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STATISTICAL DETERMINATION OF IMPACT OF PROPERTY ATTRIBUTES FOR WEAK MEASUREMENT SCALES

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Abstract

Many of the property attributes are measured on weak scales (nominal and ordinal scale). For example, land allocation in the development plan is measured on a nominal scale and such categories as proximity, equipment, access to means of communication, location, and soil and water conditions, are measured on an ordinal scale. The use of statistical measures appropriate for interval or quotient scales is wrong in such cases. Therefore, the article presents statistical measures that allow specifying the impact of the attributes on real estate prices, which can be used for the weaker scales, mainly for the ordinal scale. In the empirical illustration the proposed measures will be calculated by using the actual database of transaction prices.

Key words: *impact of the real estate attributes, Spearman coefficient, Kendall's tau-b coefficient*.

JEL Classification: *C10, R14, R30*.

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1. Background

One of the problems frequently encountered during real estate valuation is how to determine the weight fractions (or simply weights) when explaining the volatility of property prices. Generally, the interval procedure is used where properties are similar – they differ in one attribute (the one for which the weight is computed). Then the weight is an averaged fraction of the transaction price difference in a total interval of the price volatility. When there are several similar properties that differ in one attribute, the weight is averaged. This approach poses some risks. First of all, similar properties differing in only one attribute may not always be found in a set of transaction data. When this is the case, the weight for this attribute cannot be computed. Moreover, this procedure is vulnerable to random factors. When calculating weights, we can come across prices that vary significantly from the typical ones. It may also happen that a property with better valued attributes is offered at a lower unit price, which leads to negative values. The above problems can be avoided when we use statistical procedures.

Apart from the weights computing method used by real estate valuers, the paper presents the approach where weights are computed by means of some statistical measures, mainly by Spearman's rank correlation coefficient and Kendall's correlation coefficient τ_b. The measures are compared in terms of their statistical qualities as well as of their applicability in the property valuation process.

2. Literature Review

There is relatively rich of reference literature about the application of statistical methods to determine the impact of property attributes on their prices, also in the context of general considerations concerning the real estate valuation, e.g. BARAŃSKA 2004; PARZYCH, CZAJA 2015; MCALLISTER 2007. The impact of property attributes is often determined by means of multiple regression models (BENJAMIN *et al.* 2004; ISAKSON 1998). However, this approach is criticized (HOZER *et al.* 1999; HOZER *et al.* 2002). Econometric models can be used to determine the importance of property attributes only when the model is properly specified and the independent variables are introduced considering the measurement scales in a proper manner. Many failed attempts to specify such models indicate that it is not always possible.

The property attributes are usually measured on the ordinal or nominal scale. The characteristics of scales and operations that they can be subject to can be found in, (e.g. WALESIAK 2016).

We have a nominal scale when the function representing the variants of a given attribute is a bijection. This means that every variable value measured on the nominal scale corresponds to just one attribute variant, and vice versa – every attribute variant corresponds to just one value of the variable. In the case of variables measured on the nominal scale we can only count equality (inequality) associations. The remaining arithmetical operations are not admissible. In order to measure the associations among variables expressed in the nominal scale we can use e.g. Goodman-Kruskal τ or the so called information dependency coefficients (KORONACKI, MIELNICZUK 2001; DOMAŃSKI 1990; OSTASIEWICZ *et al.* 1999).

We have an ordinal scale when the function representing the attribute variants is a monotonically increasing function. This means that the more beneficial attribute category always corresponds to a higher value of the variable. The admissible arithmetical operations are: counting the equality (inequality) and majority (minority) associations.

The property price is measured on the interval scale where the variable level is represented by a (positive) linear transformation. Here the above mentioned operations are admissible along with addition and subtraction.

3. Research Methods

The study examines the associations between the unit property price and the attributes. All the variables will be expressed on the ordinal scale, so, in the case of the price, the scale will be weakened (from the interval scale to the ordinal one).

The strength and the direction of the association between two variables expressed on the ordinal scale are often measured with Spearman's rank correlation coefficient:

$$\rho_S = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)} \tag{1}$$

where

n – the number of objects,

d_i – the difference between ranks of a given object.

