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POLE PLACEMENT TECHNIQUE APPLIED IN UNMANNED AERIAL VEHICLES AUTOMATIC FLIGHT CONTROL SYSTEMS DESIGN

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ABSTRACT

Unmanned aerial vehicles are widely spread and intensively used ones both in governmental and in private applications. The standard arrangements of the commercial-off-the-shelves unmanned aerial vehicles sometimes neglect application of the automatic flight control system onboard. However, there are many initiatives to ensure autonomous flights of the unmanned aerial vehicles via pre-programmed flight paths. Moreover, automatic flight control system can ensure necessary level of the flight safety both in VFR and IFR flights. The aim of this study is to guide UAV users in set up commercial onboard autopilots available on the market. On the contrary, fitness of the autopilot to a given type of the air robot is not guaranteed, and, an extra load on users can appear in controller settings. The proposed pole placement technique is one of the proper methods eliminating difficulties, and, computer aided gain selection using MATLAB will be presented.

KEYWORDS: unmanned aerial vehicle, autopilot gain selection, pole placement technique, MATLAB script

1. Introduction

Unmanned aerial vehicles (UAV) are widely spread ones both in military and civil applications. Some UAVs famous for its robust automatic flight control systems ensuring appropriate level of the flight safety comparable to that of the manned aircraft. Regarding national regulations, there is a general rule that not necessary to apply autopilot on the board. However, if to implement, the onboard autopilot can support UAV operators in execution of the flight missions, regulating appropriate flight parameters, ensuring automation of the safe return to home, and, in case of necessity, the automated emergency landing also can be executed autonomous way. There are many sellers, trading with universal autopilots, like MP2028, MP2028g, or, Paparazzi. The universal feature of the autopilots is an advantage, i.e. they can be implemented on the board of the wide range of the different UAV types. The universality means and requires high level of skills whilst to schedule and fit it to the given UAV type. This study proposes an analytic method of gain scheduling of the autopilots, as the first steps in setting and defining PID-controllers' parameters. Thus, importance of the heuristic gain selection is reduced and replaced by the analytic one of the pole placement technique. This analytic controller design method and, computer simulation can support UAV users in gain fitting and scheduling.

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2. Preliminaries and Literature Review

Pole placement technique applied to design control law of the closed loop control systems have a long history. Many scholars, mathematicians and academicians were on the design of the closed loop control systems ensuring closed loop poles at predefined places on the complex plain, i.e. dynamic performances were ensured via closed loop poles locations. Bokor et al. (2014) deals with classical and modern control engineering. The pole placement design method is outlined in Franklin et al. (1994), in Friedland (1986), in McLean (1990), in Golten and Verwer (1991), in Ogata (1999), and, in Skelton (1988). Article of Szabolcsi (2014), deals with dynamic performances of the UAV longitudinal motion, and sets many criteria applicable during design of the autopilots. Szabolcsi (2011), gives MATLAB scripts to solve problems in modern control engineering, including gain selection for aircraft autopilots. Szabolcsi (2016), explained spatial motion dynamics model of the UAV being considered for the numerical example. The MATLAB R 2017b (2017), and MATLAB Control System Designer 10.3 (2017) computer packages have been used for design and for simulation purposes.

3. Controller Design using Pole Placement Method

The pole placement technique is based upon an idea that closed loop automatic flight control systems' pre-defined dynamic performances can be ensured by appropriate selection of the closed loop poles (Bokor et al., 2014), in other words, by selection of the state feedback gain matrix, say, K. The full state feedback gain matrix K of the closed loop control system defines closed loop control system poles.

The dynamical system being considered is supposed to be completely state controllable, the state variables being manipulated are measurable ones and supposed to be available for feedback. The same fashion, the observability of the state variables is supposed. Finally, the control input is supposed to be unconstrained.

