factor for identifying the nature of the technological progress in each of the countries.

Figure 4 Time series of *MRTS* calculated for each country during the period under observation (MRTS on the vertical axis, time *t* on the horizontal axis).



Source: own processing

Our analysis has identified a decreasing trend of *MRTS* in the case of the Czech Republic, Hungary, Austria and Poland. This means that the marginal product of capital is increasing at a faster rate than the marginal product of labour. Such development is characteristic of labour-saving technological progress. In the case of the Czech Republic and Hungary, *MRTS* decreases almost proportionally during the entire period, but in Austria, the decrease is more rapid in the second half of the period. In contrast, in the case of Poland, a rapid decline in *MRTS* in 2003–2009 slowly changes to a moderate rate of decline.

An increasing trend in *MRTS* is detected for Germany and Slovakia. This means the isoquants are shifted inward and the slope becomes steeper; in other words, the marginal product of labour increases more rapidly than the marginal product of capital and capital-saving technological progress is identified. Whereas the trend is similar, the dynamic of growth differs. Compared to Slovakia, in the case of Germany, *MRTS* increases significantly in 2010–2015.

Discussion

The results reveal that the time-augmented version of the Sato production function can be applied to the Visegrád Four countries, Austria and Germany. Generally speaking, based on the results, the countries observed can be divided into two subgroups according to their kind of technological progress. Labour-saving technological progress is found for the Czech Republic, Hungary, Poland and Austria. This means that progress is influenced by technical advances in capital equipment.

These results are in accordance with the research of Witajewskij-Baltvilks (2016), who analysed the convergence of the Polish, Hungarian and Czech economies in the period of 1995–2013. The analysis is based on the statement that countries with relatively low levels of capital per efficiency unit should experience an inflow of capital until the capital per efficiency unit matches the level in more developed countries. This should be the case for the Czech Republic, Hungary and Poland. The results prove that the capital level has converged with the higher capital level of Germany in all three countries. Witajewkij-Baltvilks (2016) also found that the impact of workers' high skills is minor, and that productivity improvement is skill-neutral. The analysis shows that improvement comes through capital (Witajewskij-Baltvilks, 2016).

The *MRTS* trend can also be explained from the point of view of the labour market. At the beginning of the 2000s, Hungary experienced structural problems. According to an EEAG (2012) analysis, in the post-transformation period, the participation of the workforce was significantly lower than in the other Visegrád countries. This situation was caused by privatisation, which influenced labour demand. Pension and benefit policies affected labour supply. All these factors led to increased competition and demand for skilled workers - EEAG (2012). Generally speaking, high-skilled workers are required in capital-intensive production, which indirectly indicates that technological progress is driven by advance in capital equipment. And if the distortions in the labour market are high, it is typical to substitute capital for labour. Blanchard (1997) proved this statement. According to the OECD (2007), further training of workers is limited in Austria, which may be a reason for why capital improvement is observed in our results. The similar shape of the *MRTS* trend curves may be related to the unemployment rate. The Hungarian unemployment rate was around 7.5 % on average and below 9 % in the Czech Republic. In Austria, the unemployment rate was between 4 % and 6 % during the entire period, but in Poland, it increased at the end of 1990s, dramatically declined in 2005 and increased again in 2008. This can be projected into the convex shape of the curve for Poland.

In Slovakia and Germany, we prove capital-saving technological progress. For the Slovak Republic, the explanation is probably connected to its labour market situation. The unemployment rate declined from 19 % to a level of below 10 % in the years between 2000 and 2008, but then increased to above 13 % in 2014. Biea (2015) mentions significant growth in labour productivity in the pre-crisis period and describes labour force participation as low, mainly due to the lower participation of people under 25, of women, and partly of people over the age of 55. This continues to contribute to the higher level of unemployment. The author concludes that reducing unemployment and increasing labour force participation is the way to compensate for the effects of population aging (Biea, 2015). Such evidence corresponds with our results with reference to improving knowledge and workforce skills. High-skilled workers contribute to higher labour force participation and subsequently to a lower unemployment rate.

Another explanation may be related to improvement in the educational level and workforce skills of the Roma population. According to the European Parliament (2016), estimated data from 2005 report 98,000 Roma inhabitants, but data from 2014 report about 402,840 Roma inhabitants in Slovakia. Their total unemployment rate is about 70 %, which significantly affects the total long-term unemployment rate. It is also mentioned that the high unemployment rate of Roma is caused by low attainment of education and their low qualifications. The OECD (2012) highlights the importance of a national policy targeted at integrating excluded Roma workers into the labour market, which is related to their need to improve their skills and educational level, but Slovak public expenditure on education remains low. However, there is evidence that during the period of 2000–2008, expenditure was increased by almost 40 %, see also Kahanec and Sedláková (2016).

Kureková (2015) states that the participation of young Roma in education has risen substantially since 2004, which reflects their greater participation in upper secondary and tertiary education. Based on an FRA survey (2014), about 5 % of Roma children do not attend school. Despite the fact that the unemployment gap between Roma and non-Roma remains large, these publications provide evidence of a slight improvement relating to the education of this group during the analysed period. Combined with a Biea (2015) statement that Slovak capital investment and capital stock was shrinking during the period 2000–2008, these findings also support our results.