Formula (1) can be used only when there are no ties, i.e. ranks that are identical for two (or more) objects (properties). In the case of properties the values of particular attributes are often identical, which means that the equation (1) yields incorrect results. Spearman's rank correlation coefficient can be computed as, e.g., Pearson's linear correlation coefficient determined for ranked variables. Successive properties analyzed due to a given attribute are assigned increasing ranks (successive integers). If the attribute variants are identical, the averaged rank (of successive integers) is determined for the properties to which these variants refer.

The drawbacks of Spearman's rank correlation coefficient also include the fact that it assumes equal distance between successive attribute variants. Obviously, all we know from the ordinal scale is that a given variant is better or worse than others, but we do not know 'how much' better or worse it is. Therefore, here the scale is artificially 'strengthened'.

Kendall's τ_b correlation coefficient (KENDALL 1955) is devoid of such flaws. It is a non-parametric measure of association based on the number of concordant, discordant and tied pairs.

The pair (i, j) is said to be concordant if for the observations (x_i, y_i) and (x_j, y_j) one of the following associations occur:

1. $x_i > x_j$ and $y_i > y_j$
2. $x_i < x_j$ and $y_i < y_j$

The pair is concordant when we analyze two properties due to their two attributes and the values of each attribute are bigger (or smaller) for a given property.

The pair of the observations (x_i, y_i) and (x_j, y_j) is said to be discordant if one of the following associations occur:

1. $x_i > x_j$ and $y_i < y_j$
2. $x_i < x_j$ and $y_i > y_j$

The pair of properties is discordant when the value of one attribute is bigger (smaller) and the value of another attribute is correspondingly smaller (bigger).

The pair of observations (x_i, y_i) and (x_j, y_j) is said to be tied if $x_i = x_j$ and/or $y_i = y_j$. This occurs when for two properties the value of at least one attribute does not change.

When examining the associations, we are considering all the two-element combinations whose number for n objects equals $N = \frac{1}{2}n(n-1)$.

For the variables with a large number of tied pairs Kendall's τ_b coefficient of the correlation between variables X and Y can be computed by means of the following formula:

$$\tau_b = \frac{P - Q}{\sqrt{(N - X_0)(N - Y_0)}} \qquad (2)$$

where

P – the number of concordant pairs,
Q – the number of discordant pairs,
X_0 – the number of tied pairs (due to the variable X),
Y_0 – the number of tied pairs (due to the variable Y).

The coefficient $\tau_b \in \langle -1; 1 \rangle$. It should be understood as the difference in the probability between the concordance and discordance of variable values in the analyzed objects (properties). This coefficient only measures the associations of increase, decrease and equality, therefore its use to in relation to variables counted on the ordinal scale is fully justified.

Further in the article, the weights of individual attributes will be computed basing on the correlation coefficients (Spearman's, Kendall's τ_b), being a product of the absolute value of the correlation coefficient (between the property price and a given attribute) and a sum of the absolute coefficient values of the correlation between the property price and each attribute:

$$w_j = \frac{|r_j|}{|r_1| + |r_2| + \cdots + |r_k|} \qquad (3)$$

where

w_j – the weight of the j–th attribute,
r_j – the coefficient of the correlation between the unit property price and the j-th attribute $(j = 1,2, \dots, k)$,
k – the number of attributes.

4. Empirical Results

The empirical study was based on the database of transaction prices of agricultural properties. The database consists of 22 transactions concluded in 2014 and 2015. Due to the absence of a clear price trend, their adjustment due to a lapse of time was abandoned[1].

The properties under analysis were categorized according to the following attributes:

1. General location: 1 – worse (over 12 km), 2 – average (8 km - 12 km), 3 – better (4 km - 8 km), 4 – best (up to 4 km). The general location was understood as the distance (in kilometers) from the local main institutions (i.e. from the local market).
2. Specific location and accessibility: 1 – worse, 2 – average, 3 – better. This attribute represented the location of the property in relation to the village center in a given administrative unit.
3. Size: 1 – small (to 15 ha), 2 – below average (15 ha - 30 ha), 3 – average (30 ha - 45 ha), 4 – above average (over 45 ha). Due to increased demand for large properties, it was assumed that higher prices were paid for large properties in one piece.

[1] There was no reason to reject a hypothesis that the direction parameter of the linear trend function is not significantly different from zero (statistical significance $\alpha = 0.05$). The prices were analyzed on a monthly basis. The prices in months for which there were no data about transactions were linearly interpolated. When there were several transactions in a given month, the arithmetic mean of the price was computed.