The dynamics of the multivariable control system can be defined using state and output equations (McLean, 1990; Szabolcsi, 2016):

$$\dot{x} = \mathbf{A}x + \mathbf{B}u;\, y = \mathbf{C}x + \mathbf{D}u\,, \qquad (1)$$

where **x** is a column state vector of length *n*, **u** is the control input vector of length *r*, **A** is an $(n\times n)$ square state matrix of constant coefficients, **B** is an $(n\times r)$ input matrix of the constant coefficients that weight input variables, **y** is a column vector of the output variables, **C** is an $(m\times n)$ the output matrix of the constant coefficients that weight the state variables, and finally, **D** is an $(m\times r)$ direct feedforward matrix of the constant coefficients that weight the system outputs. For many physical systems the matrix **D** is a null matrix. Thus, the system state and output equations can be represented in the following notation:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u;\;\; y = \mathbf{C}x \qquad (2)$$

Block diagram of the closed loop system built by equation (2) can be seen in Figure no. 1.

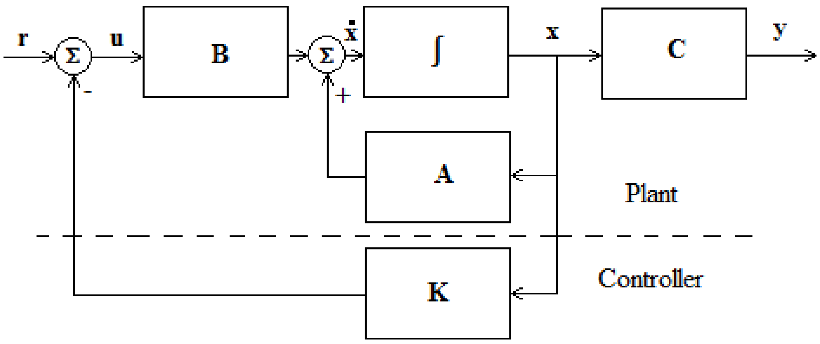

Figure no. 1: Block diagram of the closed loop control system (Ogata, 1999)

The closed loop control system is of two main parts, say, the plant to be controlled, and controller represented by the full state feedback gain matrix K.

Unmanned aerial vehicles designed by classical approaches to have conventional aerodynamic control surfaces are controlled by using one of the four control channels of elevator, ailerons, rudder, and thrust (McLean, 1990; Szabolcsi, 2016). In that sense the general equations of (1) can be rewritten as follows:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u; \; y = \mathbf{C}x \tag{3}$$

The characteristic polynomial of the open loop control system defined by equation (3) can be derived as (Ogata, 1999; Bokor et al., 2014):

$$a(s) = det[s\mathrm{I} - \mathrm{A}] = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0 \,, \tag{4}$$

where I is an (n×n) identity matrix.

Using Figure no. 1 the control law of the closed loop control system affecting closed loop poles can be derived as:

$$\mathbf{u} = r - \mathbf{K}x \,, \tag{5}$$

where r represents the reference input signal of the closed loop control system, and, the state feedback gain matrix K can be represented as follows:

$$\mathbf{K} = [k_{n-1} \quad \cdots \quad k_0] \tag{6}$$

Substituting control law of (5) into state equation of (3) yields to the closed loop control system state equation as it defined below:

$$\dot{x} = [\mathbf{A} - \mathbf{BK}]x + \mathbf{B}\mathbf{r}; y = \mathbf{C}x \tag{7}$$

The characteristic polynomial of the closed loop control system defined by equation (7) is as follows by ():

$$\alpha(s) = det[s\mathrm{I} - \mathrm{A} + \mathrm{BK}] = s^n + \alpha_{n-1}s^{n-1} + \alpha_{n-2}s^{n-2} + \cdots + \alpha_1 s + \alpha_0 \,, \tag{8}$$

where a_i coefficients of the original characteristic polynomial. If the dynamical system defined by matrices of A, B, and C is controllable, characteristic polynomial equation of the closed loop control system can be set using state feedback matrix of K.

It is well-known that all the controllable state space dynamical models can be expressed in the controllable canonical form, i.e.:

$$\dot{x} = \mathbf{A}_c x + \mathbf{B}_c u; \; y = \mathbf{C}_c x \,, \tag{9}$$

where $\mathbf{A}_c$, $\mathbf{B}_c$, and, $\mathbf{C}_c$ are matrices of the transformed system to the controllable canonical form, i.e.:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 0 & \ddots & 0 & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 1 \\ -a_{n-1} & -a_{n-2} & -a_{n-3} & -a_{n-4} & \cdots & -a_0 \end{bmatrix} \tag{10}$$

$$\mathbf{B}_c^T = [0 \quad 0 \quad \cdots \quad 0 \quad 1] \tag{11}$$

In this case, the closed loop control system state matrix can be calculated as follows:

$$\mathbf{A}_{CL} = \mathbf{A}_c - B_c\mathbf{K} =$$
$$= \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 0 & \ddots & 0 & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 1 \\ -a_{n-1}-k_{n-1} & -a_{n-2}-k_{n-2} & -a_{n-3}-k_{n-3} & -a_{n-4}-k_{n-4} & \cdots & -a_0-k_0 \end{bmatrix} \tag{11}$$