The German unemployment rate declined from 10 % in 1995 to 4.5 % in 2015. From the point of view of capital quality, Germany is considered to be a benchmark for catching up countries. Looking at labour force quality, we can find several studies showing that German policy has been focused on workforce training and education for decades. However, as the OECD (2012a) states, there have still been opportunities to improve the skills and qualifications of the labour force.

This is mainly influenced by the increasing number of immigrant workers in Germany. According to Speckesser (2013), who analysed this problem between the years 2006 and 2011, the overall immigrant participation in workforce training programmes is very low, despite the fact that there are no formal obstacles for their participation except for the fact that occupational qualifications obtained outside the EU remain unrecognized.

Speckesser (2013) mentions the Active Labour Market Policy change in 2004. This reform removed most of the present barriers and enforced the unification of the job-search regime and the assistance system that have benefited immigrants in this field. Of particular importance and help were language training and specialized programmes for workers focused on providing knowledge specific to the German environment. There is also evidence of a significant improvement in human capital in the second half of the observed period. The OECD (2012) describes reforms and improvements made in the area of secondary and tertiary education. Labour market policy is focused on human resources quality and skills in part due to migration and the effort to find ways to integrate refugees and to attract high-skilled workers from abroad.

Although our results for Slovakia and Germany have one main explanation in common, the economic background and the source of causes leading to these results is rather individual. While in Germany the capital equipment and quality are high, further technological progress is being achieved through labour force quality improvements. Mainly in the second half of the period under analysis, technological progress was also influenced by low-skilled immigrants. In Slovakia, during the analysed period, there is evidence of low capital productivity and limited capital stock (Biea, 2015), and also serious problems with a high unemployment rate connected in part to the large gap in unemployment rate between Roma and non-Roma workers. The developments in this area both in Germany and Slovakia show reasons to substantially improve labour force qualification.

In spite of the fact that the Sato version of the production function is not widely used by authors dealing with the issue of productivity, there is an example in the research of Makin and Strong (2013), who have also used the Sato production function. They also use the time variable as an input, but they differ in rather focusing on the elasticity of substitution and factor productivity development and its change as a consequence of labour, product and capital market reforms. Their results confirm that the Sato production function is suitable for analysis at the aggregate level. Based on this analysis proving that the Sato function is suitable for the output and input relationship investigation at the aggregate level, the objective of further research could be to use this specification for economic output and production gap estimation. There are several papers using the Cobb-Douglas production function for this purpose.

Conclusion

In conclusion, this paper shows how the combination of labour and capital inputs has changed during the period of 1995–2015 in the Visegrád Four, Germany and Austria. The results show that the Sato production function is applicable for analysis at the aggregate level for the conditions of selected Central European countries. The OLS and NLS procedures provide very similar models and parameters compared to the original version of the Sato production function. However, the relationship between economic output, labour and capital is not fixed during the period under observation. Therefore, the time-augmented Sato production function is proposed in order to capture the development of the relationship in each year of the period. In the case of the time-augmented Sato function, the quality of NLS models is higher in respect to the original version of the Sato function. This means that the time variable is an important input. Additionally, it is the only way to estimate macroeconomic production functions for selected periods continuously year on year. Such a model is more appropriate because it provides information about technological progress. However, linearization was not successful in the case of the augmented Sato function.

The development of the isoquant tangent line slope detects labour-saving technological progress in the Czech Republic, Hungary, Poland and Austria and capital-saving technological progress in Slovakia and Germany. In the Czech Republic, Hungary, Poland and Austria, where the results show that technological progress was achieved through capital equipment improvement, national policies should concentrate on this way of improvement and find ways and resources to support capital and technological enhancement to contribute to faster productivity growth. Similarly, in Slovakia and Germany, where the results show that technological progress is achieved through the increasing of skills and qualifications of the workforce, national policy should primarily focus on activities leading to the education of the workforce. It should also be supported by an adequate social benefits policy to avoid the lack of motivation to work. These recommendations stemming from the results are very general. To provide sufficient recommendations for national policies, it would be necessary to monitor the situation and repeat this kind of analysis periodically to verify whether the way technological progress occurs changes or not. More complex analysis could help to reveal whether there is still room for improvement in capital or labour, or whether it is exhausted and policies should rather concentrate on another input, labour in the case of the first group of countries and capital in the case of Slovakia and Germany. Additionally, analysis at the sector level would also be necessary to make policy targeting more efficient.

The results we found can be verified by other authors, but there is also space for further research. The implementation of the time-augmented production function could be extended and applied for widely used functions, e. g. the Cobb-Douglas and the CES production function. This methodology could be used for all European countries to get an overview of the type of technological progress found in each of the countries of the EU. It can be a base for further research focusing on the sector level using selected countries - this could provide valuable information for economic entities operating within those sectors. We can presume that productivity growth of tradable sectors whose outputs are traded internationally will be faster than in the case of non-tradable sectors. Further research focused on productivity development at the sectoral level for both tradable and non-tradable sectors may help prove this hypothesis and also provide the opportunity to gain deeper insight into the Balassa–Samuelson effect.

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