4. Disadvantages: 1 – more significant, 2 – less significant, 3 – absent. They include: the shape of the plot, overhead power lines, patches of wasteland or buffer strips etc.
5. Agricultural conditions: 1 – poor, 2 – average, 3 – good. This category includes: agricultural procedures, the state of development of the crop, weed infestation and stones, soil texture, fertilization rate, condition of drainage soil improvement systems, the quality of soil, the wheat-soil complex, etc.

The unit property prices and the attribute values are shown in Table 1. It is worth to emphasize that data used in the research could be much richer. This is due to lack of the possibility to obtain better information concerning prices and attributes. The properties are arranged in an ascending order according to their unit price, which is helpful when determining Kendall's τ_b coefficient.

Table 1

Unit property prices and attribute values

No	Price [zł/ha]	General location	Specific location	Size	Disadvantages	Agricultural conditions
1	11895	1	1	1	1	2
2	12783	3	1	2	2	1
3	15651	2	2	1	2	2
4	18463	3	2	2	3	2
5	18959	1	2	3	3	3
6	19496	2	2	1	2	2
7	19753	4	2	1	3	3
8	20042	4	2	2	3	3
9	20384	4	2	3	3	1
10	21341	4	1	2	3	3
11	21645	3	2	1	2	2
12	22032	3	2	4	2	1
13	24656	4	2	2	3	3
14	25225	4	2	2	3	3
15	25243	4	2	2	3	3
16	25358	4	2	1	3	3
17	25961	3	3	4	2	1
18	26209	3	2	3	3	3
19	26260	4	2	2	3	3
20	26262	3	2	2	3	3
21	26851	4	3	1	3	2
22	26859	4	2	2	3	3

Source: own study based on the property valuer's database.

In the first step the weights were computed by means of the interval procedure which is popular with property valuers (Table 2). The price range equaled: $\Delta C = C_{max} - C_{min} = 26859 - 11895 = 14964$. The weights were computed as: $(C_w - C_M)/\Delta C$, where C_W, C_M were the prices of properties differing with one attribute (for which the weight was computed), and C_w[2] was the property whose unit price had a more advantageous value of the attribute of interest

Computing the weights following the above procedure is controversial. In several cases the properties with a more advantageous value of the singled out attribute and identical values of the

[2] In the analyzed database, in all but one cases the attributes for which a weight had been determined differed in no more than one category. So, they did not represent the 'total' volatility of the attribute. Therefore, the mean weights were computed for the growth of the attributes by one category, and then thus determined mean weights were multiplied by the number of categories diminished by one (to obtain total volatility). The weights were also adjusted proportionally so that their sum could be 100%.

remaining attributes were cheaper. It was impossible to compute the weight for the attribute termed 'Disadvantages'. There were no properties that differed in terms of only that particular attribute. Moreover, in the case of the 'Agricultural conditions' the weight was determined basing on just one pair of properties and its value seems to be excessive. There is no doubt that the above procedure of computing weights requires further studies with the use of the expert method.

Table 2

Computing weights of property attributes wag

No	Attribute	Fractional weights (%)	Weight (%)
1	General location	$\frac{C_{11}-C_3}{\Delta C}=40.1$; $\frac{C_{11}-C_6}{\Delta C}=14.4$; $\frac{C_{18}-C_5}{\Delta C}=24.2$; $\frac{C_{11}-C_6}{\Delta C}$ =14.4; $\frac{C_8-C_{20}}{\Delta C}=-41.6$; $\frac{C_{13}-C_{20}}{\Delta C}=-10.7$; $\frac{C_{14}-C_{20}}{\Delta C}=-6.9$; $\frac{C_{15}-C_{20}}{\Delta C}$ =-6.8; $\frac{C_{19}-C_{20}}{\Delta C}=0.0$; $\frac{C_{22}-C_{20}}{\Delta C}=4$;	5.41
2	Specific location	$\frac{C_8-C_{10}}{\Delta C}$ =-8.7; $\frac{C_{13}-C_{10}}{\Delta C}=22.2$; $\frac{C_{14}-C_{10}}{\Delta C}=26$; $\frac{C_{15}-C_{10}}{\Delta C}=26.1$; $\frac{C_{19}-C_{10}}{\Delta C}$ =32.9; $\frac{C_{22}-C_{10}}{\Delta C}=36.9$; $\frac{C_{17}-C_{12}}{\Delta C}=26.3$;	13.45
3	Size	$\frac{C_8-C_7}{\Delta C}=1.9$; $\frac{C_{13}-C_7}{\Delta C}=32.8$; $\frac{C_{14}-C_7}{\Delta C}=36.6$; $\frac{C_{15}-C_7}{\Delta C}$ =36.7; $\frac{C_{19}-C_7}{\Delta C}=43.5$; $\frac{C_{22}-C_7}{\Delta C}=26.5$; $\frac{C_8-C_{16}}{\Delta C}=-35.5$; $\frac{C_{13}-C_{16}}{\Delta C}=-4.7$; $\frac{C_{14}-C_{16}}{\Delta C}=-0.9$; $\frac{C_{15}-C_{16}}{\Delta C}=-0.8$; $\frac{C_{19}-C_{16}}{\Delta C}$ =6; $\frac{C_{22}-C_{16}}{\Delta C}=10$; $\frac{C_{18}-C_{20}}{\Delta C}=-0.4$;	20.40
4	Disadvantages		0.00
5	Agricultural conditions	$\frac{C_{20}-C_4}{\Delta C}=52.1$	60.74