The closed loop control system characteristic polynomial will have a form of:

$$\alpha(s) = det[s\mathbf{I} - \mathbf{A}_c + \mathbf{B}_c K] = s^n + (a_{n-1} + k_{n-1})s^{n-1} + \cdots + (a_1 + k_1)s + (a_0 + k_0) \tag{12}$$

Let us denote the closed loop control system poles as $p_1, \ p_2, \ p_3, \ \cdots \ p_n$. Using closed loop poles the characteristic polynomial can be calculated as:

$$\alpha(s) = (s - p_1)(s - p_2)\ (s - p_3)\cdots(s - p_n) = s^n + \alpha_{n-1}s^{n-1} + \cdots \alpha_1 s + \alpha_0 \ , \tag{13}$$

where α_i are constant coefficients of the modified characteristic polynomial.

The relationship between coefficients of k_i, a_i, and α_i is as follows below:

$$\alpha_i = a_i + k_i; i = 0, 1, 2, \cdots, n-1 \tag{14}$$

From equation (14) elements of the state feedback gain matrix K can be calculated using following formula:

$$k_i = \alpha_i - a_i; i = 0, 1, 2, \cdots, n-1 \tag{15}$$

If the dynamical system is the controllable one, however, it is represented not in the first companion form, the dynamical system model can be expressed in the controllable canonical form using nonsingular transformation matrix of T. Let us introduce the following Bass-Gura transformation (Ogata, 1999):

$$\mathbf{x} = \mathbf{T}\hat{\mathbf{x}} \tag{16}$$

$$\frac{d\hat{\mathbf{x}}}{dt} = \mathbf{T}^{-1}\frac{d\mathbf{x}}{dt} \tag{17}$$

$$\frac{d\hat{\mathbf{x}}}{dt} = \mathbf{T}^{-1}(\mathbf{A}\mathbf{x} + \mathbf{B}u) \tag{18}$$

$$\frac{d\hat{\mathbf{x}}}{dt} = (\mathbf{T}^{-1}\mathbf{A}\mathbf{T})\hat{x} + (\mathbf{T}^{-1}\mathbf{B})u \tag{19}$$

Using Bass-Gura approach the transformation matrix T must be chosen such that matrix $\mathbf{T}^{-1}\mathbf{A}\mathbf{T}$ will be given in the first companion form, or, in the controllable canonical form. For that select following form of the transformation matrix T:

$$\mathbf{T} = \mathbf{M}\mathbf{W}\ , \tag{20}$$

where M represents the controllability matrix of the form:

$$\mathbf{M} = [\mathbf{B} \quad \mathbf{A}\mathbf{B} \quad \mathbf{A}^2\mathbf{B} \quad \cdots \quad \mathbf{A}^{n-1}\mathbf{B}] \tag{21}$$

and W is as follows:

$$\mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & \cdots & \ddots & \ddots & 0 \\ \vdots & \cdots & \ddots & \cdots & \vdots \\ a_1 & 1 & \cdots & \cdots & \vdots \\ 1 & 0 & \cdots & \cdots & 0 \end{bmatrix} \quad (22)$$

The control law of the closed loop control system, for r = 0, can be defined as:

$$u = -\hat{\mathbf{K}}\hat{\mathbf{x}} = -(\hat{\mathbf{K}}\,\mathbf{T}^{-1})\mathbf{x} = -\mathbf{K}\mathbf{x} \quad (23)$$

The control law synthesis consists of the following steps (Bokor et al., 2014):

- using dynamical system state space model defined by equation (3) check the open loop system controllabity: find controllability matrix M;
- using equation (4) find coefficients $a_i{}'s$ of the original dynamical system;
- find transformation matrix T defined by equation (20);
- find coefficients, $\alpha_i{}'s$ of the characteristic polynomial of the transformed system, using equation (13);
- using equation (15) find elements of the full static feedback gain matrix K.