Source: own study.

Spearman's rank correlation coefficient was counted as Pearson's linear correlation coefficient for ranked variables. As it has been mentioned before, Formula (1) cannot be applied for tied ranks. The ranks, being successive integers, were arranged in a descending order, form the largest to the smallest variable values. The ranks, the values of Spearman's rank correlation coefficient and thus computed attribute weights are shown in Table 3.

Table 3

Ranks, values of Spearman's coefficient of rank correlation between price and individual attributes, attribute weights[3]

No	Price [zł/ha]	General location	Specific location	Size	Disadvantages	Agricultural conditions
1	22	21.5	21	19	22	15,5
2	21	15	21	10.5	18.5	20,5
3	20	19.5	11	19	18.5	15,5
4	19	15	11	10.5	8	15,5
5	18	21.5	11	4	8	6,5
6	17	19.5	11	19	18.5	15,5
7	16	6	11	19	8	6,5
8	15	6	11	10.5	8	6,5
9	14	6	11	4	8	20,5
10	13	6	21	10.5	8	6,5

[3] Ranks can be computed by means of Excel: RANK.AVG().

11	12	15	11	19	18.5	15,5
12	11	15	11	1.5	18.5	20,5
13	10	6	11	10.5	8	6,5
14	9	6	11	10.5	8	6,5
15	8	6	11	10.5	8	6,5
16	7	6	11	19	8	6,5
17	6	15	1.5	1.5	18.5	20,5
18	5	15	11	4	8	6,5
19	4	6	11	10.5	8	6,5
20	3	15	11	10.5	8	6,5
21	2	6	1.5	19	8	15,5
22	1	6	11	10.5	8	6,5
Spearman		0,518	0.551	0.205	0.466	0.364
Weights (%)		25%	26%	10%	22%	17%

Source: own study.

It is clearly seen that there are plenty of tied ranks. Basing on Spearman's coefficient we can say that between the price and the attributes there were moderate (or weak) associations. The largest weight (determined on the basis of (3)) was computed for the categories of specific (26%) and general location (25%), while the lowest weight – for the size category (10%).

In order to compute Kendall's correlation coefficient τ_b we are to determine the number of concordant, discordant and tied pairs. To this end we use the data about variable values (Table 1)[4]. Ranking of the variables is not necessary.

The method of determining concordant, discordant and tied ranks is exemplified by the correlation between the unit price and the general location (Table 4). The pairs between the price and the remaining attributes are determined in the same way.

Table 4

Number of concordant, discordant and tied pairs for unit price and general location

No	Price	General location	Concordant pairs	Discordant pairs	Tied pairs
1	11,895	1	20	0	1
2	12,783	3	11	3	6
3	15,651	2	17	1	1
4	18,463	3	11	2	5
5	18,959	1	17	0	0
6	19,496	2	16	0	0
7	19,753	4	0	5	10
8	20,042	4	0	5	9
9	20,384	4	0	5	8
10	21,341	4	0	5	7
11	21,645	3	7	0	4
12	22,032	3	7	0	3
13	24,656	4	0	3	6
14	25,225	4	0	3	5
15	25,243	4	0	3	4
16	25,358	4	0	3	3
17	25,961	3	3	0	2

[4] In order to determine the number of concordant pairs we can use the Excel function: COUNTIFS().