Summing up main results of this chapter it is easy to agree that finding full state feedback gain matrix K is based upon mathematical operations executed over matrices. Computer packages like MATLAB supports solution of such problems.

The advantage of the closed loop system analytical design that the closed loop control system will be stable. The disadvantage of the pole placement method is that location of the closed loop poles on the complex plain is quiet difficult, and requires high level of skills in establishing relationship between poles and dynamic performances of the closed loop control system, and often the heuristic setting of poles is implemented.

4. Allocation of the Closed Loop Poles

The transfer function of the nth order dynamical system can be derived as follows:

$$W(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0} \quad (24)$$

where, for the proper systems takes place $n \geq m$.

The behavior of the dynamical system described by equation (24) can be evaluated in time domain. It there are no multiply roots of the characteristic polynomial, the impulse response function of the dynamical system is as follows:

$$w(t) = \mathcal{L}^{-1}\{W(s)\} = C_1^0 e^{p_1 t} + C_2^0 e^{p_2 t} + \dots + C_n^0 e^{p_n t} = \sum_{k=1}^{n}\{C_k^0 e^{p_k t}\}\ , \quad (25)$$

where C_i^0's coefficients are calculated regarding initial conditions, and p_i's are solutions of the characteristic polynomial. From equation it is easy to conclude that the system response is the algebraic sum of time responses derived by the roots of the characteristic polynomial.

For higher order systems, say, $n \geq 5$, using Galois-theorem, there is no closed formula to find roots of the polynomials. It means that the impulse response function of the dynamical system, mostly used to evaluate stability of the dynamical system, cannot be derived directly, i.e. approximated roots must be calculated and used latter.

In some control applications, the nth order dynamical system is represented with its second order approximation. It means that nth order s-polynomial of the denominator of equation (24) is reduced to that of the second order one.

In many control applications, like in electrical systems, the R-L-C circuits, and, in mechanical systems the spring-mass-damper system can be described by the second order ordinary differential equations.

If to find Laplace-transforms of those systems described above, one can get the model of the second order dynamical systems, with two energy storages.

4.1. Dominant Pole Approximation

This method proposes approximation of the nth order dynamical system with the most important two roots, which are mostly complex conjugates ones. The remaining roots are supposed to be located at large distances from the pair of complex conjugate roots, i.e. theirs contribution to building up system responses is neglected.

The following expression of the second order system's simplified model is used:

$$W(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0} \cong \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{1}{T^2 s^2 + 2\xi T s + 1}, \qquad (26)$$

where $T^2 = \frac{a_2}{b_0}$, and, $2\xi T = \frac{a_1}{b_0}$.

Finding roots of the characteristic polynomial of the system defined by equation (26) will give:

$$p_{1,2} = -\sigma_0 \pm \omega_d = -\frac{\xi}{T} \pm j\frac{1}{T}\sqrt{1-\xi^2}, \qquad (27)$$

where $\omega_0 = \frac{1}{T}$ is the natural frequency, $\sigma_0 = \xi\omega_0$, $\omega_d = \omega_0\sqrt{1-\xi^2}$, and, $\omega_0 = \sqrt{\sigma_0^2 + \omega_d^2}$.

The allocation of the roots defined by equation (27) on the complex plain can be seen in Figure no. 2.

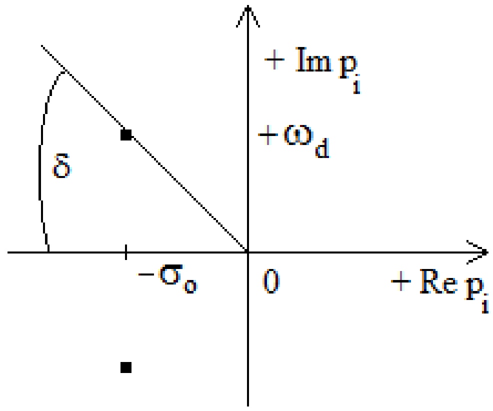

Figure no. 2: Block diagram of the closed loop control system

Using formulas defined above, the second order dynamical system transfer function can be represented as follows below:

$$W(s) = \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{1}{T^2 s^2 + 2\xi T s + 1} = \frac{\omega_0^2}{s^2 + 2\sigma_0 s + \omega_0^2} \qquad (28)$$