18	26,209	3	3	0	1
19	26,260	4	0	1	2
20	26,262	3	2	0	0
21	26,851	4	0	0	1
22	26,859	4	0	0	0
	Sum		114	39	78

Source: own study.

The properties are arranged in the ascending order by their unit price. For example, the first property is assigned with twenty concordant pairs, zero discordant pairs and one tied pair. The number of twenty concordant pairs means that there were twenty properties whose price was higher than the first property on the list (11895 zł/ha) and, simultaneously, their location was more valuable than 1. Those were all the properties excluding the first one and the property No 5 whose location value is also 1. In the case of the remaining twenty properties the price and location had higher values that the first property on the list.

The absence of discordant pairs for the property No 1 means that there were no properties whose price was higher than 11895 zł/ha and whose general location value was less than 1. It is understandable because the general location determined for the first property on the list is the lowest possible.

In Table 4 the tied pairs are determined for the general location only. No pair fell into the price category as there were no identical unit prices. One tied pair assigned do the property No 1 means that, apart that one, one more property had the general location valued at 1. It was the property No 5.

In a similar way the concordant, discordant and tied pairs are determined for the remaining properties and attributes.

Table 5

Number of concordant pairs determined for price and individual attributes

No	General location	Specific location	Size	Disadvantages	Agricultural conditions
1	20	19	15	21	12
2	11	19	5	15	17
3	17	2	14	15	12
4	11	2	5	0	12
5	17	2	2	0	0
6	16	2	12	13	11
7	0	2	12	0	0
8	0	2	4	0	0
9	0	2	2	0	11
10	0	12	3	0	0
11	7	2	9	9	8
12	7	2	0	9	9
13	0	2	2	0	0
14	0	2	2	0	0
15	0	2	2	0	0
16	0	2	5	0	0
17	3	0	0	5	5
18	3	1	0	0	0
19	0	1	0	0	0
20	2	1	0	0	0
21	0	0	1	0	1

22	0	0	0	0	0
Sum	114	79	95	87	98

Source: own study.

Table 6

Number of discordant pairs determined for price and individual attributes

No	General location	Specific location	Size	Disadvantages	Agricultural conditions
1	0	0	0	0	4
2	3	0	6	0	0
3	1	1	0	0	3
4	2	1	5	4	3
5	0	1	13	4	6
6	0	1	0	0	3
7	5	1	0	3	5
8	5	1	3	3	5
9	5	1	10	3	0
10	5	0	3	3	4
11	0	0	0	0	2
12	0	0	9	0	0
13	3	0	2	1	2
14	3	0	2	1	2
15	3	0	2	1	2
16	3	0	0	1	2
17	0	4	5	0	0
18	0	0	4	0	1
19	1	0	1	0	1
20	0	0	1	0	1
21	0	1	0	0	0
22	0	0	0	0	0
Sum	39	12	66	24	46

Source: own study.

Table 7

Number of tied pairs determined for individual attributes

No	General location	Specific location	Size	Disadvantages	Agricultural conditions
1	1	2	6	0	5
2	6	1	9	5	3
3	1	16	5	4	4
4	5	15	8	14	3
5	0	14	2	13	11
6	0	13	4	3	2
7	10	12	3	12	10
8	9	11	7	11	9
9	8	10	1	10	2
10	7	0	6	9	8
11	4	9	2	2	1

12	3	8	1	1	1
13	6	7	5	8	7
14	5	6	4	7	6
15	4	5	3	6	5
16	3	4	1	5	4
17	2	1	0	0	0
18	1	3	0	4	3
19	2	2	2	3	2
20	0	1	1	2	1
21	1	0	0	1	0
22	0	0	0	0	0
Sum	78	140	70	120	87

Source: own study.

The numbers of tied pairs are determined separately for each variable. The tied pairs indicate how many properties had the same variable value as the analyzed property. The price of each property was different, therefore there were no tied pairs in reference to the price variable.

The values of Kendall's τ_b coefficient and Spearman's coefficient, along with the attribute weights computed on their basis are shown in Table 8.