Dynamic performances derived using second order system model are as follows:

$$tg\delta = \frac{\omega_d}{\omega_0} = \sqrt{1-\xi^2} \tag{29}$$

damping ratio: $\xi = cos\delta$ (30)

peak time: $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_0\sqrt{1-\xi^2}}$ (31)

settling time: $t_s = \frac{3}{\sigma_0} = \frac{3}{\xi\omega_0}$ (32)

Dynamic performances introduced above are very important in solution of the controller synthesis problems. In spite of lack of the dynamic performances of the UAV automatic flight control systems, often the general set of performance criteria is implemented.

4.2. General Case of the Closed Loop Poles

Eliminating transients derived by poles of the characteristic polynomial can lead to more simple dynamical system as defined by equation (26). If such simplification results in meaningful errors in system responses this method of approximation of more complex system must be avoided, and, all roots of the system must be considered. It the closed loop system is designed using pole placement techniques the s-plane techniques is available to lean on.

5. Design of the Closed Loop Control System of the Unmanned Aerial Vehicle Using Pole Placement Technique

The short period longitudinal motion of the Boomerang-60 UAV can be derived as follows below (Szabolcsi, R.c., 2016):

$$\dot{\mathrm{x}} = \boldsymbol{A}\mathbf{x} + \boldsymbol{B}u = \begin{bmatrix}\dot{w}\\ \dot{q}\end{bmatrix} = \begin{bmatrix}-0{,}9966 & 19\\ -3{,}9794 & -12{,}991\end{bmatrix}\begin{bmatrix}w\\ q\end{bmatrix} + \begin{bmatrix}-1{,}2965\\ -18{,}7890\end{bmatrix}\delta_e \;, \tag{33}$$

where w is the vertical speed, q is the pitch rate, and $\boldsymbol{\delta_e}$ is the elevator deflection.

$$\dot{\boldsymbol{\vartheta}} = \boldsymbol{q} \tag{34}$$

The model represented by equation (33) is changed to that of able to represent pitch angle control system. For that one can define kinematic relationship between pitch rate, and pitch angle using Euler-formula as follows below:

Using equations (33) and (34) one can set up the UAV longitudinal short period rotational motion state equation in the following form:

$$\dot{\mathrm{x}} = \begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\vartheta}\end{bmatrix} = \boldsymbol{A}\mathbf{x} + \boldsymbol{B}u = \begin{bmatrix}-0{,}9966 & 19 & 0\\ -3{,}9794 & -12{,}991 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \vartheta\end{bmatrix} + \begin{bmatrix}-1{,}2965\\ -18{,}7890\\ 0\end{bmatrix}\delta_e \tag{35}$$

The poles of the dynamical system defined by equation (35) have been calculated and they are represented in Figure no. 3 (MATLAB, 2017).

Extending order of the dynamical system with the Euler-formula defined by equation (34) leads to the introduction of the pole located in the origin of the complex plain. The system has a pair of complex conjugate roots providing overshoot of 3,05 %.

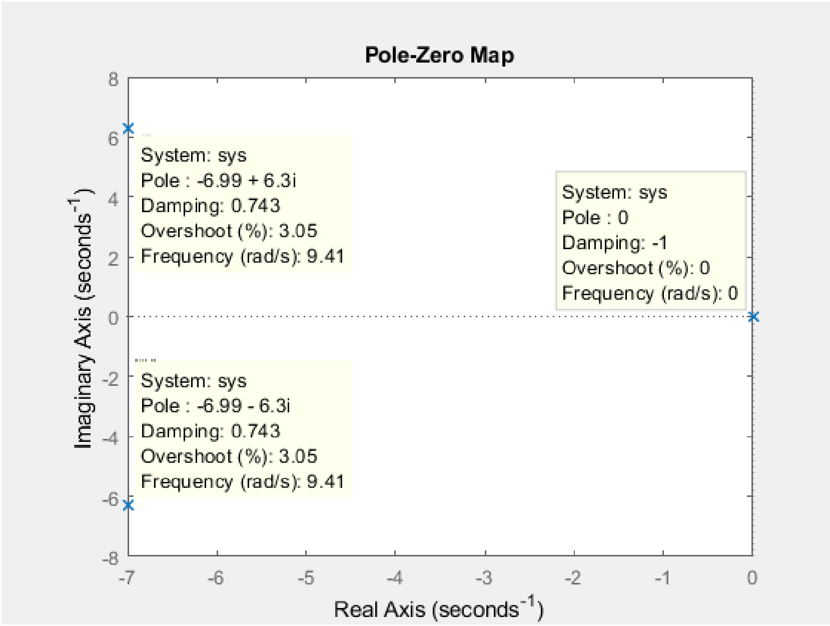

Figure no 3: Poles of the dynamical model of the Boomerang-60 UAV
(MATLAB script: author)

The open loop dynamical model of the Boomerang-60 UAV has been tested in time domain. The impulse response function of the UAV dynamics can be seen in Figure no. 4.

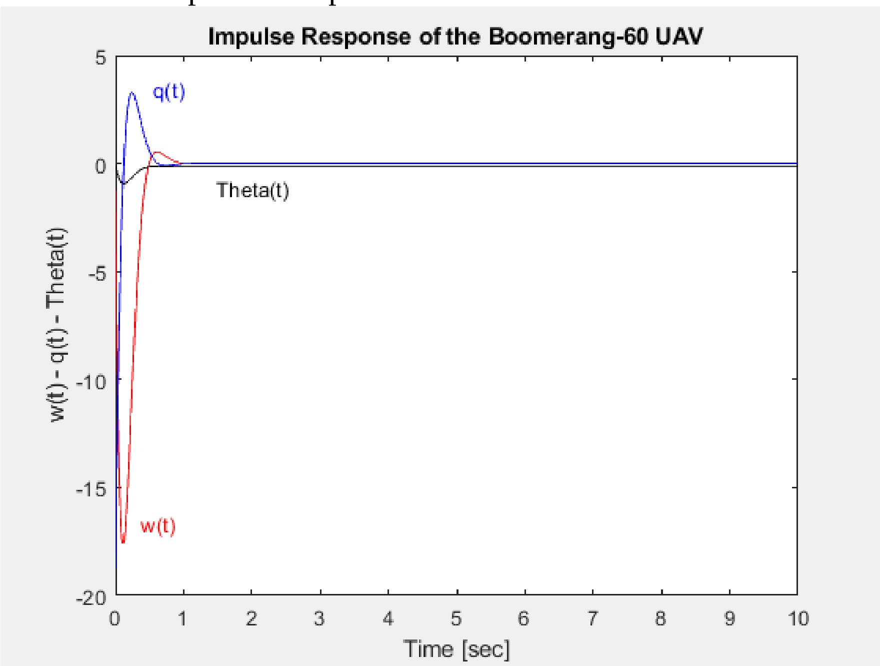

Figure no. 4: Impulse response of the UAV (MATLAB script: author)

From Figure no. 4 it is easily can be determined that the open loop system is the stable one, i.e. all transients will die out as $t \to \infty$.

The UAV time domain behavior was tested using unit step input. Results of the computer simulation can be seen in Figure no. 5.