Table 8

Values of Spearman's rank correlation coefficient and Kendall's τ_b coefficient of correlation between price and attributes with attribute weights computed on their basis

Coefficient	General location	Specific location	Size	Disadvantages	Agricultural conditions
Spearman	0.518	0.551	0.205	0.466	0.364
Weights (%)	25%	26%	10%	22%	17%
Kendall	0.399	0.462	0.150	0.393	0.285
Weights (%)	24%	27%	9%	23%	17%

Source: own study.

Generally, the weights computed on the basis of Kendall's τ_b are similar to the weights based on Spearman's rank correlation coefficient. The largest weight, oscillating around 25%, was computed for the specific and general location. The weight of app. 20% was determined for the categories of disadvantages and agricultural conditions. The lowest weight (about 10%) was computed for the size category.

5. Conclusions

One of the elementary difficulties in the real estate valuation is how to compute the weights of property attributes. The approach traditionally applied by valuers poses many problems. First of all, it does not always allow the valuer to determine the attribute weight. The database of transaction prices may not contain information about properties differing in terms of just one attribute value. The determined differences in prices are subject to marked random fluctuations. This problem does not occur when statistical procedures are applied.

The Author proposes a method for estimating the property attribute weights by means of Spearman's rank correlation coefficient and Kendall's τ_b coefficient. Although the weights computed basing on these coefficients were similar, it seems recommendable to use Kendall's τ_b. What is the drawback of Spearman's coefficient is the fact that it assumes identical distances between individual variants of the attributes, which should be regarded as artificial strengthening of the scale. When we operate on the ordinal scale, we only know which variant is better and which is worse, but we do not know the volume of disparity between the variants.

6. Reference

BARAŃSKA A., 2004, *Wybór cech nieruchomości do modelowania matematycznego wartości rynkowej na przykładzie kilku baz nieruchomości gruntowych,* (Selection of Real Estate Features for Mathematical Modelling of the Market Value Exemplified by Several Databases of ground Properties) (in Polish), Geodezja, Vol. 10. No. 1. pp. 31-38.

BENJAMIN J. D., RANDALL S., GUTTERY R. S., SIRMANS C. F., 2004, *Mass Appraisal: An Introduction to Multiple Regression Analysis for Real Estate Valuation*, Journal of Real Estate Practice and Education, Vol. 7, No. 1, pp. 65-77.

DOMAŃSKI C., 1990, *Testy statystyczne*, (Statistical tests) (in Polish), PWE, Warszawa.

HOZER J. (ED.), 1999, *Ekonometryczny algorytm masowej wyceny nieruchomości gruntowych*, (Econometric algorithm of mass property valuation) (in Polish), Katedra Ekonometrii i Statystyki Uniwersytetu Szczecińskiego, Szczecin.

HOZER J., KOKOT S., KUŹMIŃSKI W., 2002, *Metody analizy statystycznej rynku w wycenie nieruchomości*, (Methods of statistical market analysis in a property valuation) (in Polish), PFSRM, Warszawa.

ISAKSON H. R., 1998, *The Review of Real Estate Appraisals Using Multiple Regression Analysis*, Journal of Real Estate Research, Vol. 15, Issue 2, pp. 177-190.

KENDALL, M.G., 1955, *Rank Correlation Methods*, New York: Hafner Publishing Co.

KORONACKI J., MIELNICZUK J., 2001, *Statystyka*, (Statistics) (in Polish), Wydawncitwa Naukowo – Techniczne, Warszawa.

MCALLISTER P., 2007, *Valuation Accuracy: A Contribution to the Debate*, Journal of Property Research, Vol. 12, Issue 3, pp. 203-216.

OSTASIEWICZ W. (ED.), 1999, *Statystyczne metody analizy danych*, (Statistical methods of data analysis) (in Polish), Wydawnictwo Akademii Ekonomicznej im. Oskara Langego we Wrocławiu, Wrocław.

PARZYCH P., CZAJA J., 2015, *Szacowanie rynkowej wartości nieruchomości*, (Estimation of real estate market value) (in Polish), Wydawnictwa AGH, Kraków.

WALESIAK M., 2016, *Uogólniona miara odległości GDM w statystycznej analizie wielowymiarowej z wykorzystaniem programu R*, (Generalized Distance Measure GDM in statistical multivariate analysis with the R program) (in Polish), Wydawnictwo Uniwersytetu Ekonomicznego we Wrocławiu, Wrocław.