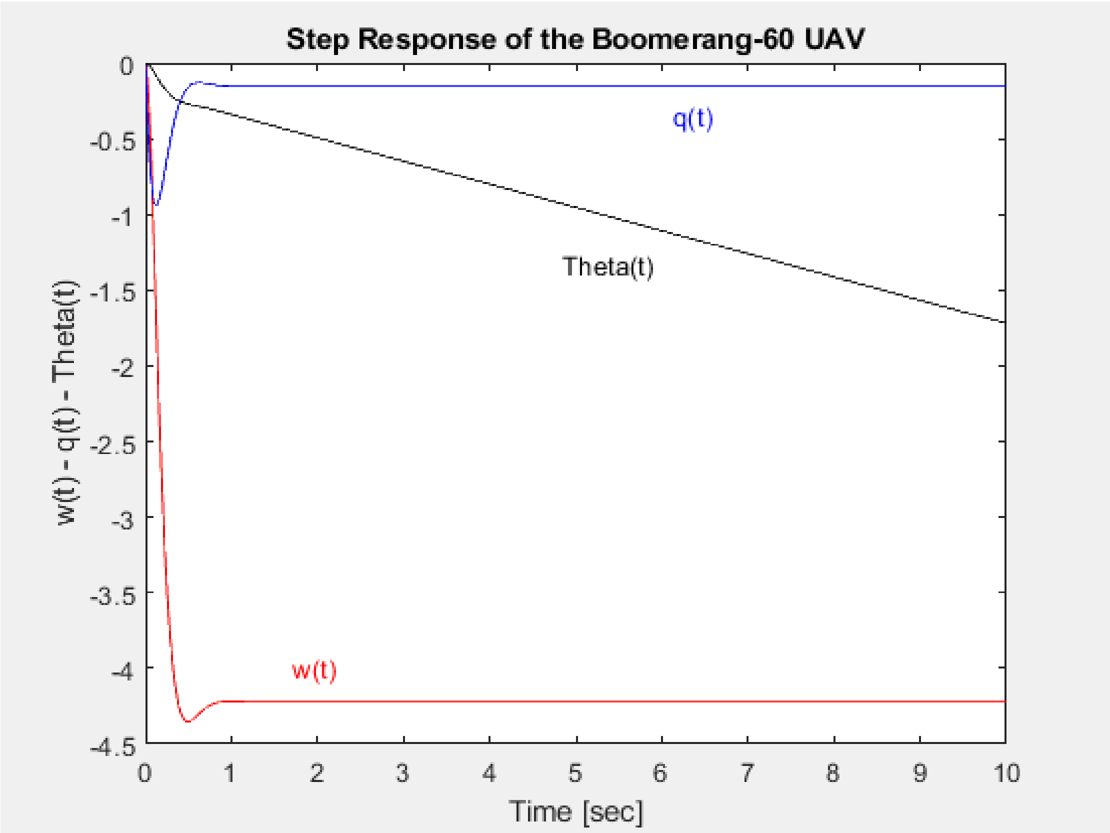

Figure no. 5: Step response of the UAV (MATLAB script: author)

From Figure no. 5 it can be deduced that two state variables, say, the vertical speed and the pitch rate, have bounded response to the bounded unit step input. The pitch angle behaves as integral of the pitch rate, defined by equation (34).

Let us find control vector, in other words, find the full state feedback gain matrix using pole placement method ensuring closed loop control system properties as follows:

damping ratio:

$$\xi = cos\delta = (0{,}6 \cdots 0{,}8) \qquad (36)$$

settling time: $t_s = \frac{3}{\sigma_0} = \frac{3}{\xi\omega_0} \leq 2\ sec$ (37)

The roots for the first trial have been chosen to be:

$$p_{1,2} = -6 \pm 6 * j \qquad (38)$$

$$p_3 = -10 \qquad (39)$$

Using MATLAB and Control System Toolbox for gain selection result in the following full state feedback gain matrix (MATLAB Control System Designer, 2017):

$$\boldsymbol{K} = [2.4123 \quad -0.5929 \quad -53.0745] \qquad (40)$$

The closed loop system time domain behavior was tested to evaluate dynamic performances during stabilizing unit step function of $\vartheta_{ref}(t) = 1(t)\ deg$. Results of the computer simulation can be seen in Figure no. 6.

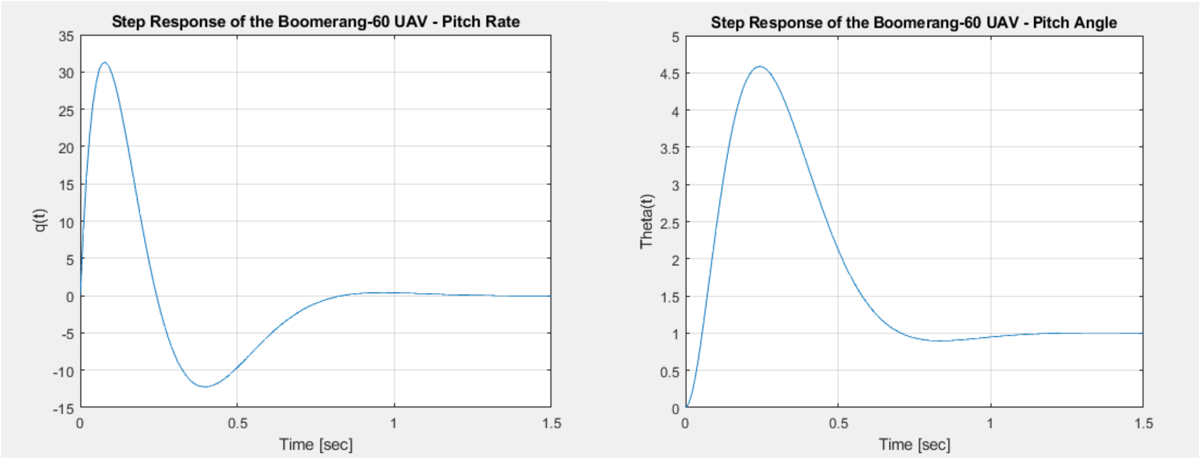

Figure no 6: Step responses of the UAV Closed loop control system (MATLAB script: author)

The closed loop control system poles derived by the static gain matrix K defined by equation (40) can be seen in Figure no. 7.

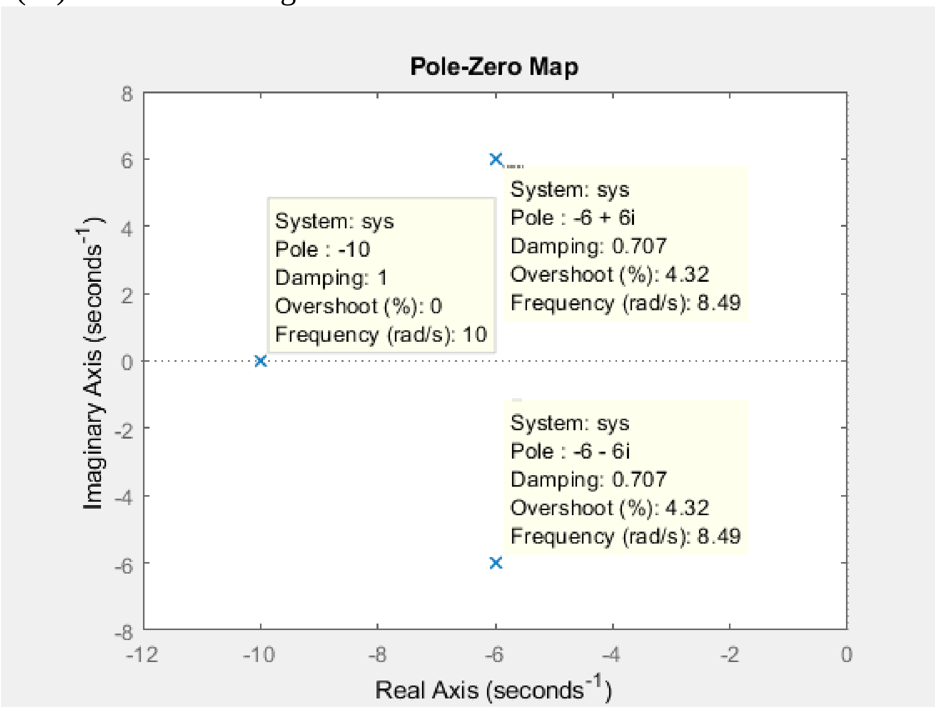

Figure no. 7: UAV flight control system closed loop poles (MATLAB script: author)

From Figure no. 7 it is easy determine that control strategy expressed in dynamic performances of the closed loop control system is met. If dynamic performances unconsidered still are important and, there is a need to calculate with, the roots must be replaced on the complex plain such that the set of performance criteria will be met. So as to minimize energy needed for the control process, it is proposed to select closed loop poles close to those of the open loop uncontrolled dynamical system's poles.

6. Conclusions

The pole placement technique applied to select static gains of the full state feedback gain matrix is able to reduce workload of the designers setting closed loop system parameters. This analytic approach can minimize needs in heuristic setting of the closed loop poles, which is an old-fashion method, however, still engineering experiences, skills and knowledges can support that activity.

The computer aided gain selection of the static feedback gain matrix will ensure fast and attractive solutions, and, if the heuristic setting of the closed loop poles is still applied, it will accelerate finding proper solution to a given UAV automatic flight control system control law design.

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